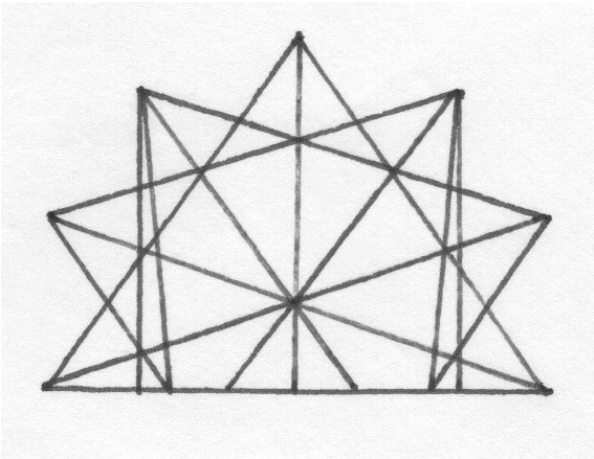
The Meantone Series of Cyclical Temperaments

Diagrams



Siemen Terpstra

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Figure 1: The Tuning Nucleus — The Six Tuning Directions

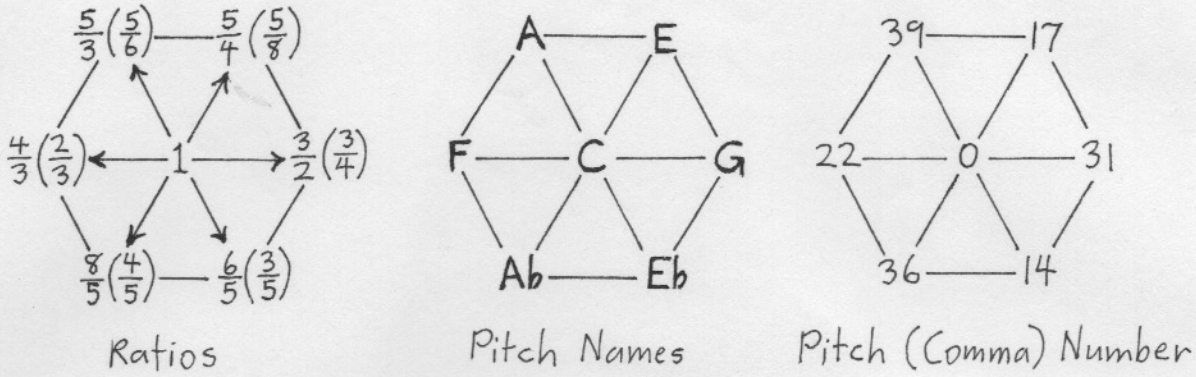
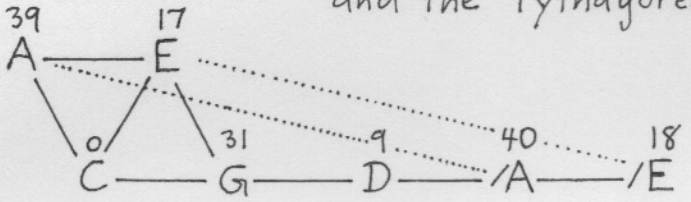


Figure 2: Generation of the Pythagorean Major Sixth and the Pythagorean Major Third



/A − A = Syntonic Comma

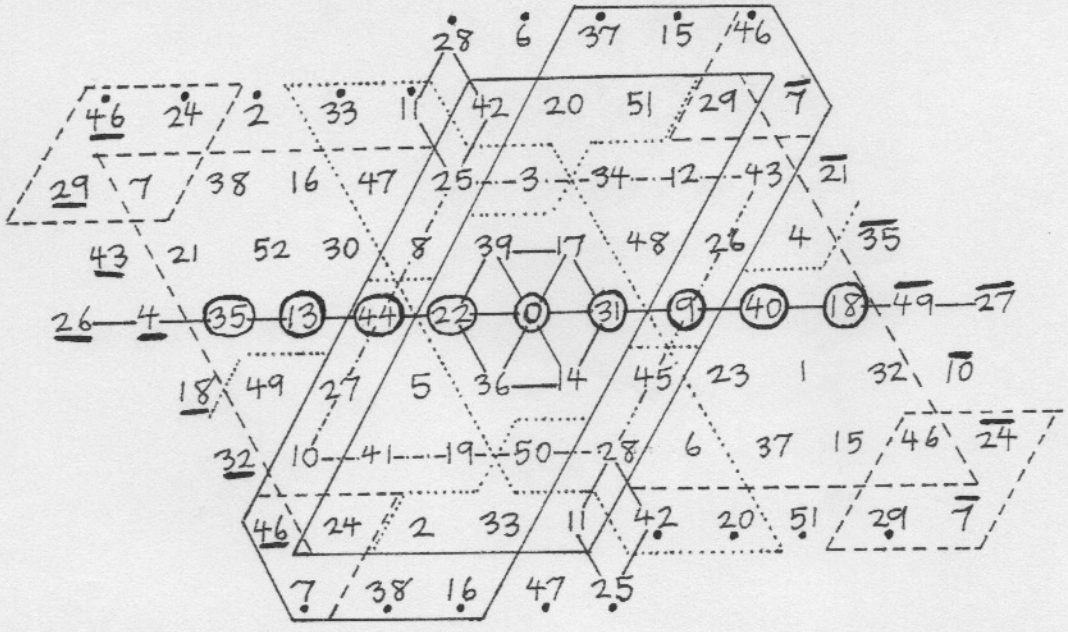
$\frac{27}{16} \div \frac{5}{3} = \frac{81}{80}$

and

/E − E = Syntonic Comma

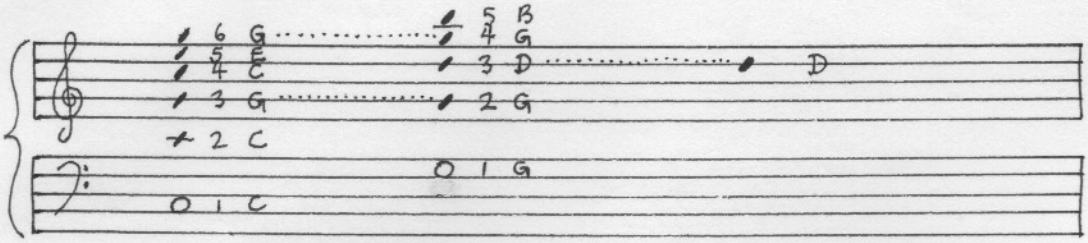
$\frac{81}{64} \div \frac{5}{4} = \frac{81}{80}$

Figure 3: Symmetrical Tuning Expansion along the Horizontal (3:2) Axis, ie. by pure musical Fifths and Fourths:



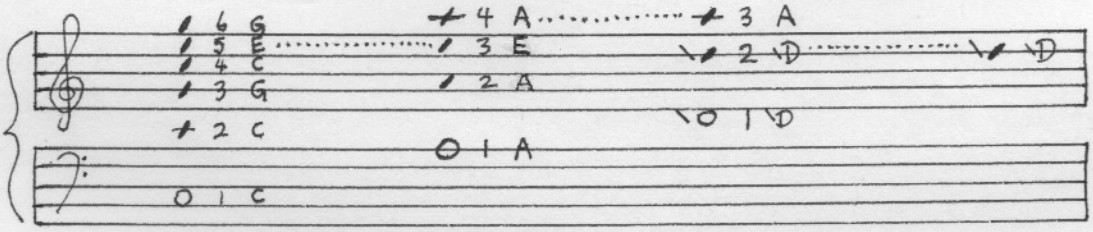
How to tune a Syntonic Comma (81:80) : -using four strings **Figure 4 :**

① Beginning with reference pitch Tenor C, tune the G string so that there is a beatless unison between the 3rd Harmonic of the C string and the 2nd Harmonic of the G string. Now G is a Just Fifth (3:2) from C. Sound the third Harmonic on the G String in order to produce the tone D :



② From reference pitch Tenor C, tune the A string so that there is a beatless unison between the 5th Harmonic of the C String and the 3rd Harmonic of the A string. Now A is a Just Major Sixth (5:3) interval above C.

③ From our A string, tune a Just Fourth (4:3) to ˇD by forming a beatless unison between the 4th Harmonic of the A string and the 3rd Harmonic of the ˇD string. Sound the 2nd Harmonic on the ˇD string to sound the octave ˇD. Compare this ˇD with the D produced above. They relate by a comma.



In ratios: $C \rightarrow G \rightarrow D$ means $1 \times \frac{3}{2} \times \frac{3}{1} = \frac{9}{2}$ (2 octaves above $\frac{9}{8}$)

$C \rightarrow A \rightarrow$ ˇD means $1 \times \frac{5}{3} \times \frac{4}{3} \times \frac{2}{1} = \frac{40}{9}$ (2 octaves above $\frac{10}{9}$)

Relation between D and ˇD is $\frac{9}{2} \div \frac{40}{9} = \frac{81}{80}$ (Syntonic Comma)

Here is this tuning pictured on the tuning graph :

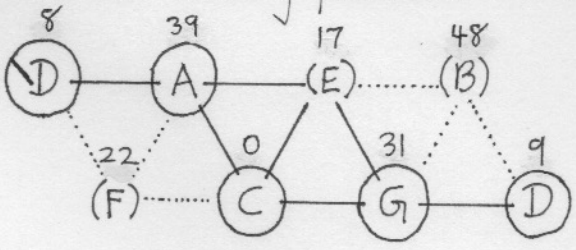


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Figure 5: The Pythagorean Chromatic Scale and its schisma altered variations:

$0 — 31 — 9 — 40 — 18 — \overline{49} — \overline{27} — \overline{5} — \overline{36} — \overline{14} — \overline{45} — \overline{23} \cdots\cdots (\overline{1})$

is the same as:

$$\begin{array}{ccccccccccccc} & & & & & & 0 & — & 31 & — & 9 & — & 40 & — & 18 \\ \overline{49} & — & \overline{27} & — & \overline{5} & — & \overline{36} & — & \overline{14} & — & \overline{45} & — & \overline{23} & & (\overline{1}) \end{array}$$

is the same as:

$$\begin{array}{ccccccccccccccc} & & & & & & 0 & — & 31 & — & 9 & — & 40 & — & 18 & — & \overline{49} & — & \overline{27} \\ \overline{5} & — & \overline{36} & — & \overline{14} & — & \overline{45} & — & \overline{23} & & (\overline{1}) \end{array}$$

Here is one (of many) schisma-altered variations:

$$\begin{array}{ccccccccccccccc} & & & & & & 0 & — & 31 & — & 9 & — & 40 & — & 18 \\ 49 & — & 27 & — & 5 & — & 36 & — & 14 & — & 45 & — & 23 \end{array}$$

Here is a Pythagorean Chromatic Scale oriented around its inherent Ptolemaic Ogdoad:

$39 — 17 — 48 — 26 — 4 — \overline{35} \,/\, 13 — 44 — 22 — 0 — 31 — 9$

which is the same as:

$$\begin{array}{ccccccccccccccc} & & & & & & 39 & — & 17 & — & 48 & — & 26 & — & 4 & — & \overline{35} \\ 13 & — & 44 & — & 22 & — & 0 & — & 31 & — & 9 \end{array}$$

Generalized patterns of Pythagorean-Type Chromatic Scales:

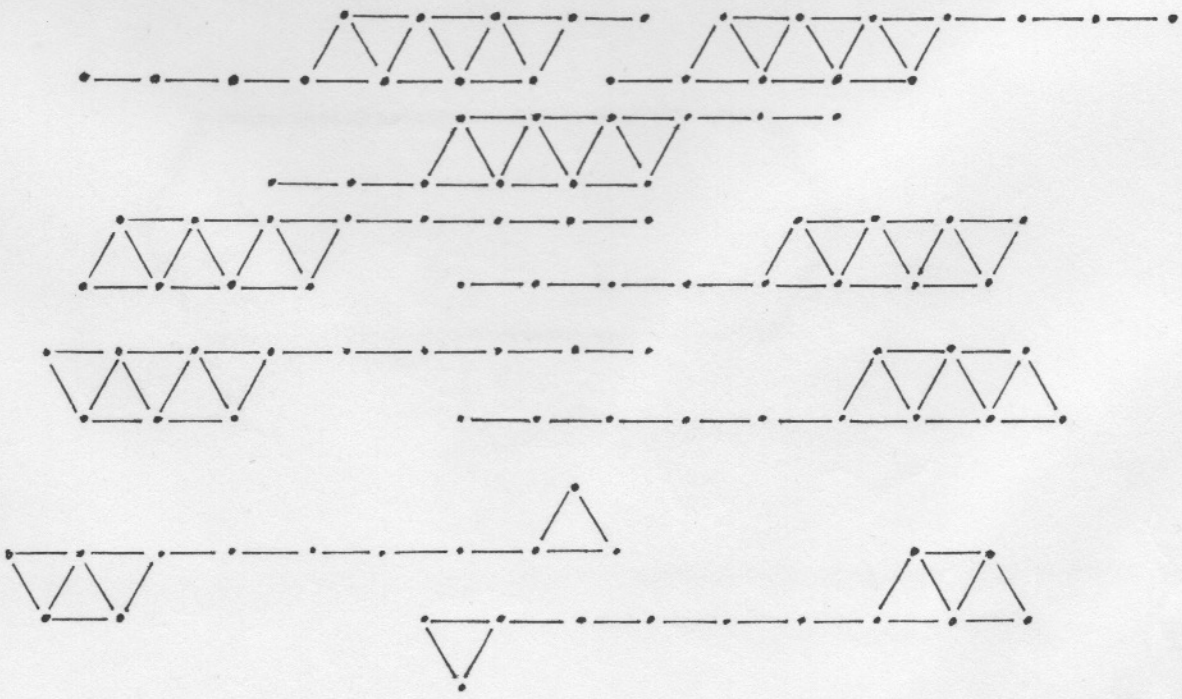
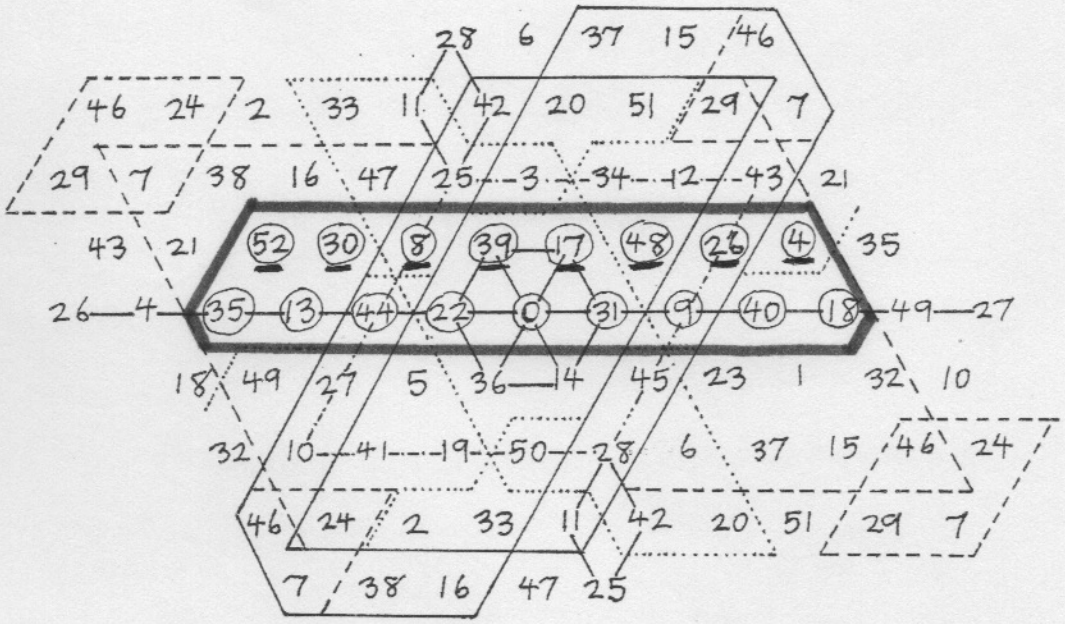


Figure 6: Prominant extensions of the Pythagorean Sequence:

17-Tone Mediaeval Arabic Tuning (9th Century Bhagdad)



25-Tone Pythagorean Sequence (which embodies the Hindu field of 22 śrutis)

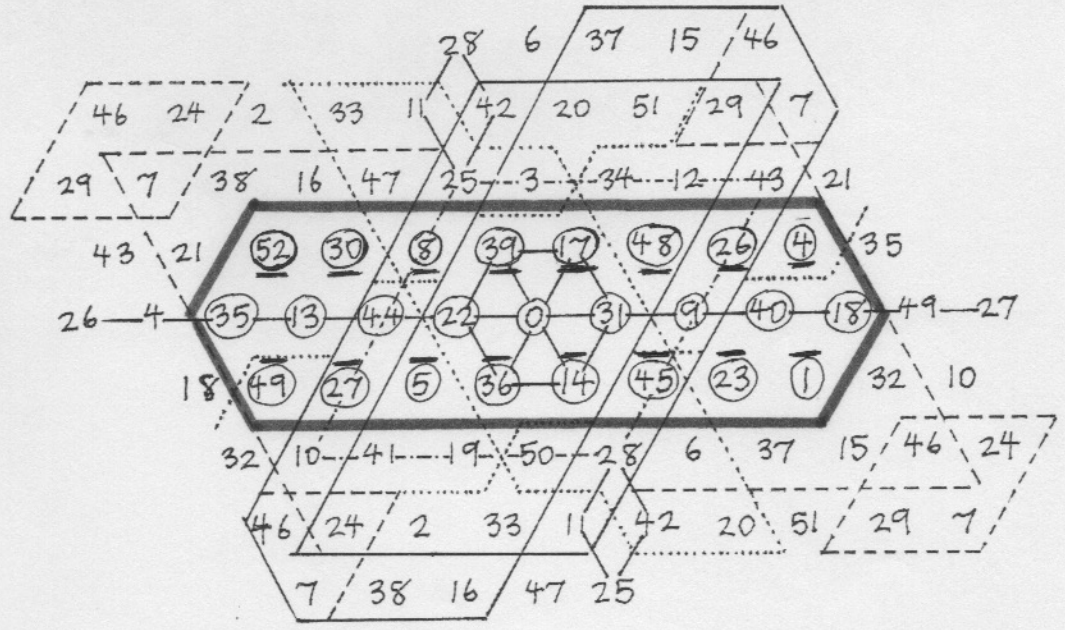
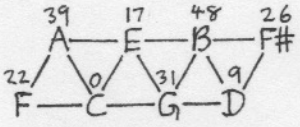


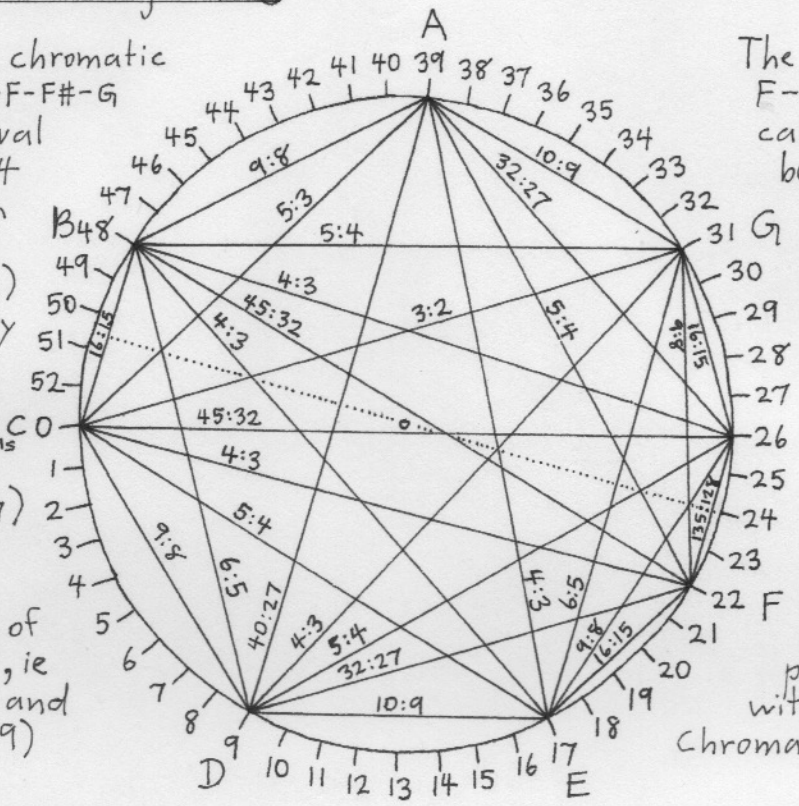
Figure 12:

Circular graph of the Ptolemaic Ogdoad (1):



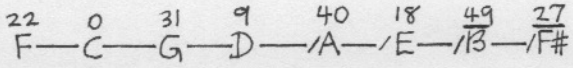
Note the chromatic segment E-F-F#-G The interval F-F# is 4 commas in size (in Cents: 92.2) flanked by Diatonic semitones of 5 commas in size (in Cents 111.7)

Note the two sizes of wholetone, ie Major (9:8) and Minor (10:9)



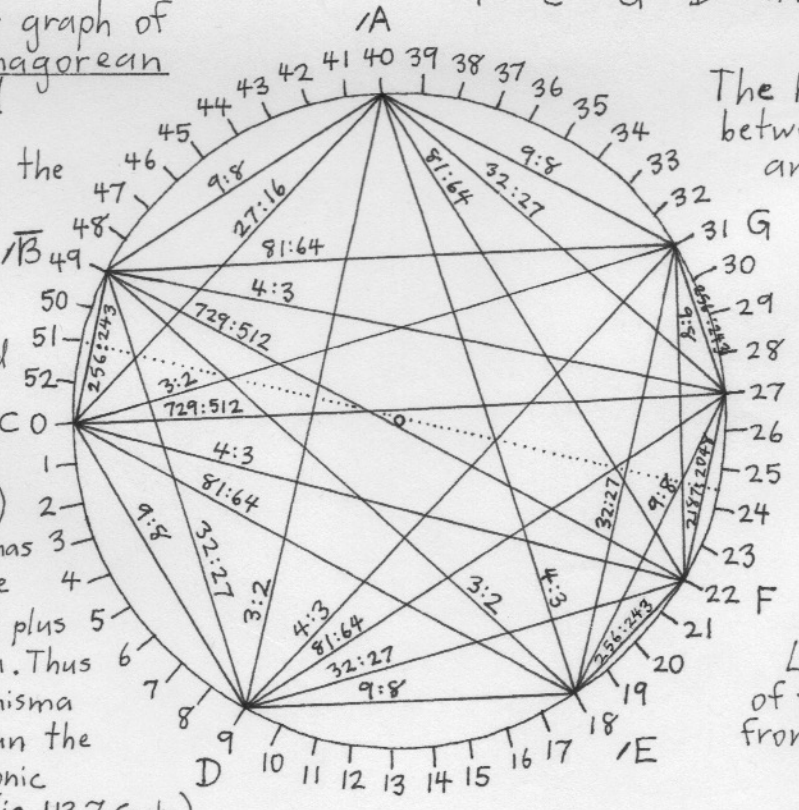
The interval between F-F# is sometimes called the Larger Limma because it is the defect of a Fourth, increased by a Diatonic Semitone, from a Fifth.

Note that these two graphs show Just and Pythagorean forms of the same Harmonic pattern. Both of these patterns are present within the Pythagorean Chromatic scale.



Circular graph of the Pythagorean Ogdoad

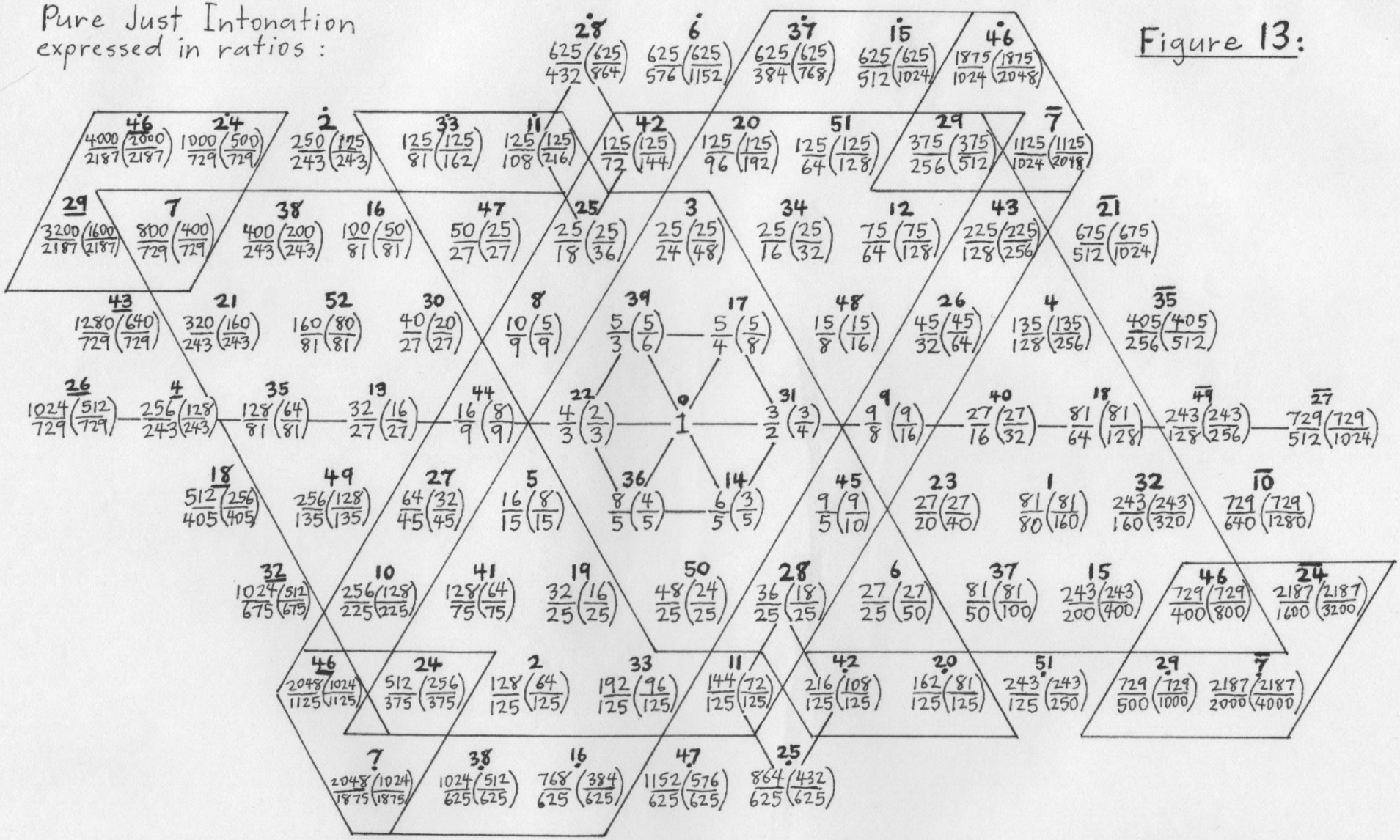
Contrast the chromatic segment /E-F-/F#-G with the one pictured above. The interval F-/F# (ratio: $\frac{2187}{2048}$) is $\bar{5}$ commas in size, ie 5 commas plus a schisma. Thus it is a schisma larger than the 16:15 Diatonic Semitone (ie. 113.7 Cents)



The Pythagorean Semitone between /B-C, /E-F, and /F#-G (ratio 256:243) is 4 commas in size, minus a schisma (ie. 90.2 Cents). Contrast this interval with the Larger Limma above (92.2 Cents). This Pythagorean Semitone is sometimes named the Pythagorean Limma (the defect of two Major Tones from the Fourth).

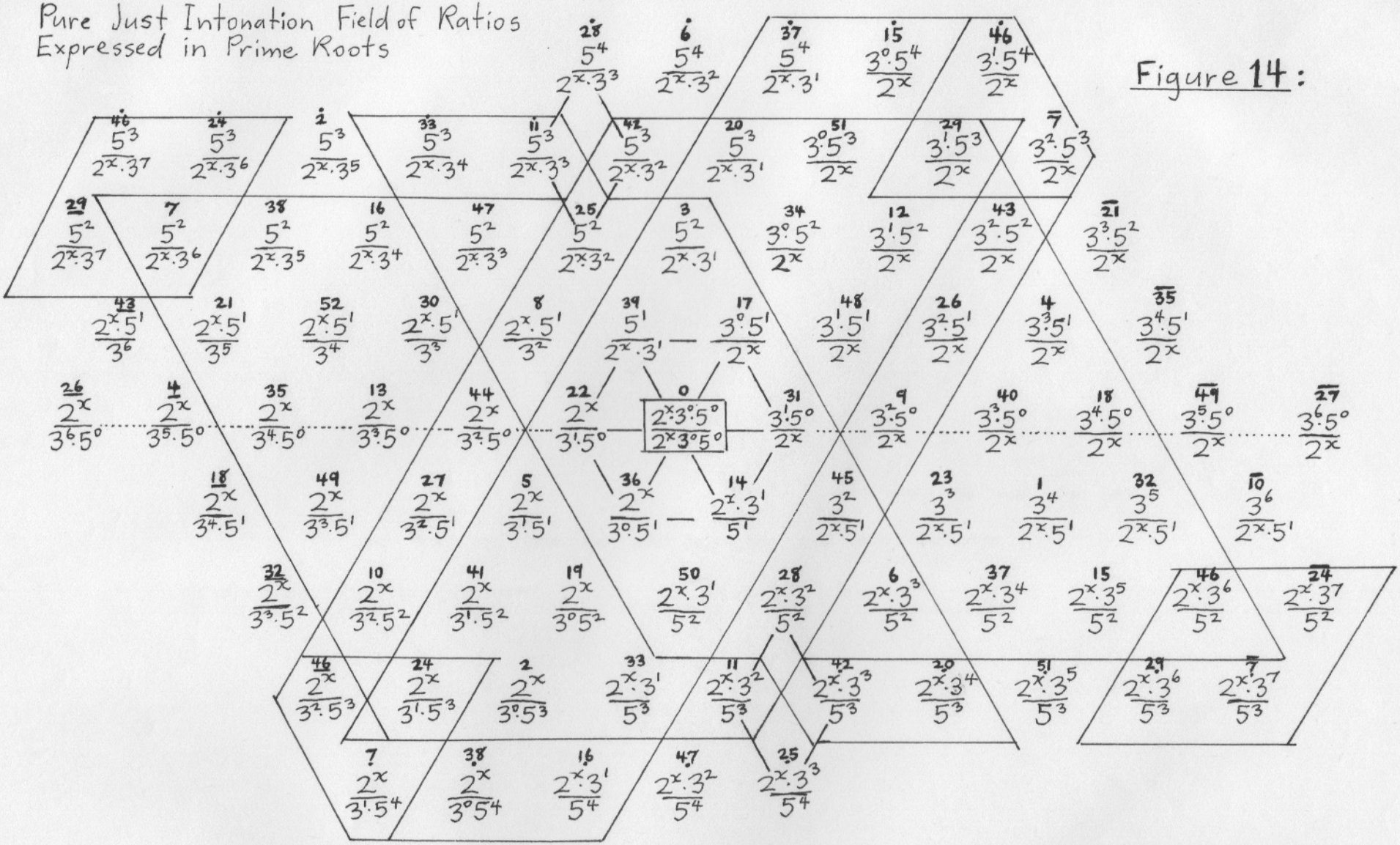
Pure Just Intonation
expressed in ratios :

Figure 13:



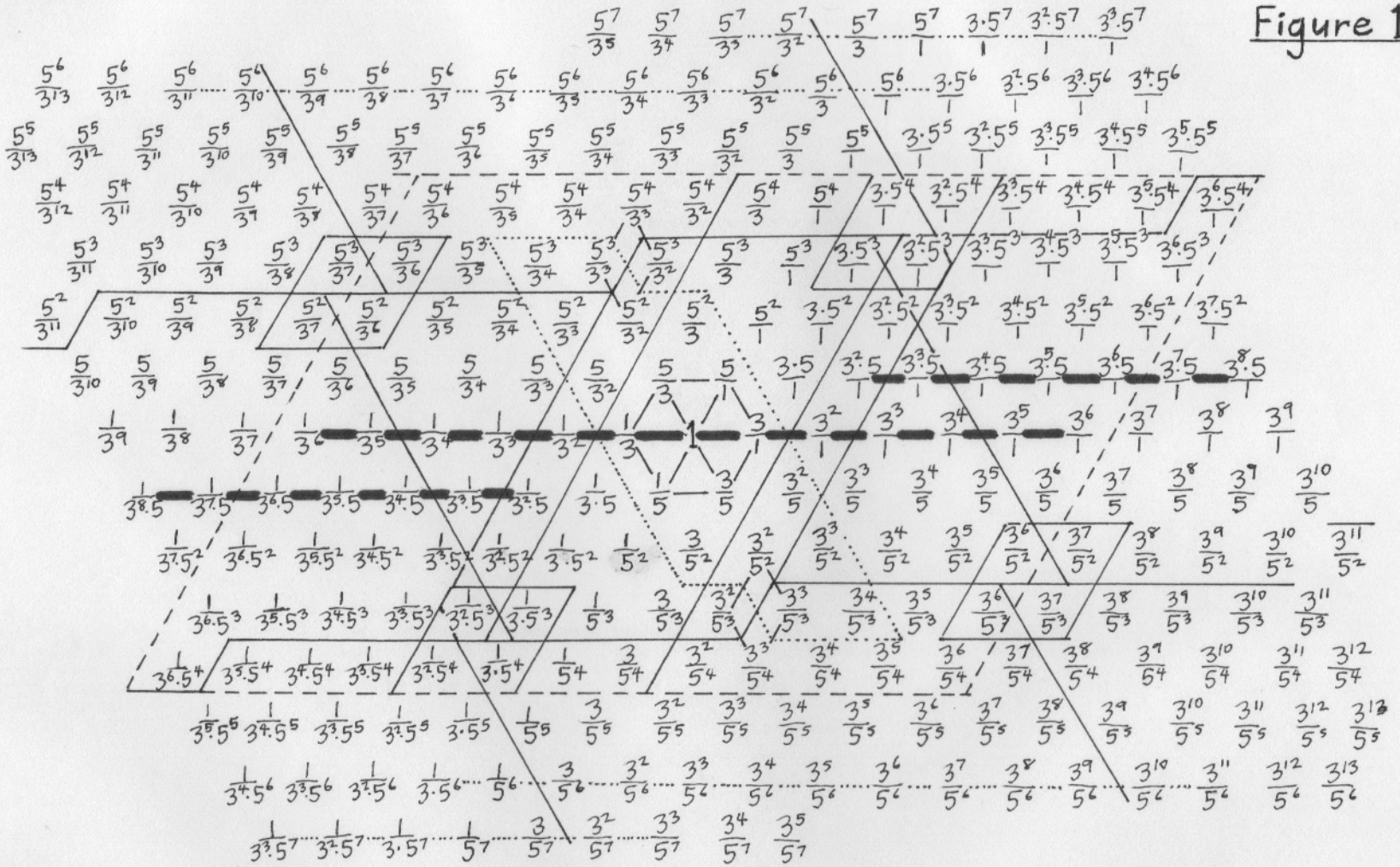
Pure Just Intonation Field of Ratios
Expressed in Prime Roots

Figure 14:



Extended Field of Pure Just Intonation – Ratios expressed as the Prime Roots 3 and 5, ie. the 3 x 5 Multiplication Table. Powers of 2 are hidden.

Figure 15:



53-E.T. (Quasi-Just Intonation) Field:

showing deviation from wider **pure** Just Intonation field

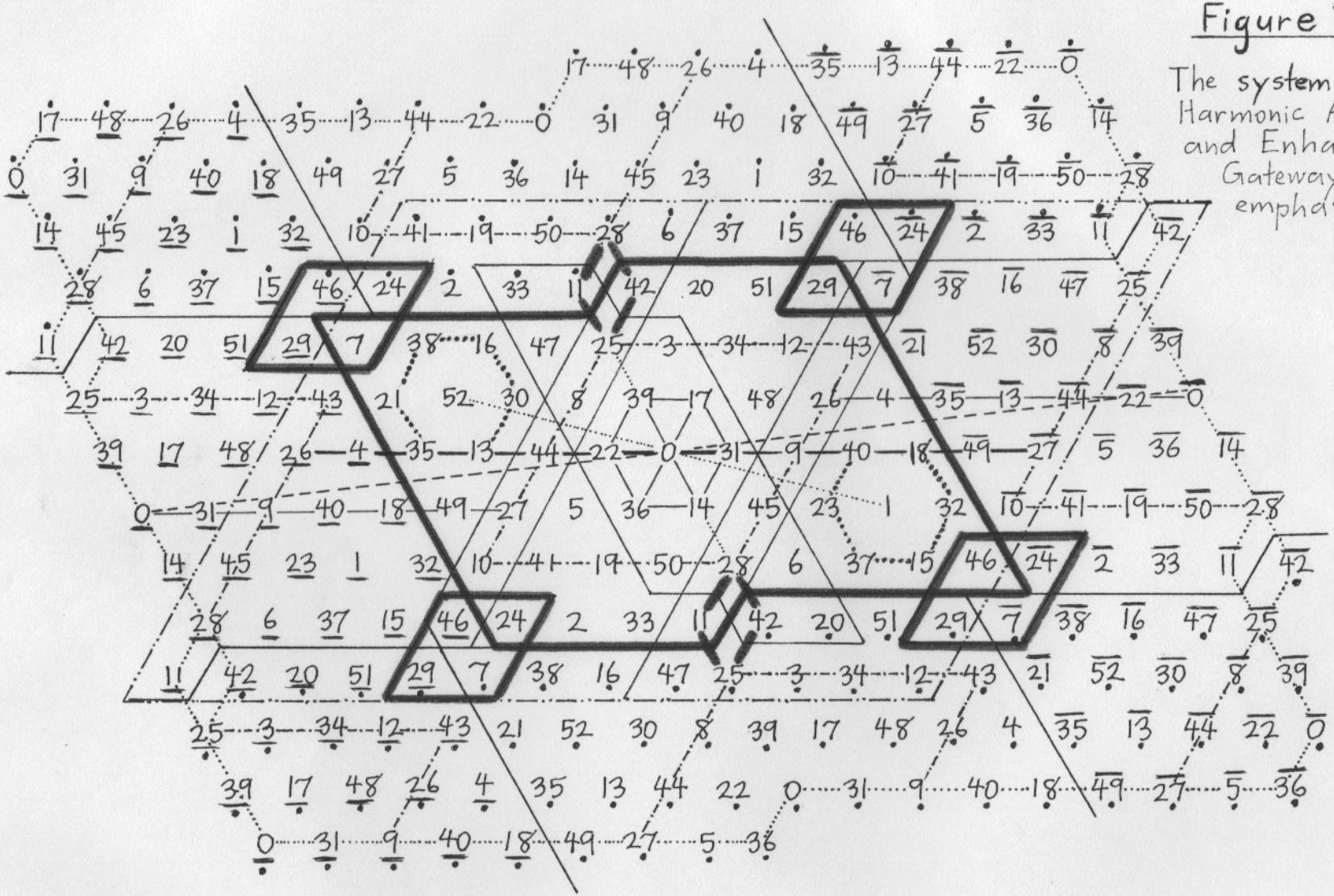


Figure 16:

The system boundary, Harmonic Antipodes, and Enharmonic Gateway are emphasized

Graph showing **deviation** (in Cents) of 53-E.T. from Just Intonation. Just Ratio expressed in Cents is shown above the circled Comma-Number notation. Tempered Ratio expressed in Cents is shown below the circled notation. Deviation is shown below that. Just Intonation Field is extended to just beyond the 53-E.T. boundary which is marked with the dotted line.

All calculations are rounded off to one decimal point of a Cent.

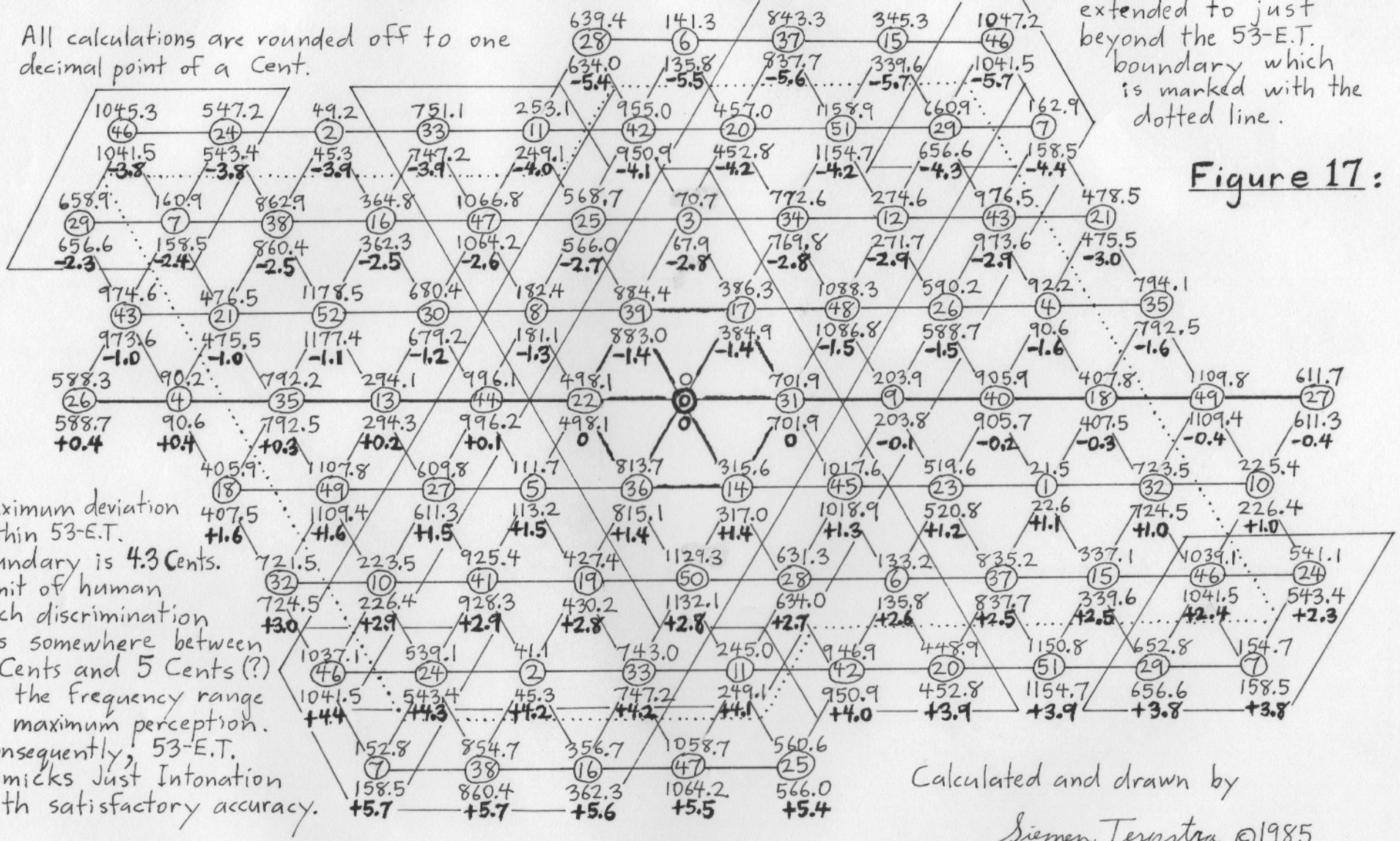


Figure 17:

Maximum deviation within 53-E.T. boundary is 4.3 Cents. Limit of human Pitch discrimination lies somewhere between 2 Cents and 5 Cents (?) for the frequency range of maximum perception. Consequently, 53-E.T. mimicks Just Intonation with satisfactory accuracy.

Calculated and drawn by

Siemen Terpstra ©1985

53-E.T. as a Field of Pitches

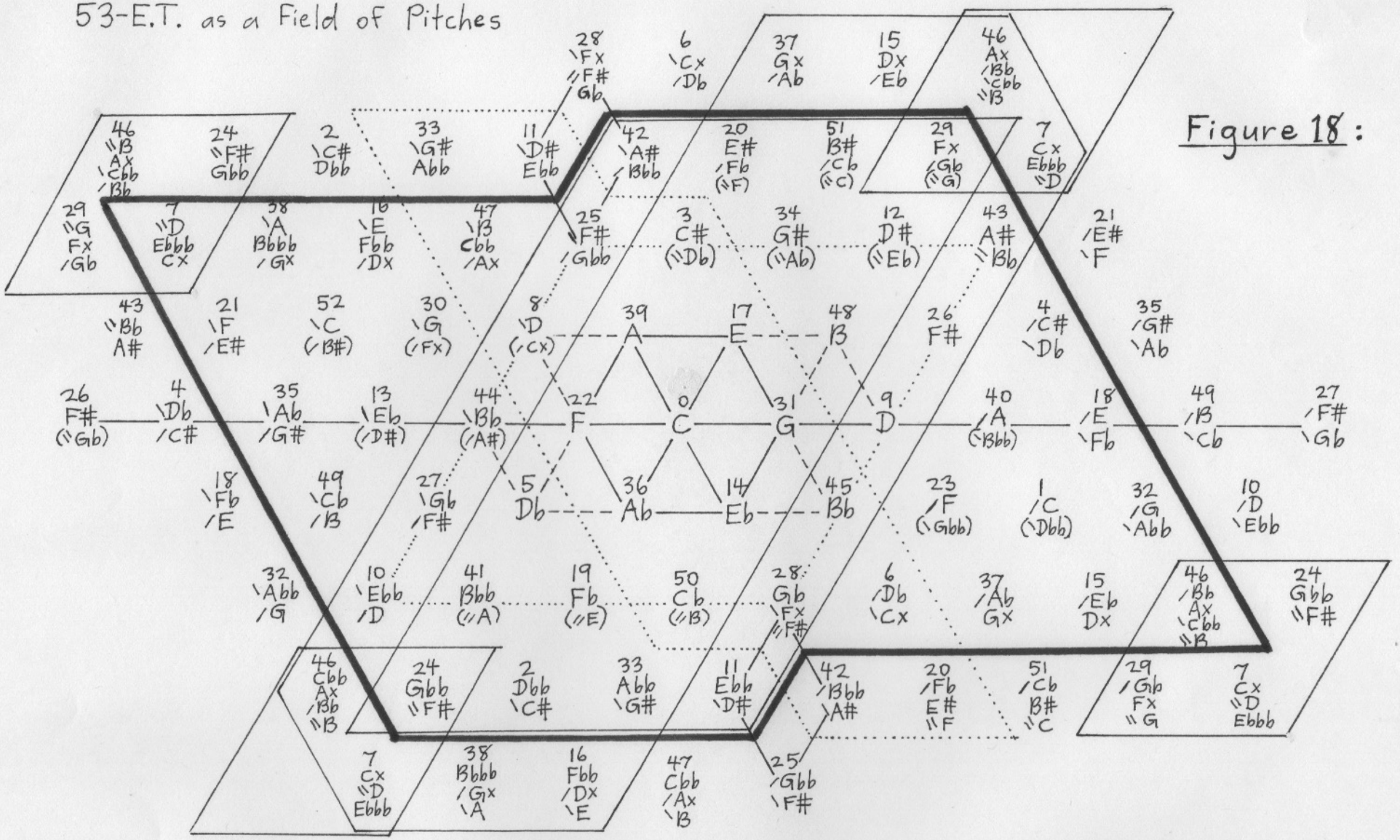


Figure 18:

53-E.T. as a Field of Functions

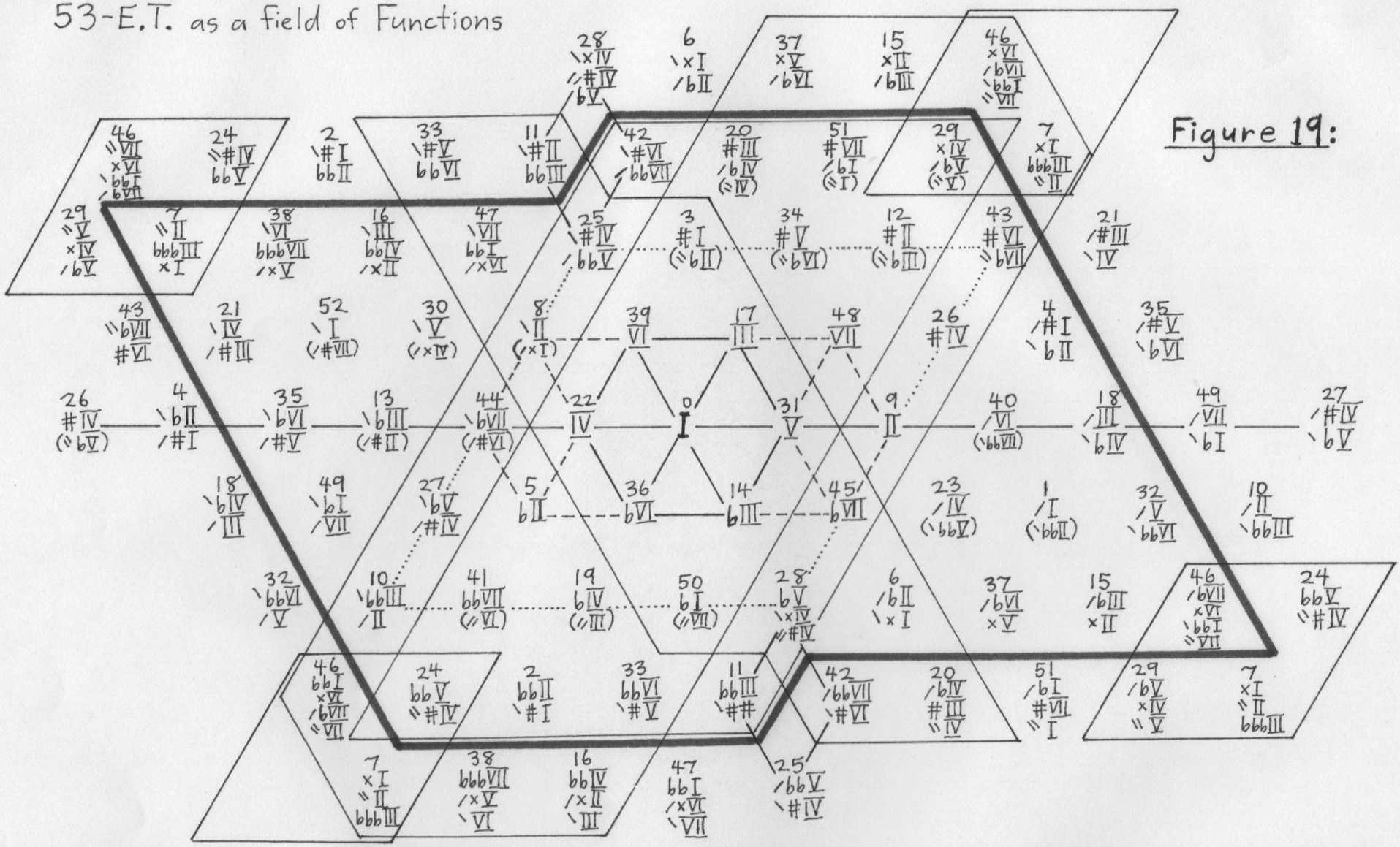


Figure 19:

12-E.T. (One-Twelfth Ditonic Comma Meantone) Field:

showing deviation from Pure 3-Limit Just Intonation in fractional parts of a Comma.

Figure 20:

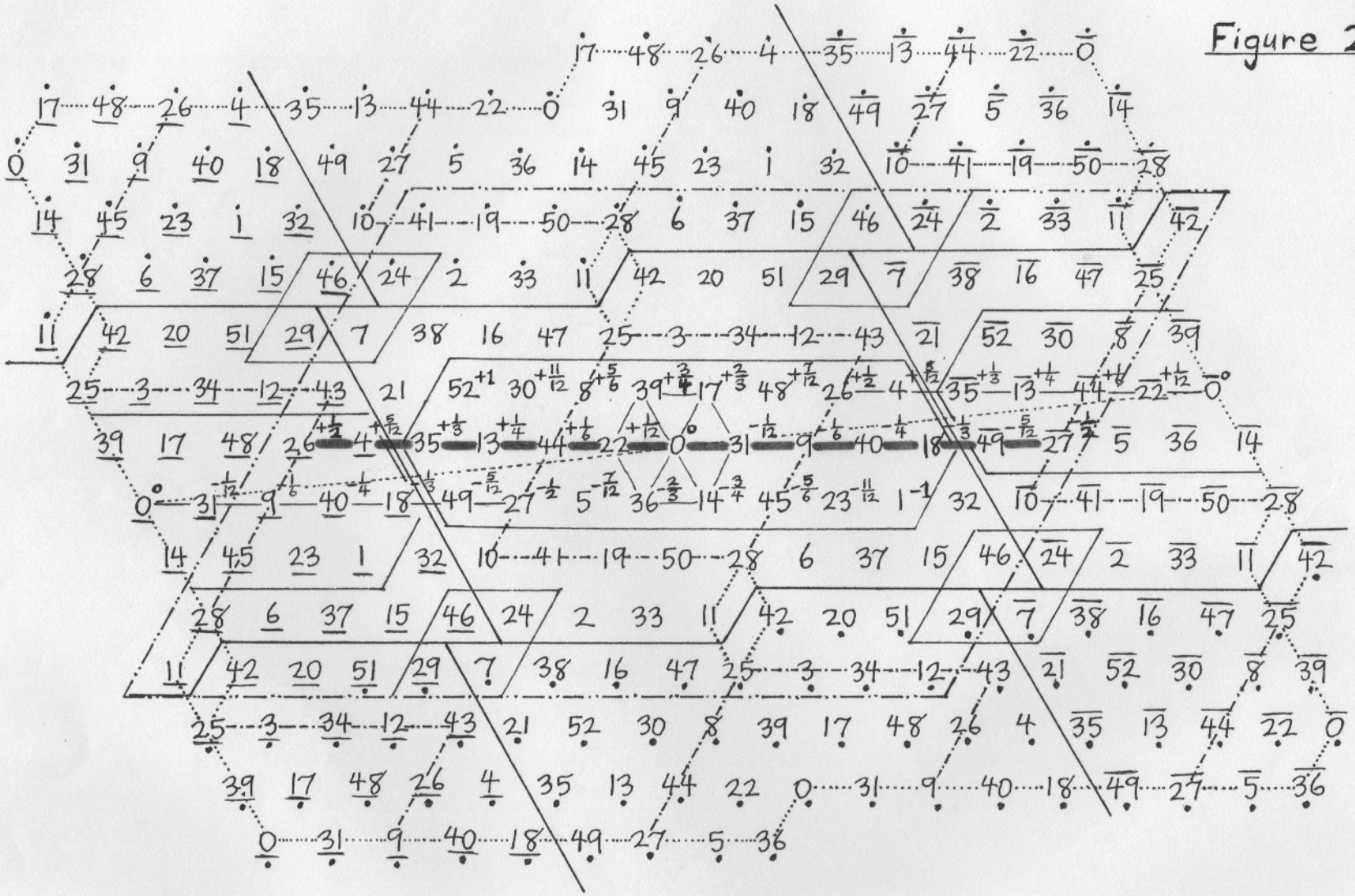
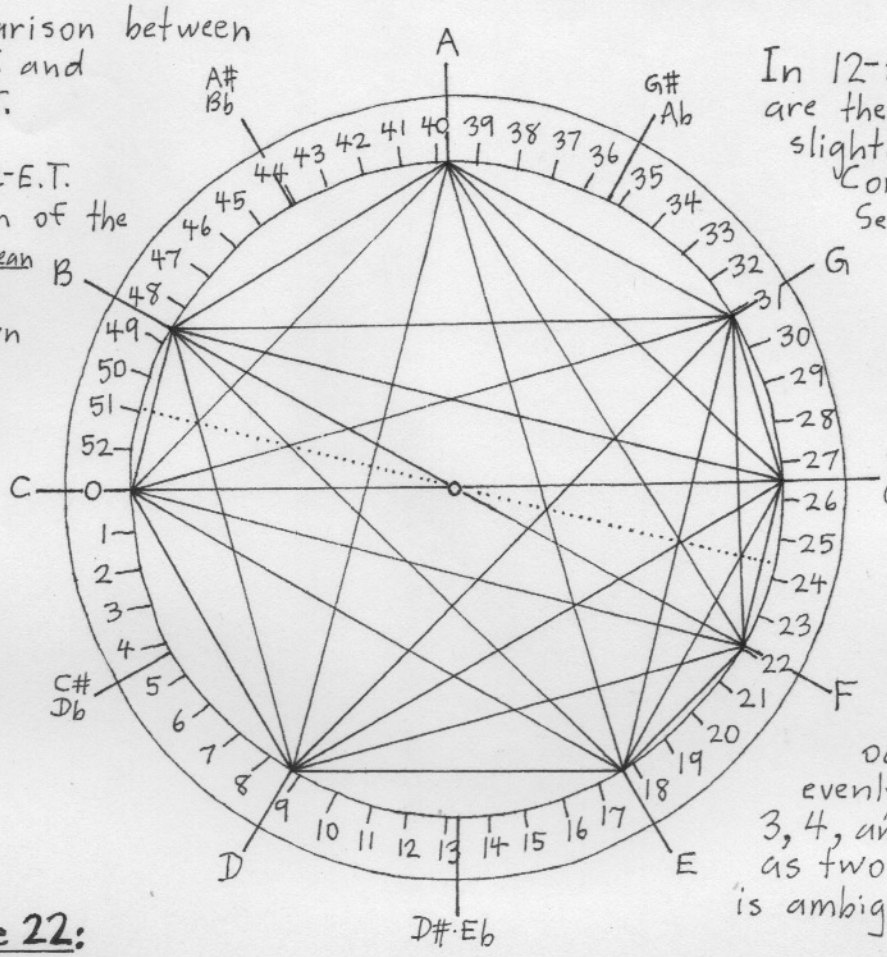


Figure 21:

F—C—G—D—A—E—B—F#

Comparison between 53-E.T. and 12-E.T.

The 12-E.T. version of the Pythagorean Ogdoad is shown

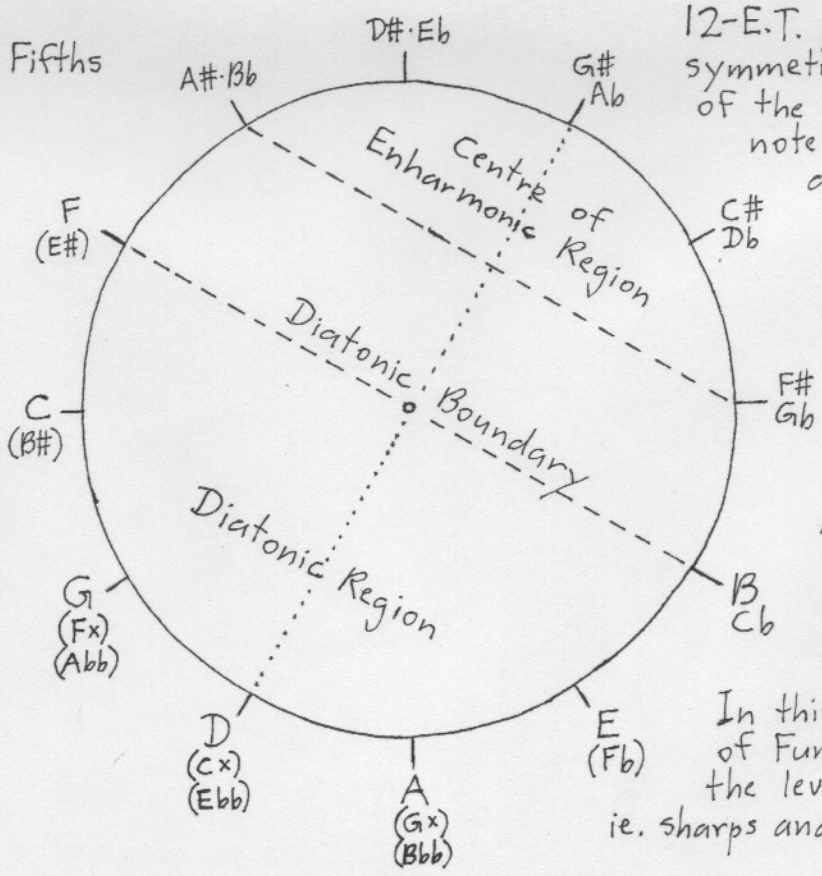


In 12-E.T. all semitones are the same size, ie. slightly closer to 4 Commas (Pythagorean Semitone) than to 5 Commas (Just Diatonic Semitone)

The most important feature of the system is the Tritone ($\sqrt{2}$) which is the Geometric Mean of the Octave. Being composite, the octave is also evenly divisible by 3, 4, and 6 – as well as two. The result is ambiguity of Function.

Figure 22:

12-E.T.
Circle of Fifths



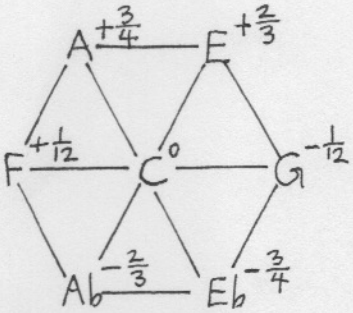
12-E.T. preserves the symmetry around "D" of the seven "natural" notes – as if it were a Pythagorean expansion. The tuning system must be considered as a tempering of the Pythagorean Sequence rather than a tempering of the Just Sequences. An essentially Pythagorean character is preserved.

In this system, ambiguity of Function exists at the level of the Enharmonic, ie. sharps and flats are confused.

Figure 23: Essential Structure of 12-E.T.:

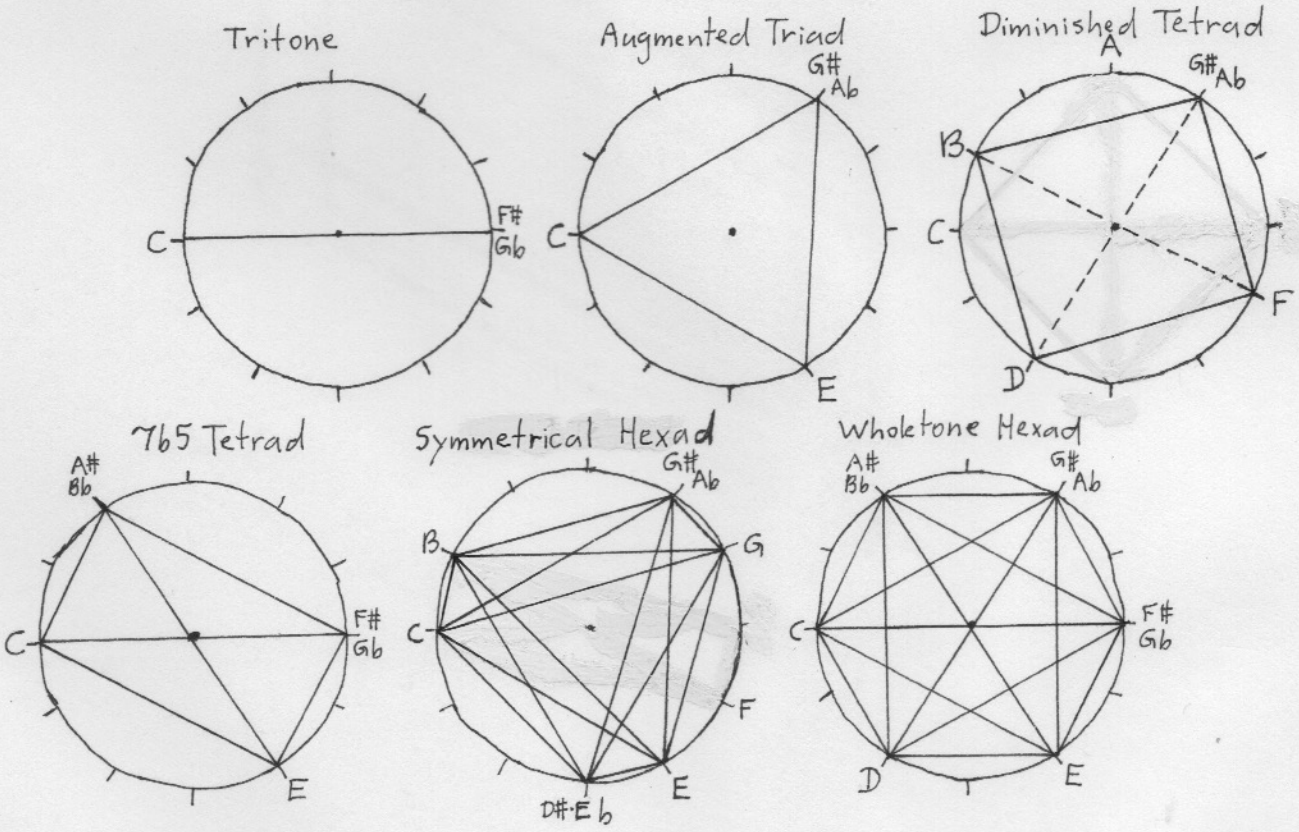
	26	4	35	13	44	22	0	31	9	40	18	49	27
3-Limit Just Intonation	F#	Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#
Amount of Comma Alteration	$+\frac{1}{2}$	$+\frac{5}{12}$	$+\frac{1}{3}$	$+\frac{1}{4}$	$+\frac{1}{6}$	$+\frac{1}{12}$	0	$-\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{5}{12}$	$-\frac{1}{2}$
12-E.T. Function	#IV / bV	bII	bVI	bIII	bVII	IV	I	V	II	VI	III	VII	#IV / bV

Deviation of Harmonic Heptad from Just Intonation for 12-E.T.:



In general, Fifths and Fourths are excellant, while Thirds and Sixths are mediocre. Septimal consonances are very poorly represented.

Figure 24: Some important "reductionist" patterns of 12-E.T.:



All of these patterns have multiple axes of symmetry.

31-E.T. (Quasi-One-Quarter Comma Meantone) Field:

showing deviation from Pure Just Intonation in Fractional parts of a Comma:

Figure 25:

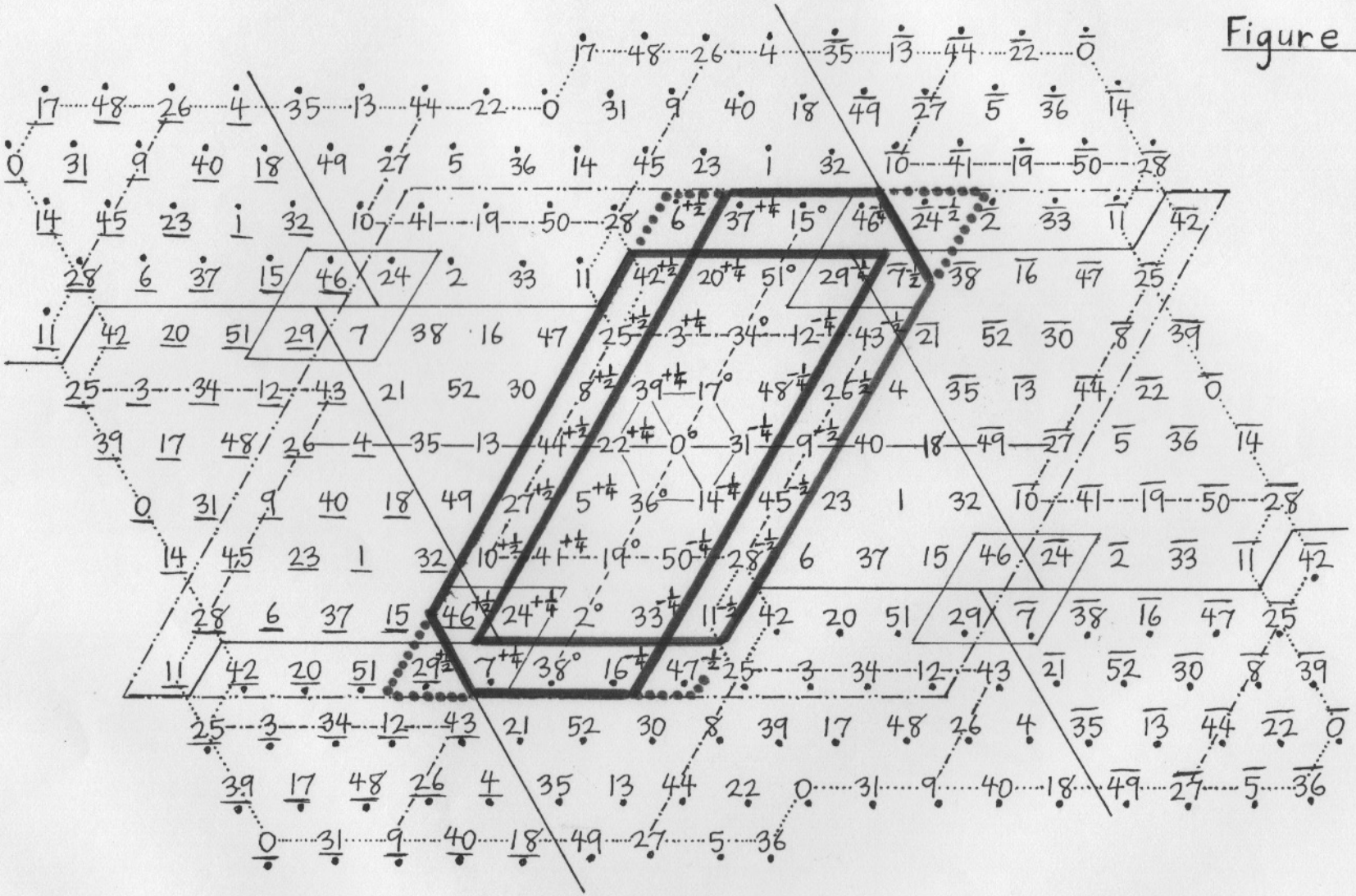


Figure 26:

Pitch Field : 31-E.T.

Notational Signs:
♯ = Semi-sharp
= Sharp
= Sesqui-sharp
x = Double-sharp
♭ = Semi-flat
b = Flat
♭♭ = Sesqui-flat
bb = Double-flat

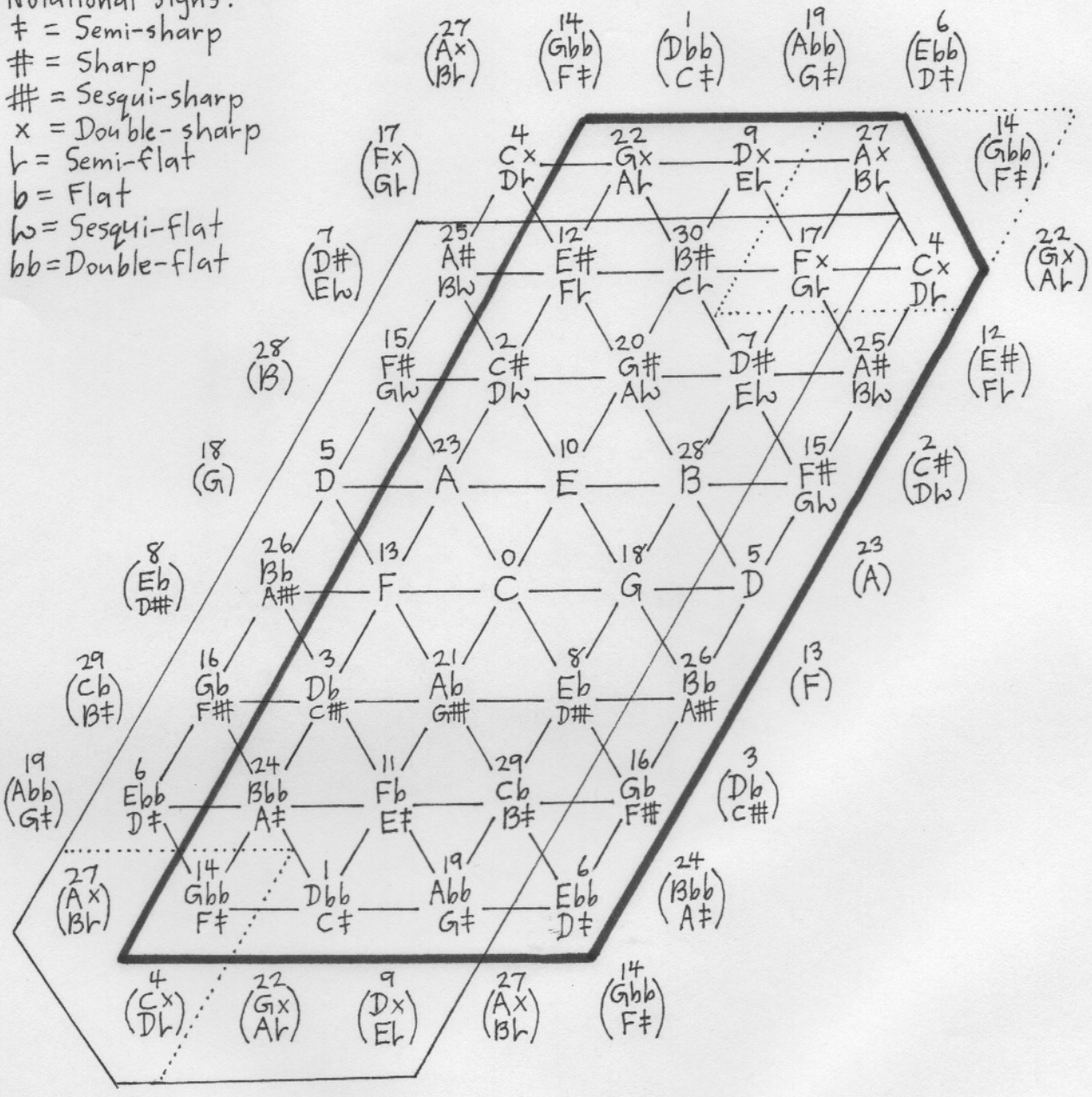
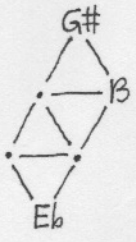


Figure 27: "Wolf" Major Triad

ie. G#-C-E♭
Just G# chord is
G#-B#-D#

"Wolf" Minor Triad



ie G#-B-E♭
Just G# chord is
G#-B-D#

Figure 28:

31-E.T. The REGIONS of the Functional Field :

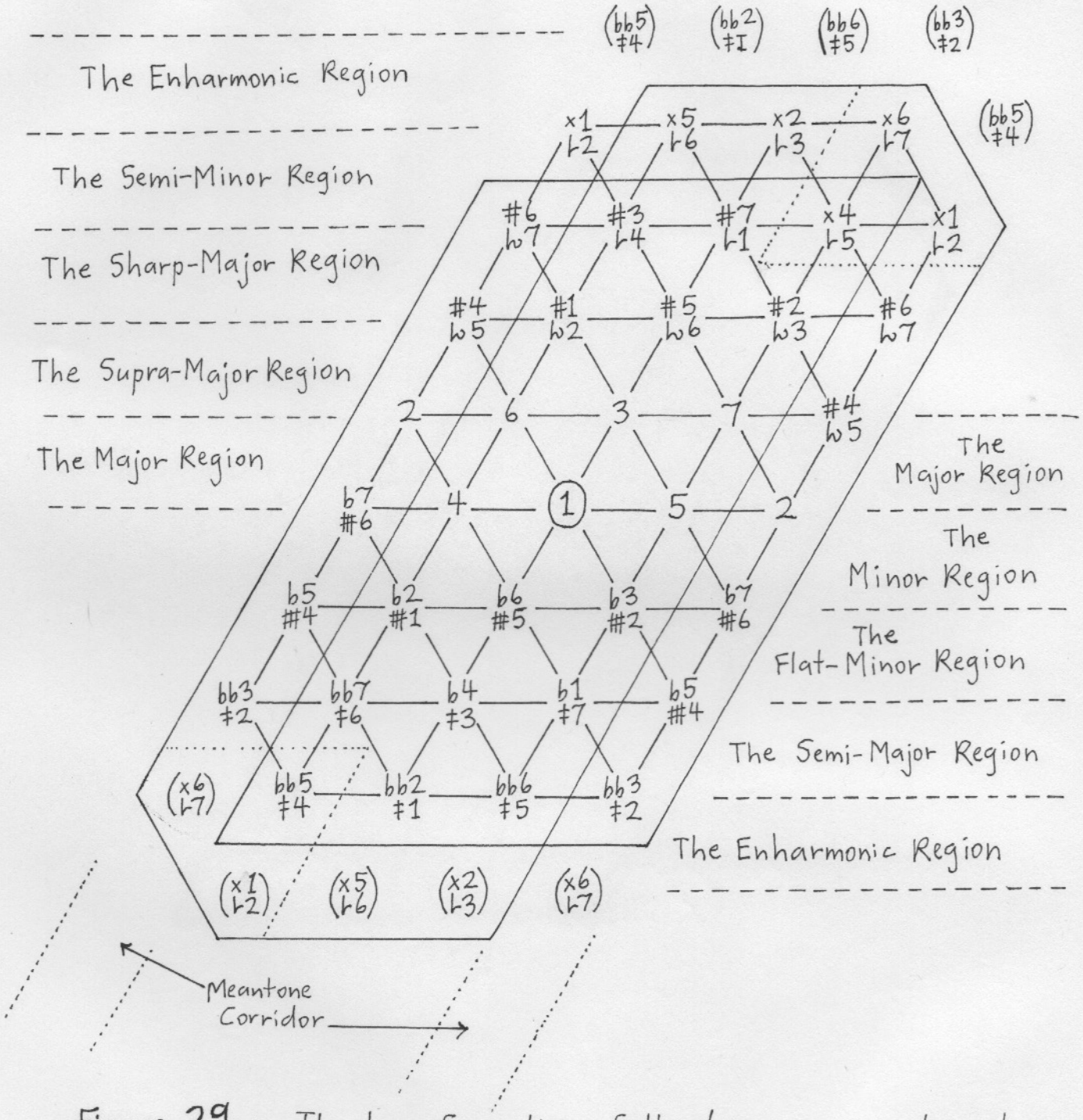
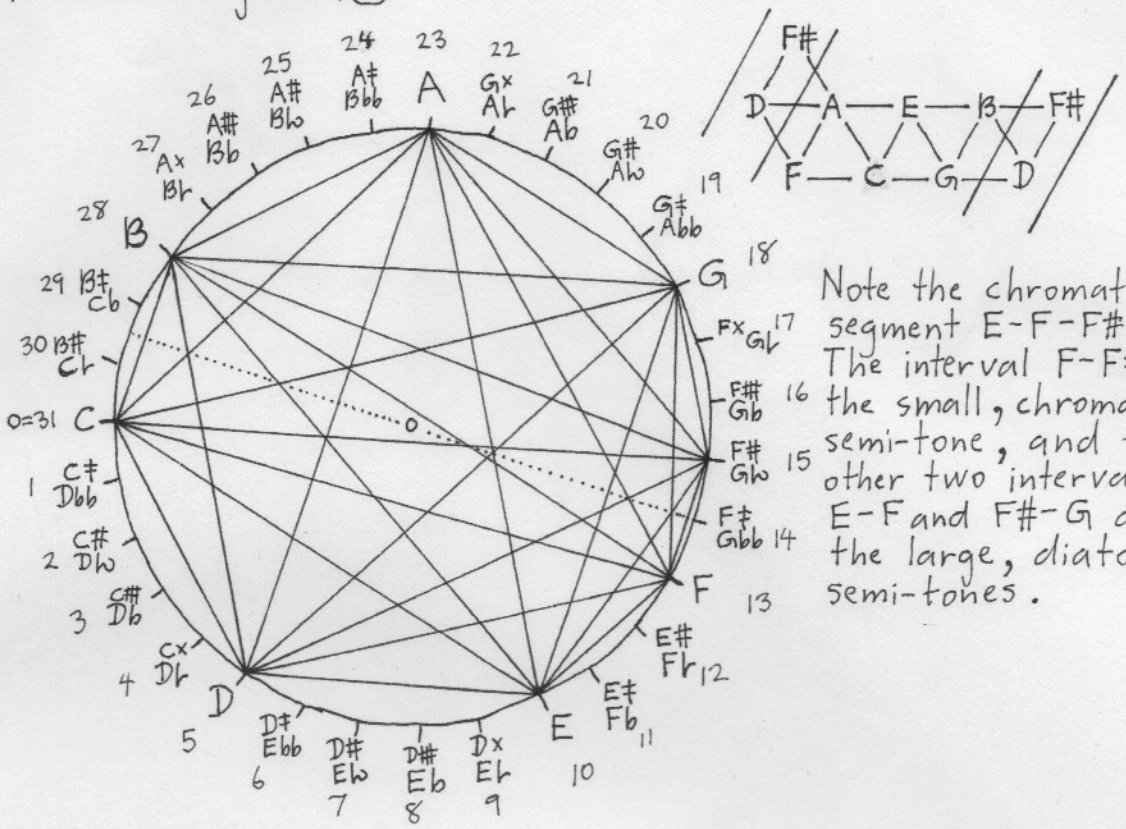


Figure 29: The transformation of the two non-symmetrical Ptolemaic Heptads (Just Diatonic Scales) into a single, symmetrical Diatonic Heptad:

ie. 6—3—7 / 4—1—5—2 = 2—6—3—7 / 4—1—5

Figure 34:

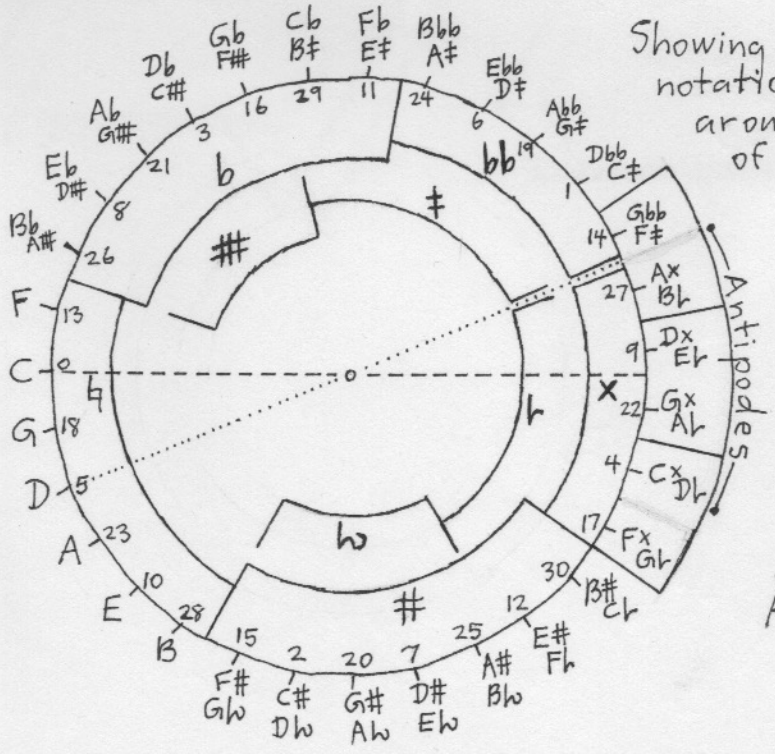
Circular graph in 31-E.T. showing the tempered form of the Ptolemaic Ogdoad ②



Note the chromatic segment E-F-F#-G. The interval F-F# is the small, chromatic semi-tone, and the other two intervals E-F and F#-G are the large, diatonic semi-tones.

Figure 35:

31-E.T. Cycle of Fifths-Fourths:



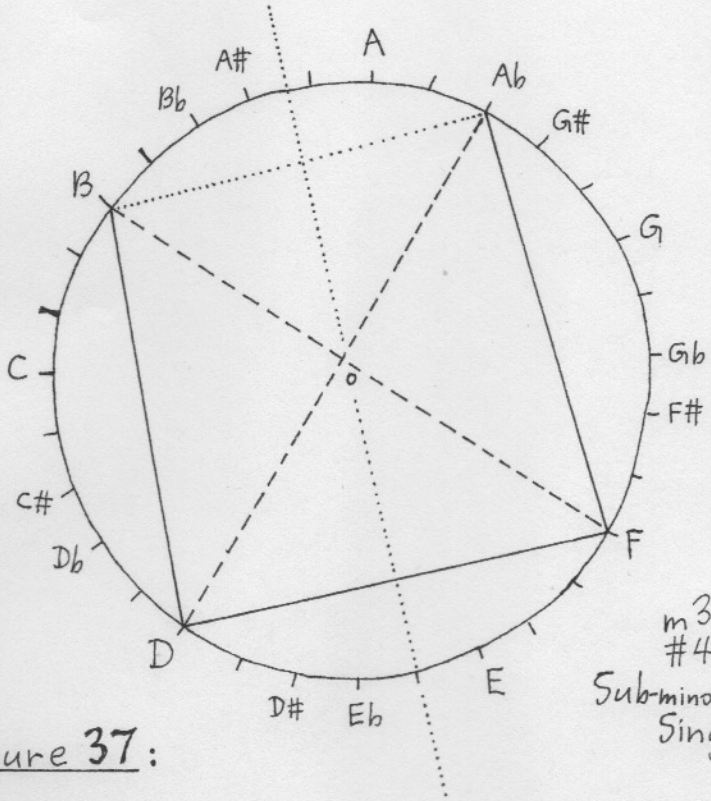
Showing the logic of the Fokker notation with its symmetry around pitch D (middle of the diatonic spectrum.)

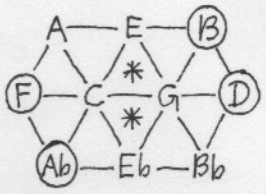
Seven naturals, seven sharps, seven flats, five double-sharps and five double-flats — which are re-named semi-flats and semi-sharps.

The pitches associated with the Harmonic Antipodes are marked.

Figure 36:

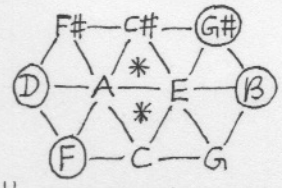
31-E.T. The Diminished Tetrad, showing integrity of Function, ie. no ambiguity in its resolution





Natural resolution is to C Major or C Minor ("Key of C").

A resolution to Key of A requires G# rather than Ab:



m3rd-M6th ————————————

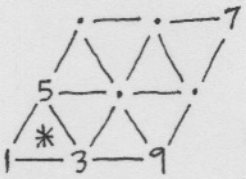
#4th-b5th - - - - - - - - - - - - - - -

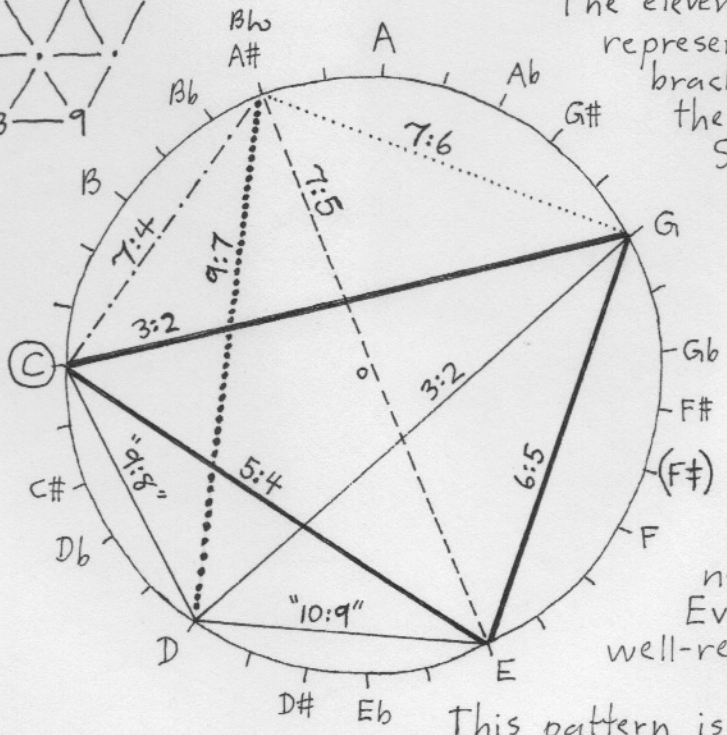
Sub-minor 3rd-Septimal M6th .....

Single Axis of Symmetry

Figure 37:

31-E.T.: The Harmonic Series Pentad, being that chord which is made up of the first 10 Harmonics (Primes 1, 3, 5, 7, "9")





The eleventh Harmonic of C is well represented by F♯ (which is bracketed) and would tranform the pattern into the Harmonic Series Hexad, also well approximated by 31-E.T.

The most innaccurate Harmonic is the Ninth, being one-half comma flat of a 9:8 ratio.

Intervals from the 5th and 7th Harmonics are very good, third Harmonic not as good but still worthy. Even the 11th Harmonic is well-represented.

This pattern is an example of a non-symmetrical structure—whose compliment is the Sub-harmonic Series Pentad

Figure 38:

The association of $\frac{1}{4}$ Comma Meantone (and 31-E.T.) with $\sqrt{5}$, the **Golden Mean Ratio**, and the **Fibonacci Series**:

The Meantone itself (ratio $\frac{\sqrt{5}}{2}$) is the geometrical mean of the Just Major Third (5:4). The arithmetic and harmonic means are the Just Intonation wholetones 9:8 and 10:9.

The $\frac{1}{4}$ Comma-flattened Fifth (ratio $\sqrt[4]{5}$) can be found as follows:

$$\begin{aligned} &\text{Major Third} = \text{ratio } 5:4 \\ &\text{Wholetone} = \text{Geometrical Mean of } 5:4 = \frac{\sqrt{5}}{2} \\ \therefore\ &\text{Ninth} = \frac{\sqrt{5}}{2} \times 2 = \sqrt{5} \\ \therefore\ &\text{Fifth} = \sqrt{\sqrt{5}} = \sqrt[4]{5}. \end{aligned}$$

The Square Root of Five ($\sqrt{5}$) is closely associated in geometrical constructions with the Golden Section or Golden Mean Ratio which is:

$$\frac{1+\sqrt{5}}{2} \text{ and } \frac{1-\sqrt{5}}{2} = \phi \text{ and } \frac{1}{\phi}$$

Although the ϕ ratio does not appear directly, it appears in altered forms in the 31-E.T. fabric.

eg. given that pitch C is ratio 1/1 (reference ratio)

then pitch C# $= \frac{5\phi - 6}{2}$

pitch E# $= \frac{\phi + 1}{2}$

pitch F# $= \frac{\sqrt{5}}{\phi}$

pitch Abb $= \frac{4}{\phi + 1}$

All of the ratios of the extended $\frac{1}{4}$ Comma Meantone Fabric may <u>ideally</u> be expressed through powers of 2 and 5. I say ideally because 31-E.T. is a slight alteration of these ratios, an alteration below the sonic threshold of pitch discrimination. See Figure 39.

Finally, 31-E.T. is part of a Fibonacci Series of good cyclical temperaments which include

$$(2), (5), (7), \underbrace{\mathbf{12, 19, 31, 50}}_{\text{useful zone}}, (81), (131) \ldots$$

The Harmonic Axes of Just Intonation relates to the Fibonacci Series: 1, 1, <u>2, 3, 5, 8</u>, 13 ....

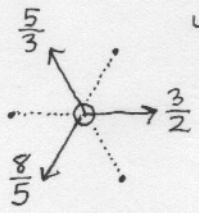


Figure 39:

The fabric of Extended ¼ Comma Meantone Tuning expressed as a field of Ratios which stem from powers of 2 (Octave) and 5:

The Field of Harmony is expressed through the exclusive use of the "human" number 5. The Tuning system 12-E.T. expresses harmony through exclusive use of the "divine" number 3. Just Intonation uses both.

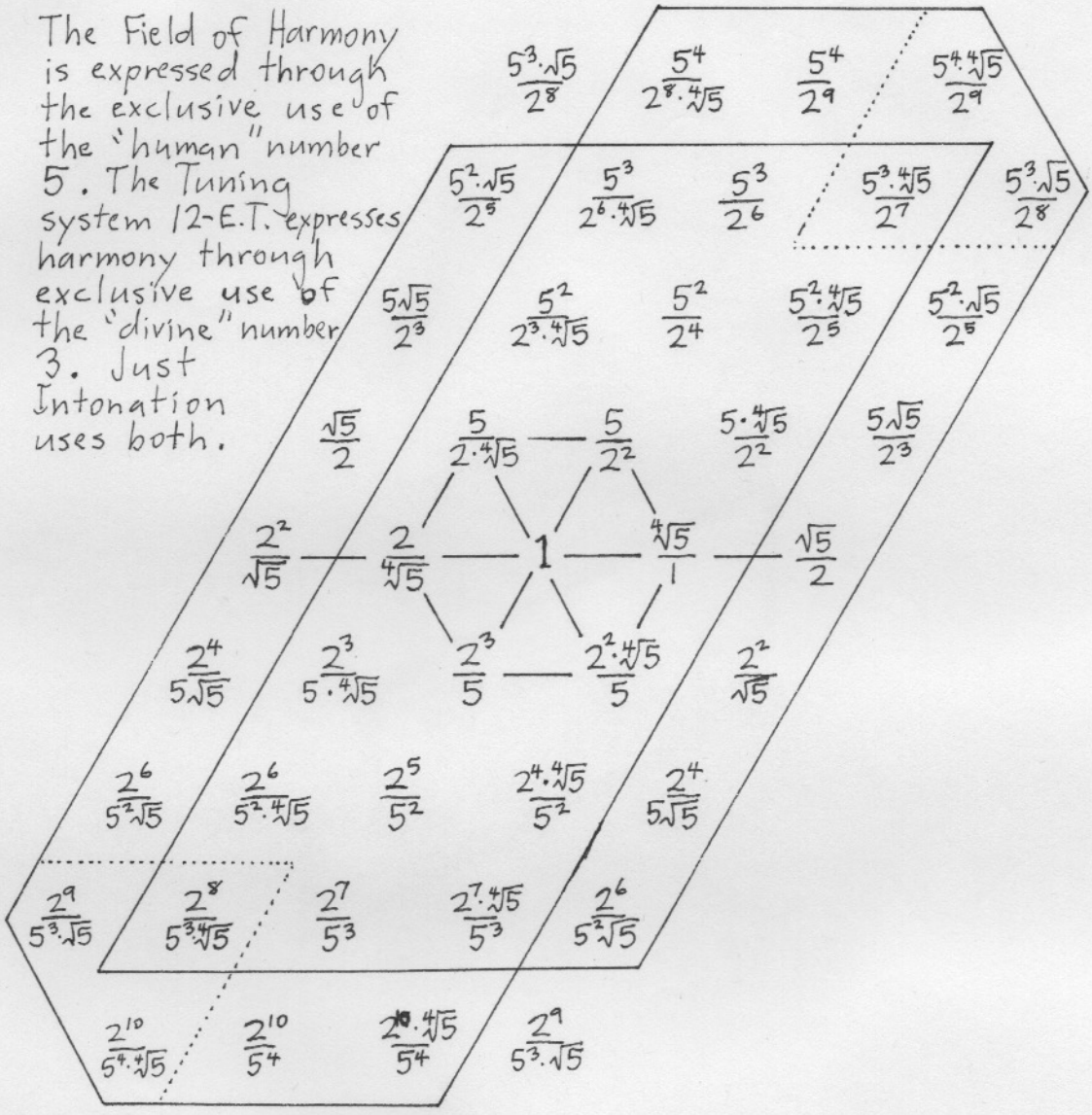


Figure 40: 31-E.T. and important Septimal Ratios:

Ratio (Cents)	31-E.T. (Cents)	Difference (Cents)
7:4 — 968.83	A# — 967.74	−1.09
7:5 — 582.51	F# — 580.65	−1.86
7:6 — 266.87	D# — 270.97	+4.10
14:9 — 764.92	G# — 774.19	+9.27
8:7 — 231.17	D♯ — 232.26	+1.09
10:7 — 617.49	Gb — 619.35	+1.86
12:7 — 933.13	A♯ — 929.03	−4.10
9:7 — 435.08	E♯ — 425.81	−9.27

19-E.T. (Quasi-One-Third Comma Meantone)

showing deviation from pure Just Intonation in fractional parts of a Comma

Figure 41

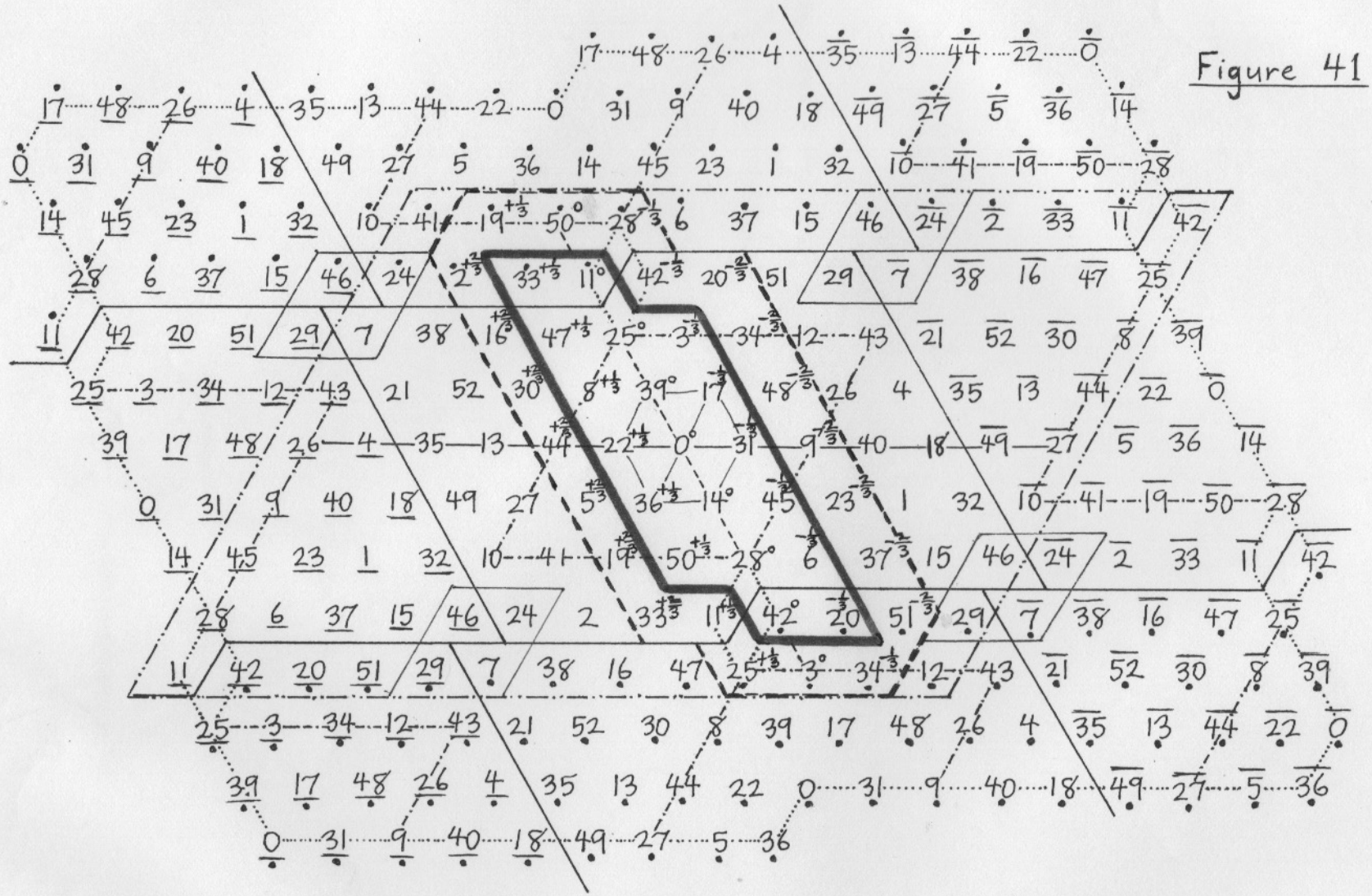


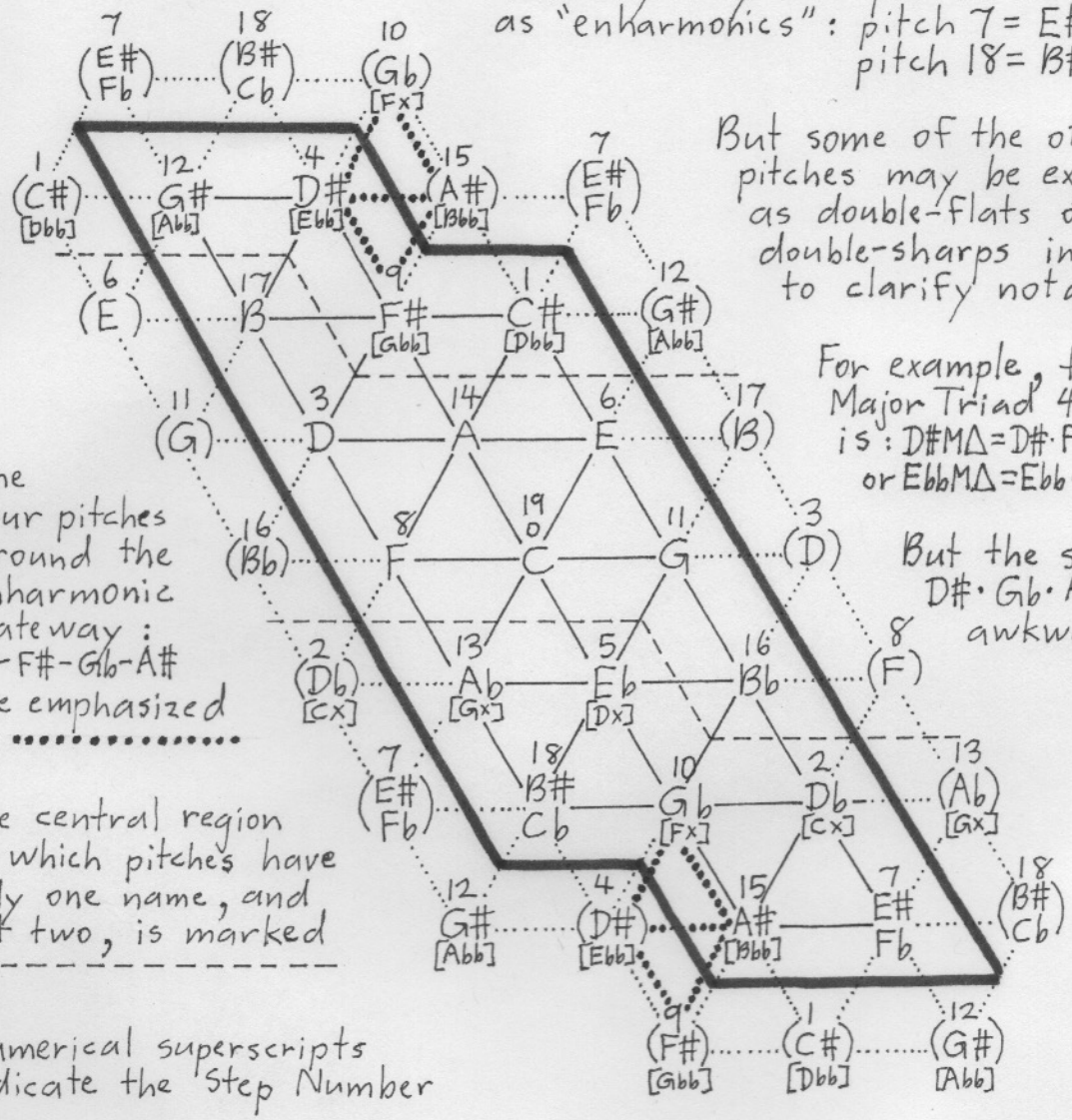
Figure 42: 19-E.T. Pitch Field:

Generally only two pitches are shown as "enharmonics": pitch 7 = E# · Fb
pitch 18 = B# · Cb

But some of the other pitches may be expressed as double-flats or double-sharps in order to clarify notation.

For example, the Major Triad 4-10-15 is: D#MΔ = D# · Fx · A#
or EbbMΔ = Ebb · Gb · Bbb

But the spelling D# · Gb · A# is awkward.



The four pitches around the Enharmonic Gateway: D#-F#-Gb-A# are emphasized by ••••••••••••••

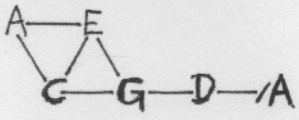
The central region in which pitches have only one name, and not two, is marked by - - - - - - - - - - - -

Numerical superscripts indicate the Step Number

Figure 43:

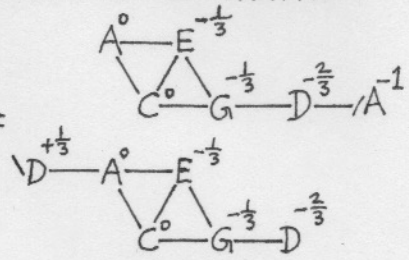
The logical source of the $\frac{1}{3}$ Comma Tempering of Fifths:

Just Intonation:



The tuning of three Just Fifths (3:2) from C results in /A (the dissonant, comma-raised form of the interval).

Meantone Intonation:



Therefore, flatten each Fifth by $\frac{1}{3}$ of a comma, so that /A becomes A. Then the tuning orients along the Major Sixth (5:3) Axis.

Figure 44: 19-E.T. Functional Field showing the **Six** Regions

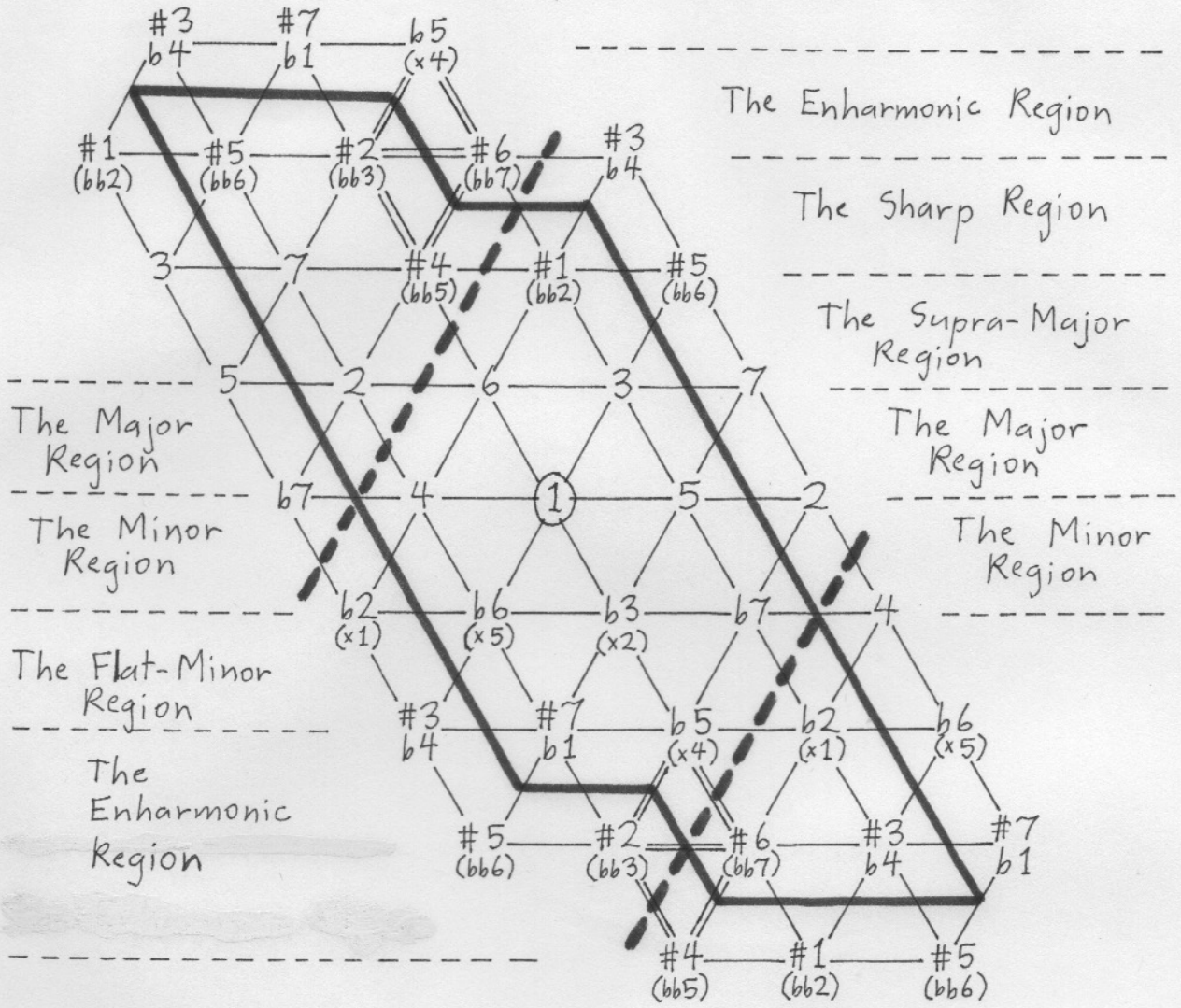


Figure 45: The Circle of 4ths-5ths showing the Regions as well as the Boundary Triads:

The triads are paired according to the Major-Relative Minor Relation.

Boundary triads are marked in asterisks.

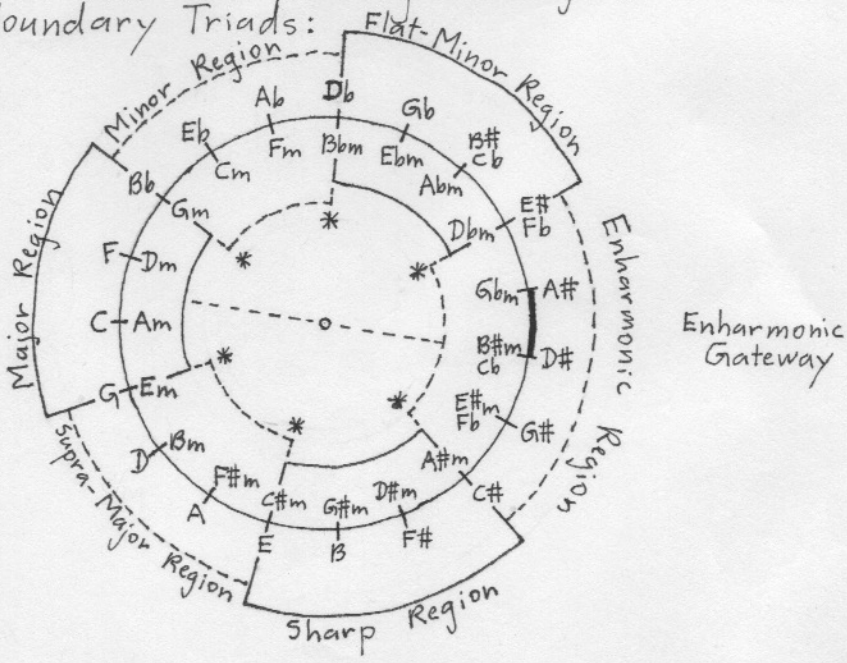
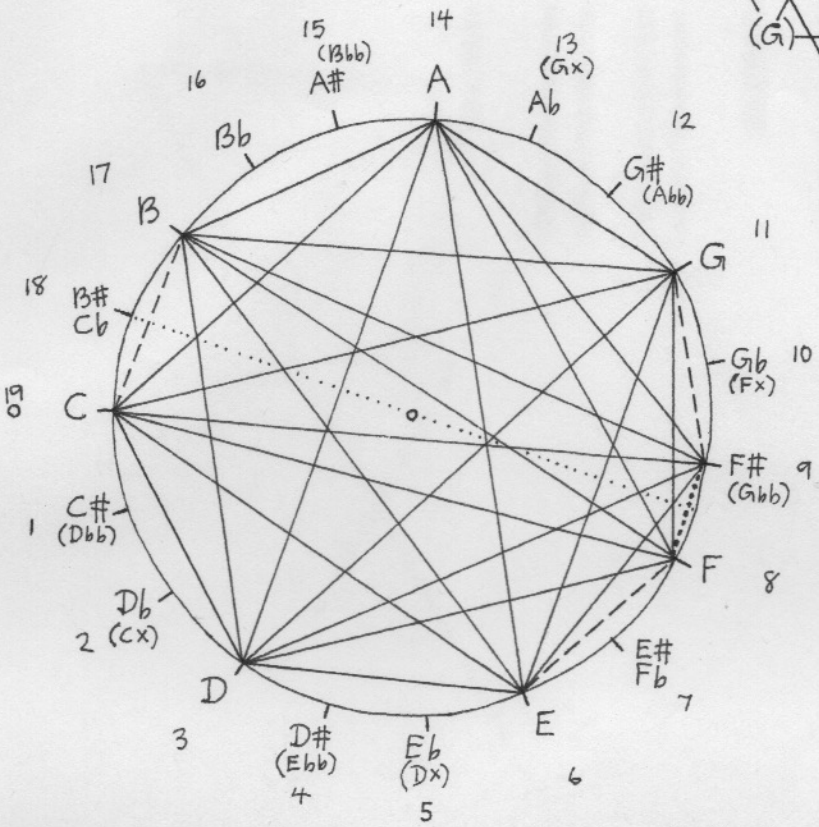
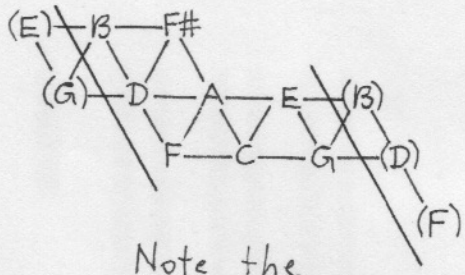


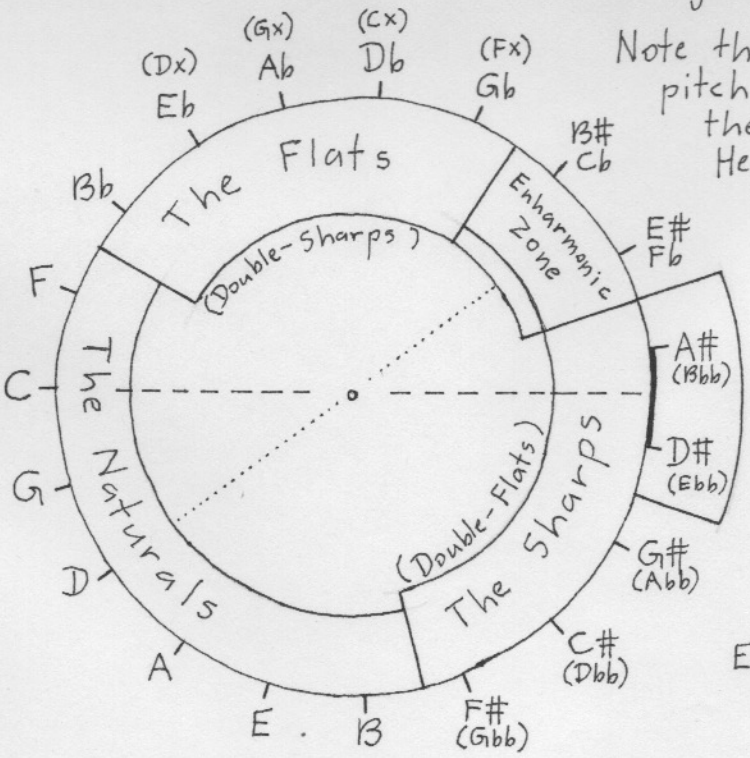
Figure 46: 19-E.T. Circular Graph showing the approximation of the Ptolemaic Ogdoad:





Note the sharp contrast in size between the Diatonic Semitone (eg. E–F or F#–G) and the Chromatic Semitone (eg. F–F#) The one is about twice as big as the other.

Figure 47: The 19-E.T. Circle of Fifths-Fourths showing the logic of the notation.

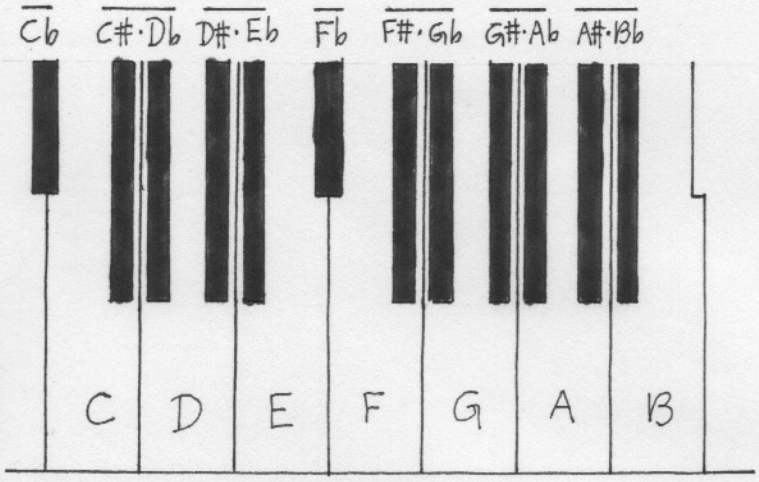


Note the symmetry around pitch "D" which positions the middle of the Diatonic Heptad (the Natural notes). Two tones have equal status as sharps and flats. Other sharps and flats have subsidiary notations as double-flats and double-sharps.

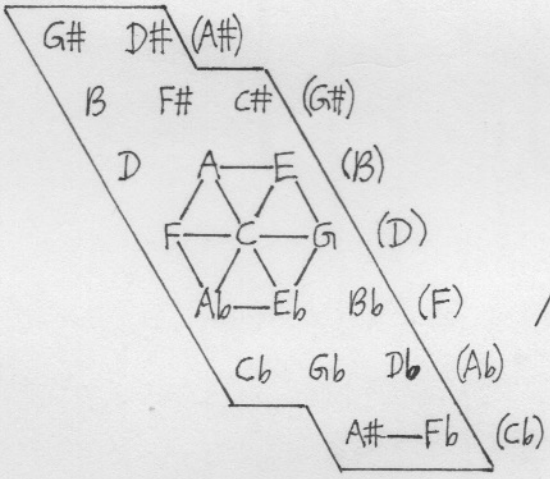
Note the "opposition" of the generator tone "C" to the two tones associated with the Enharmonic Gateway — ie. D# and A#.

Figure 48: Zarlino's 19 note cembalo :

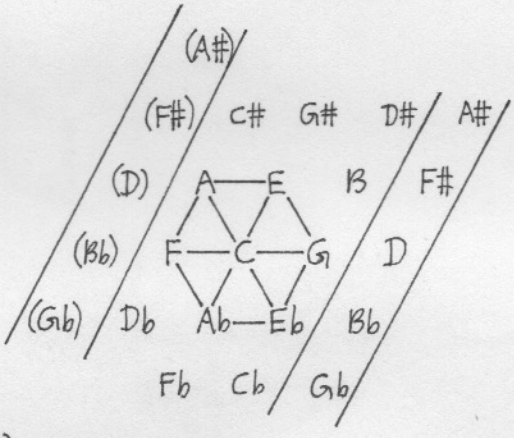
The keyboard was tuned in 1/3 Comma Meantone or in 1/4 Comma Meantone, as shown in the Field diagrams shown below, indicating the pitch names of the pattern:



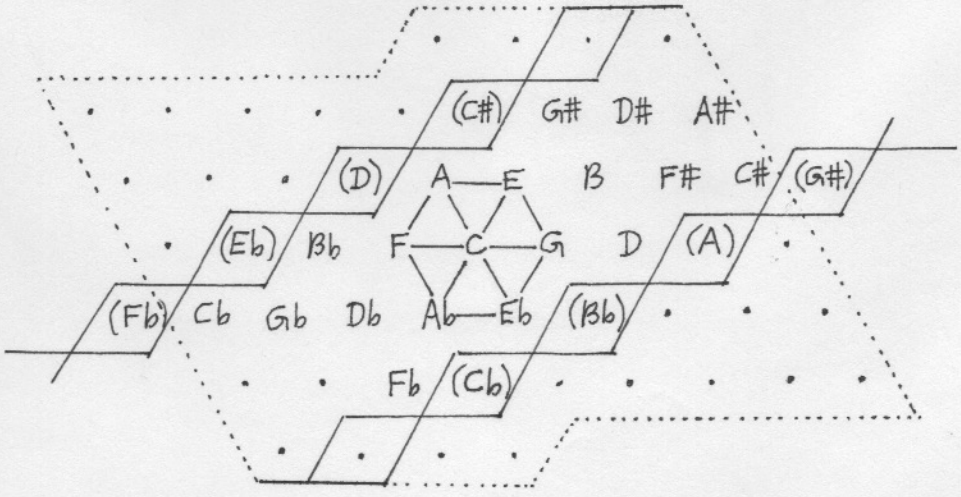
1/3 Comma Meantone (19-E.T.)



1/4 Comma Meantone (31-E.T.)



The keyboard could also have been tuned in a 19-tone segment of <u>any</u> of the other Meantone Temperaments. Below is shown the Field pattern for 1/5 Comma Meantone (43-E.T.)



43-E.T. (Quasi-One-Fifth Comma Meantone) Field:

showing deviation from Pure Just Intonation in fractional parts of a Comma

Figure 49:

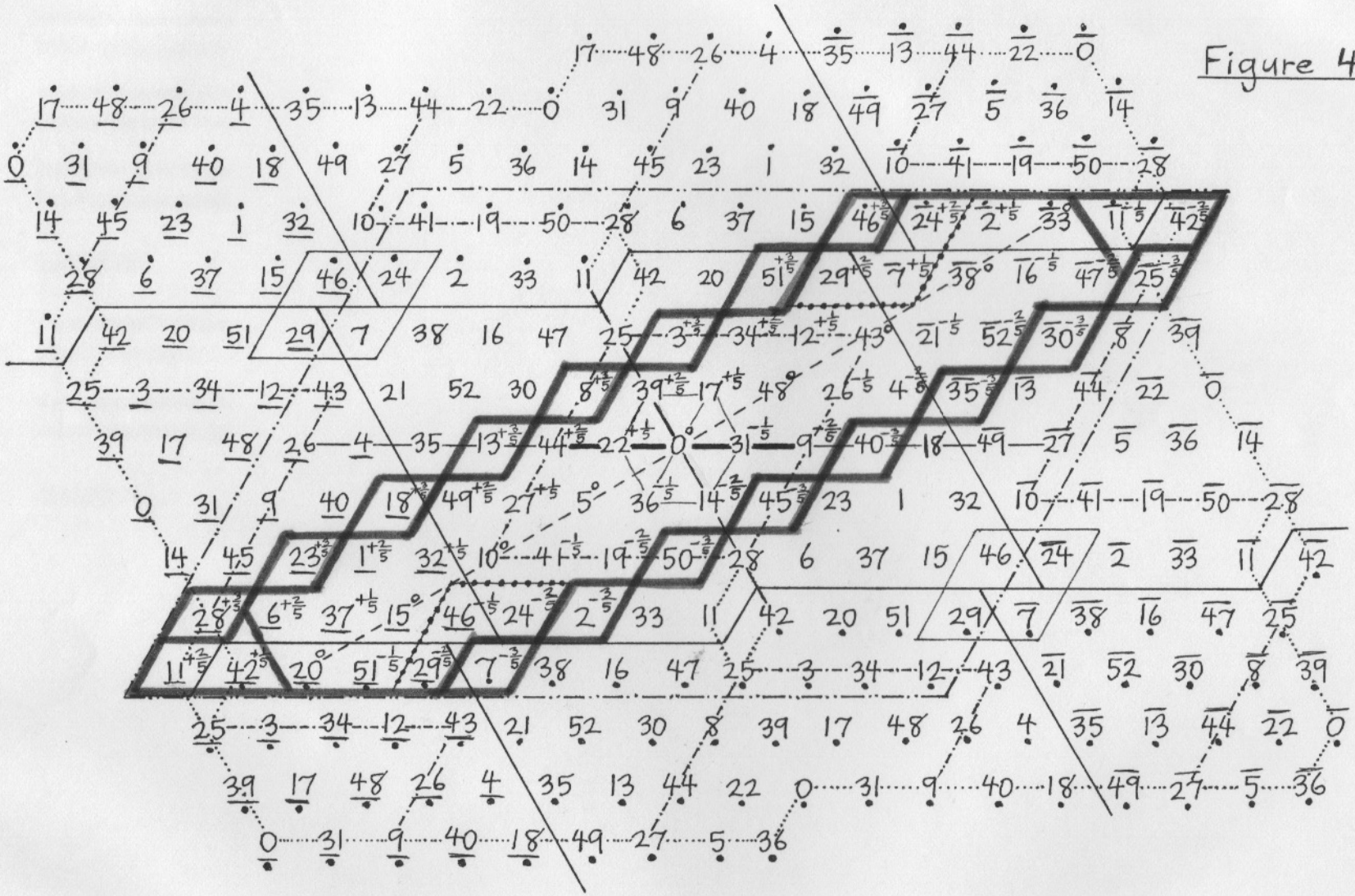


Figure 50: 43-E.T. Pitch Field :

The step number is shown above the pitch name.

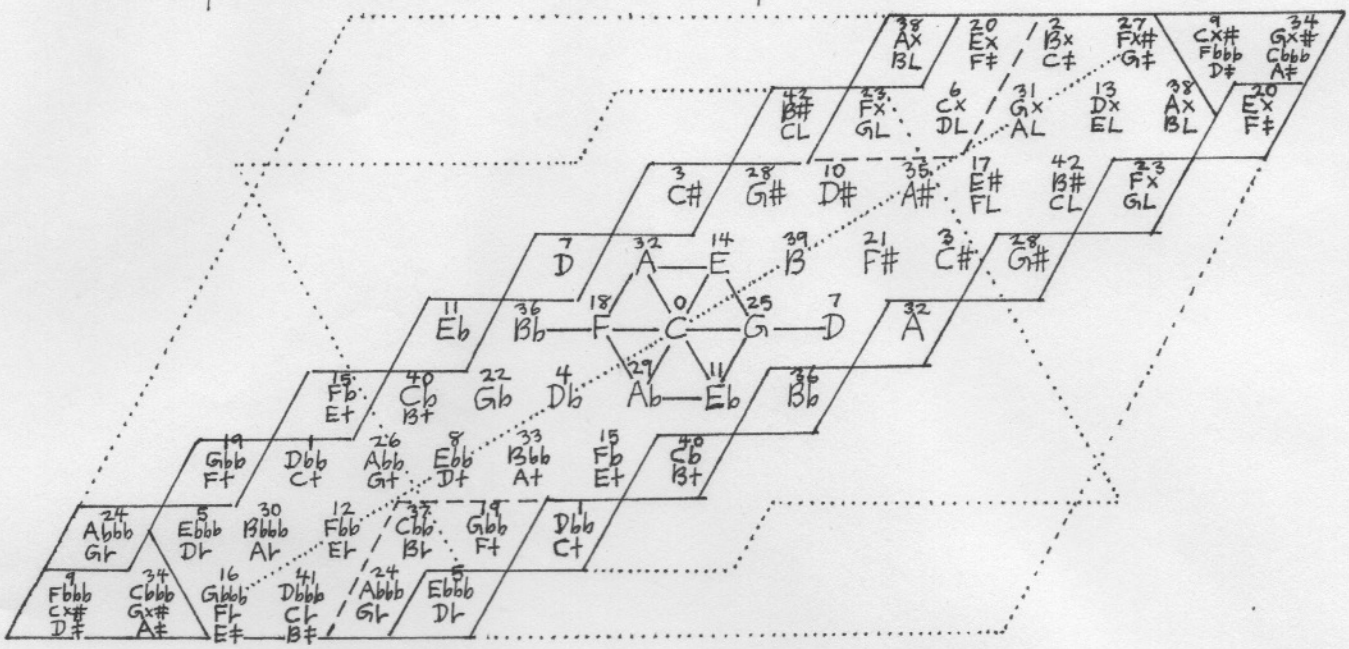
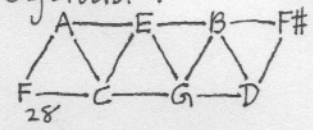


Figure 51: Circular Graph showing pattern of the Ptolemaic Ogdoad :



The smallest step is an "oversized comma" of about 27.9 Cents.

Note the difference in size between the Diatonic Semitone (eg. E-F) and the Chromatic Semitone (eg. F-F♯) They are approaching the same size, the difference being only one step.

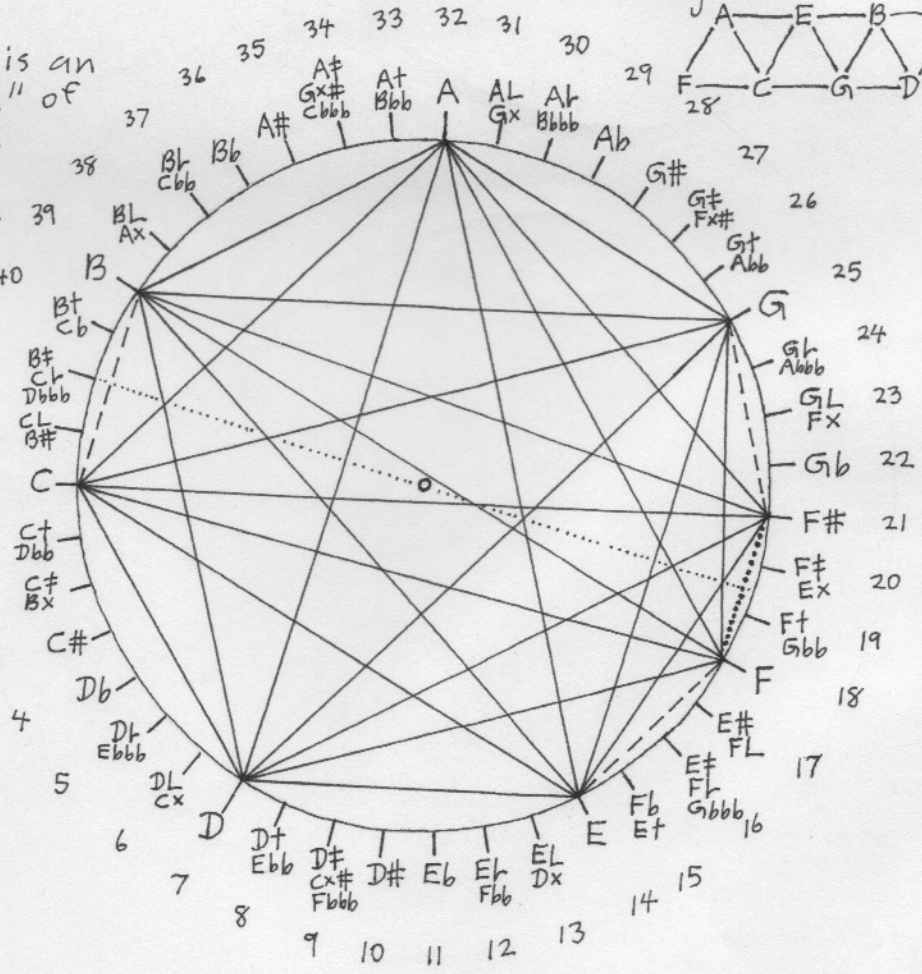


Figure 52: 43-E.T. Functional Field, showing the Eight Regions:

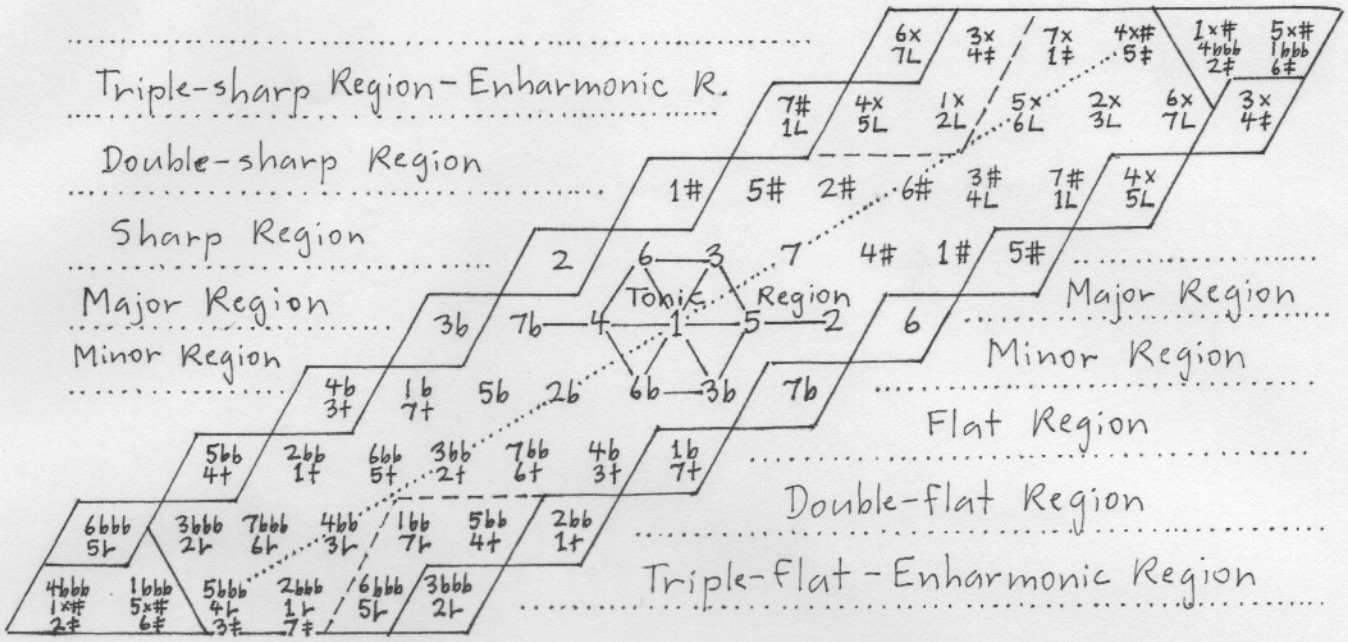


Figure 53: The Regions shown on the Circle of Fifths (Function Notation) and (Step number)

Only the Major Boundary Triads are marked. Their relative minors are understood. For example, as in 31-E.T., the Functions b7-2-4 form the Major Triad which borders the Tonic Region (in 31-E.T. the Major Region) and the Minor Region. Its relative Minor, the minor dominant 5-b7-2 also borders the same two regions. Note the extreme positions of the two Enharmonic Region Triads at the "ends" of the Field.

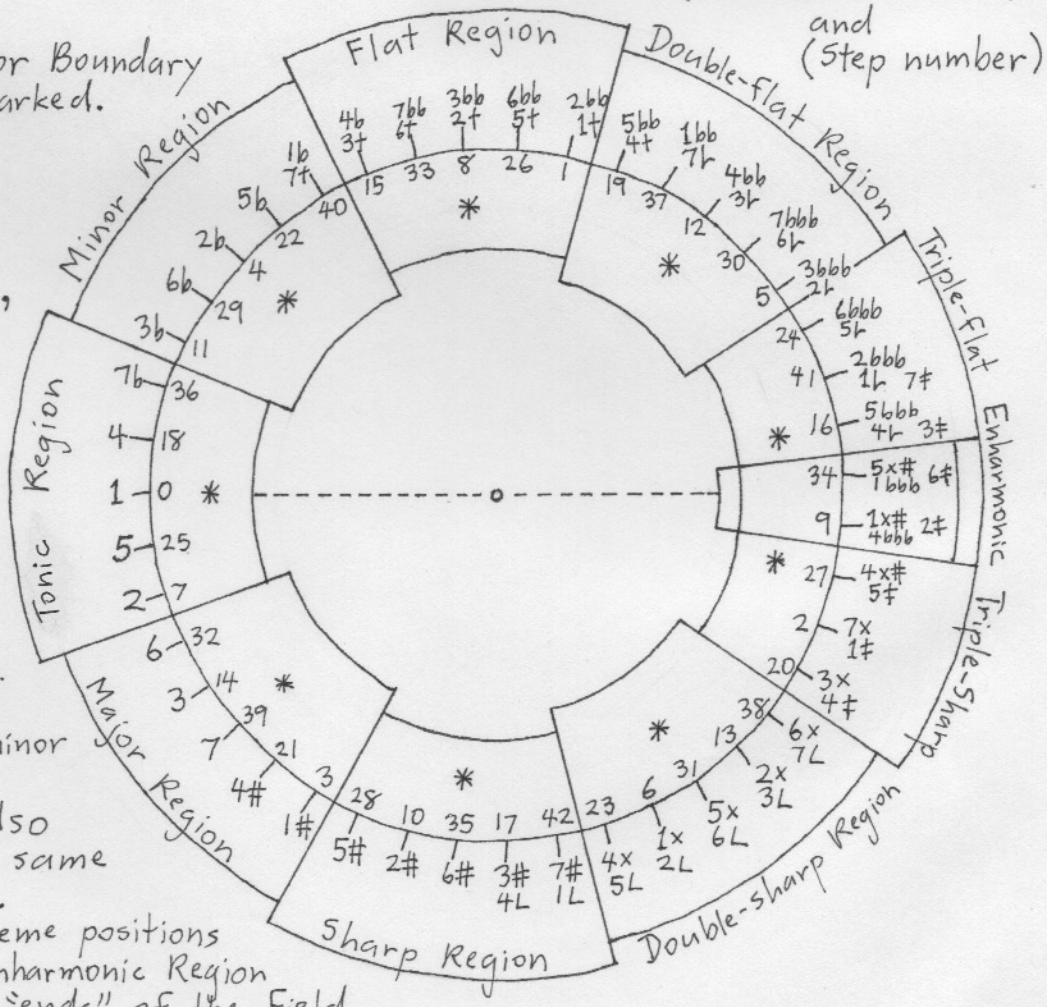
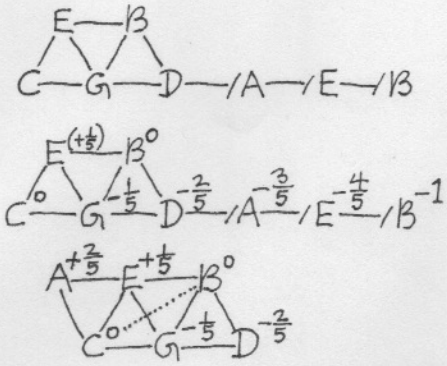


Figure 54: The logical source of the proposal to flatten each Fifth by 1/5 Comma:



The tuning of five pure Fifths results in a Major Seventh which is dissonant and sharp by a comma.

Therefore, flatten each fifth by 1/5 Comma so that /B becomes B. As a result /E is flattened 4/5 Comma, which is the same as E raised by 1/5 Comma.

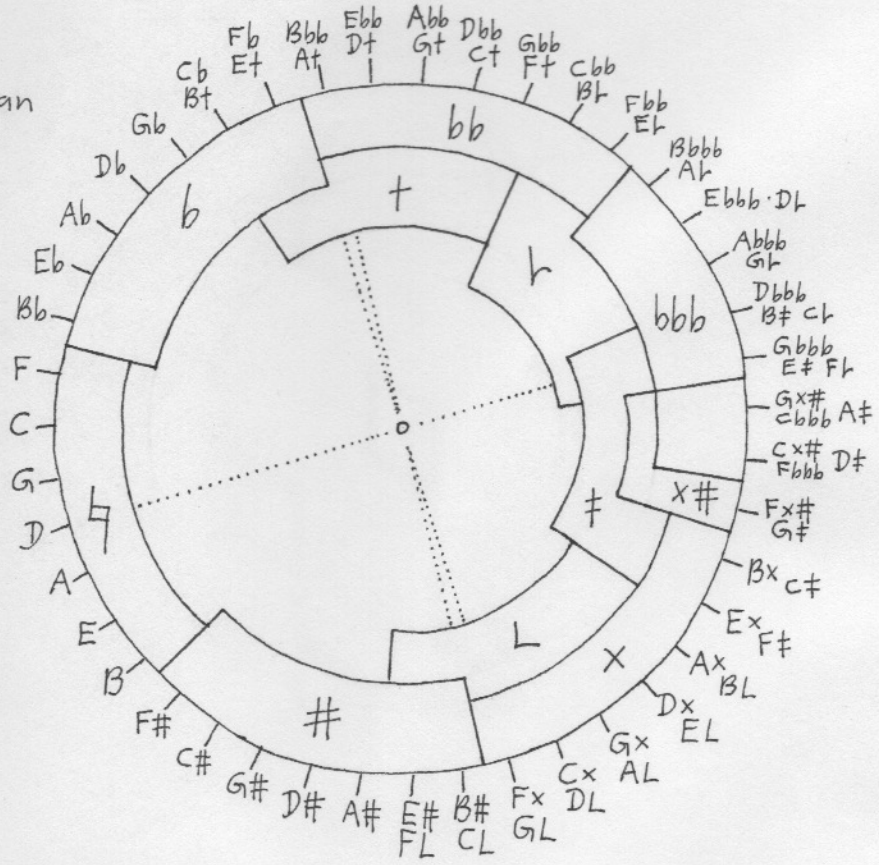
The resulting tuning orients along an axis running through C and B. Other pitches are raised or lowered by either 1/5th Comma (about 4.3 Cents)
2/5 Comma (about 8.6 Cents)

Figure 55: 43-E.T. Circle of Fifths-Fourths showing the logic of the notation

The notation, of course, must be more complex than that of 12, 19, or 31-E.T.
The notation is more complex than that for 53-E.T. in the respect that more new signs are needed.

Signs of sharpening: ♮, †, ‡, #

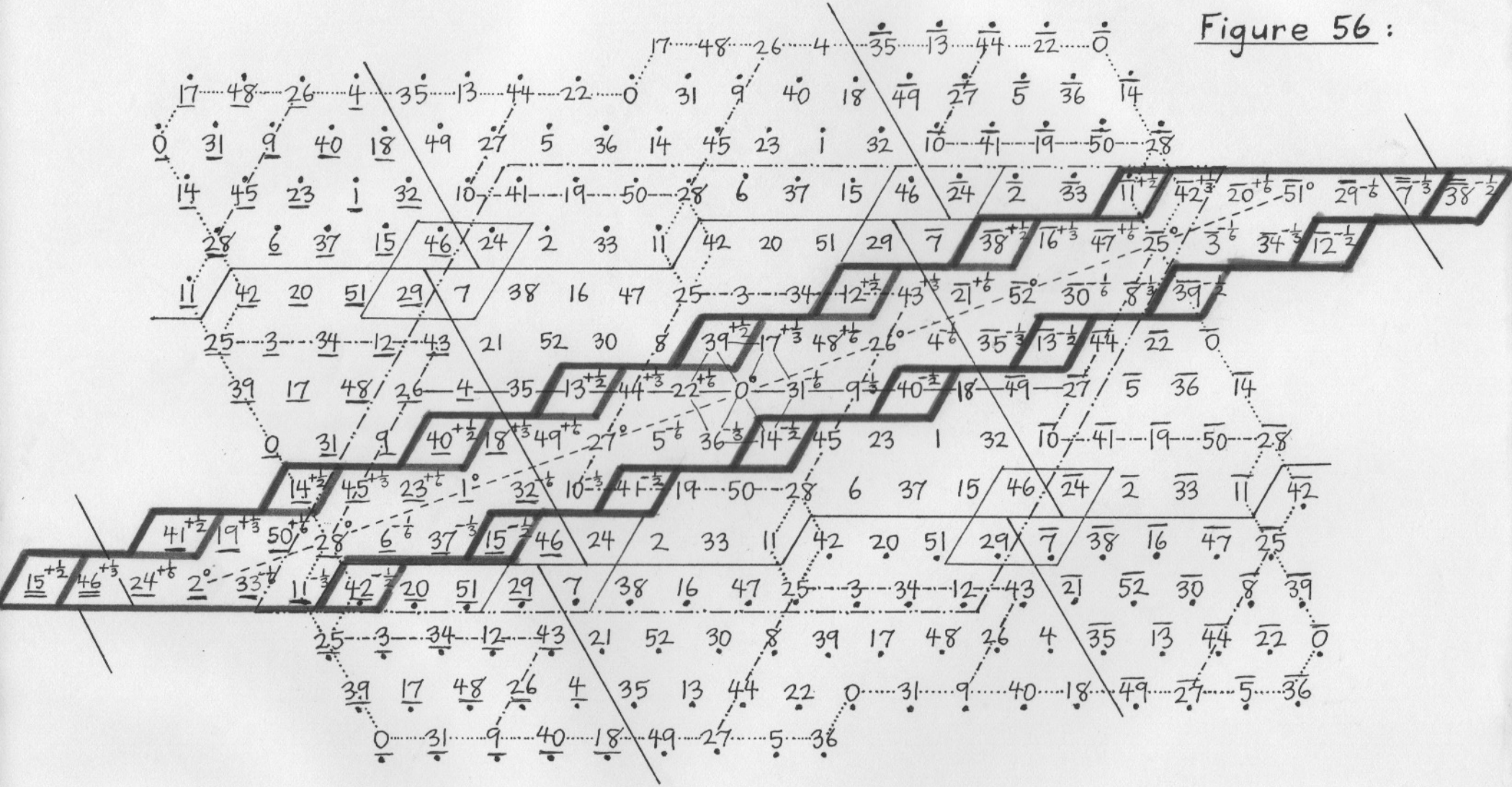
Signs of flattening: ♮, L, ♭, b



55-E.T. (Quasi-One-Sixth Comma Meantone) Field :

showing deviation (in fractional parts of a Comma) from Just Intonation :

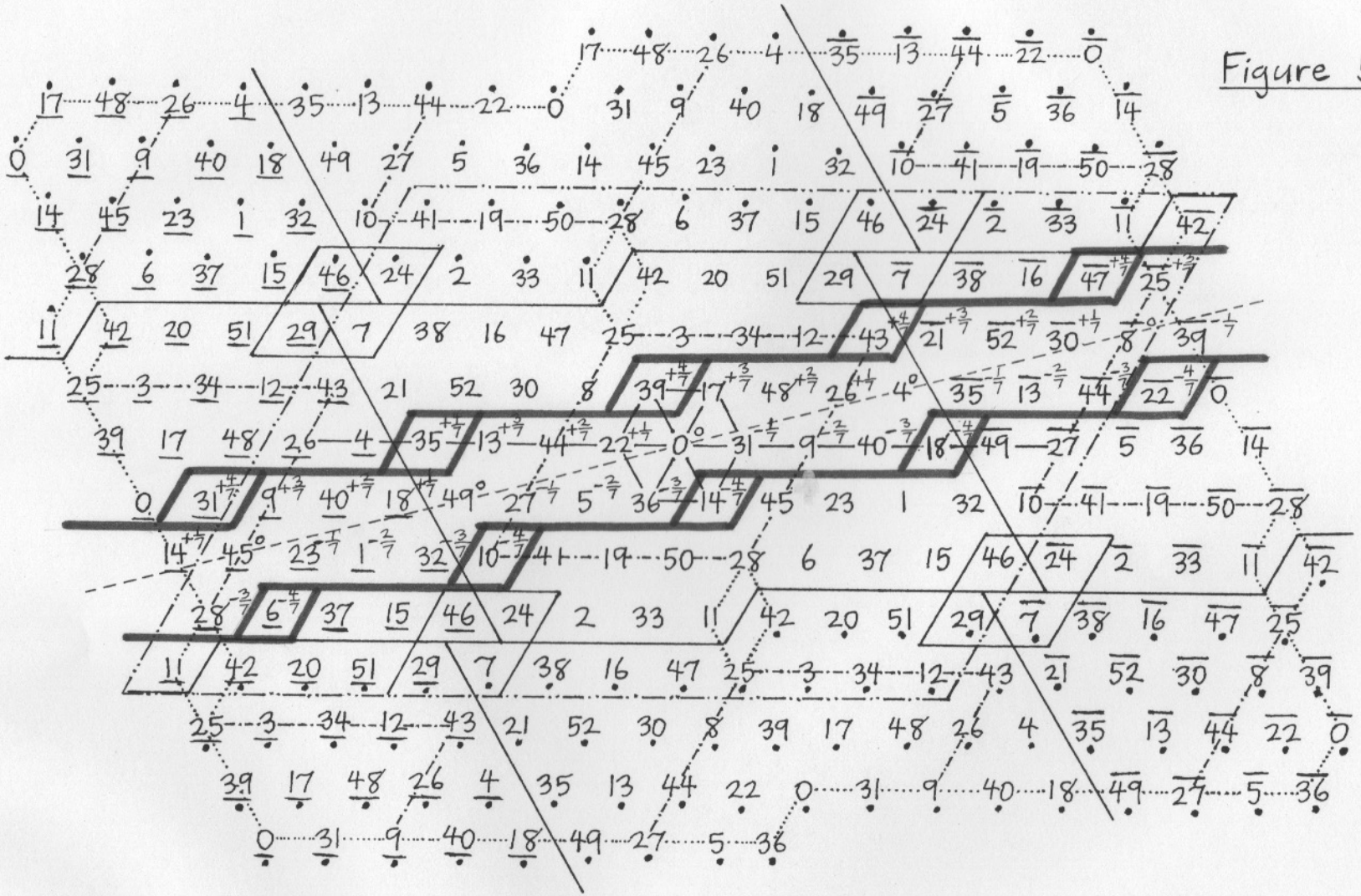
Figure 56 :



91-E.T. (Quasi-One-Seventh Comma Meantone)

showing deviation from Pure Just Intonation in fractional parts of a Comma

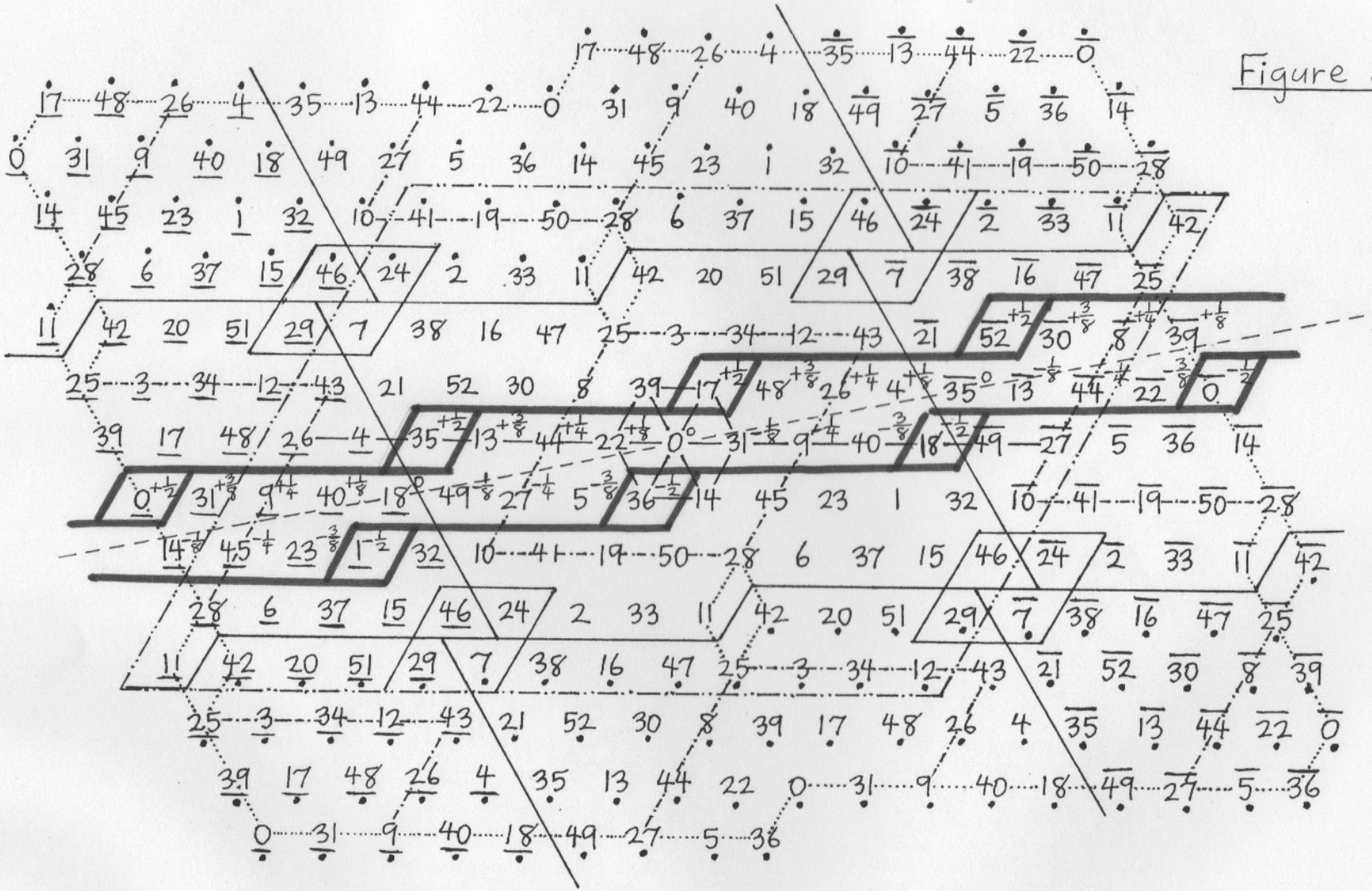
Figure 57:



139-E.T. (Quasi-One-Eighth Comma Meantone)

Showing deviation in fractional parts of a Comma from Just Intonation

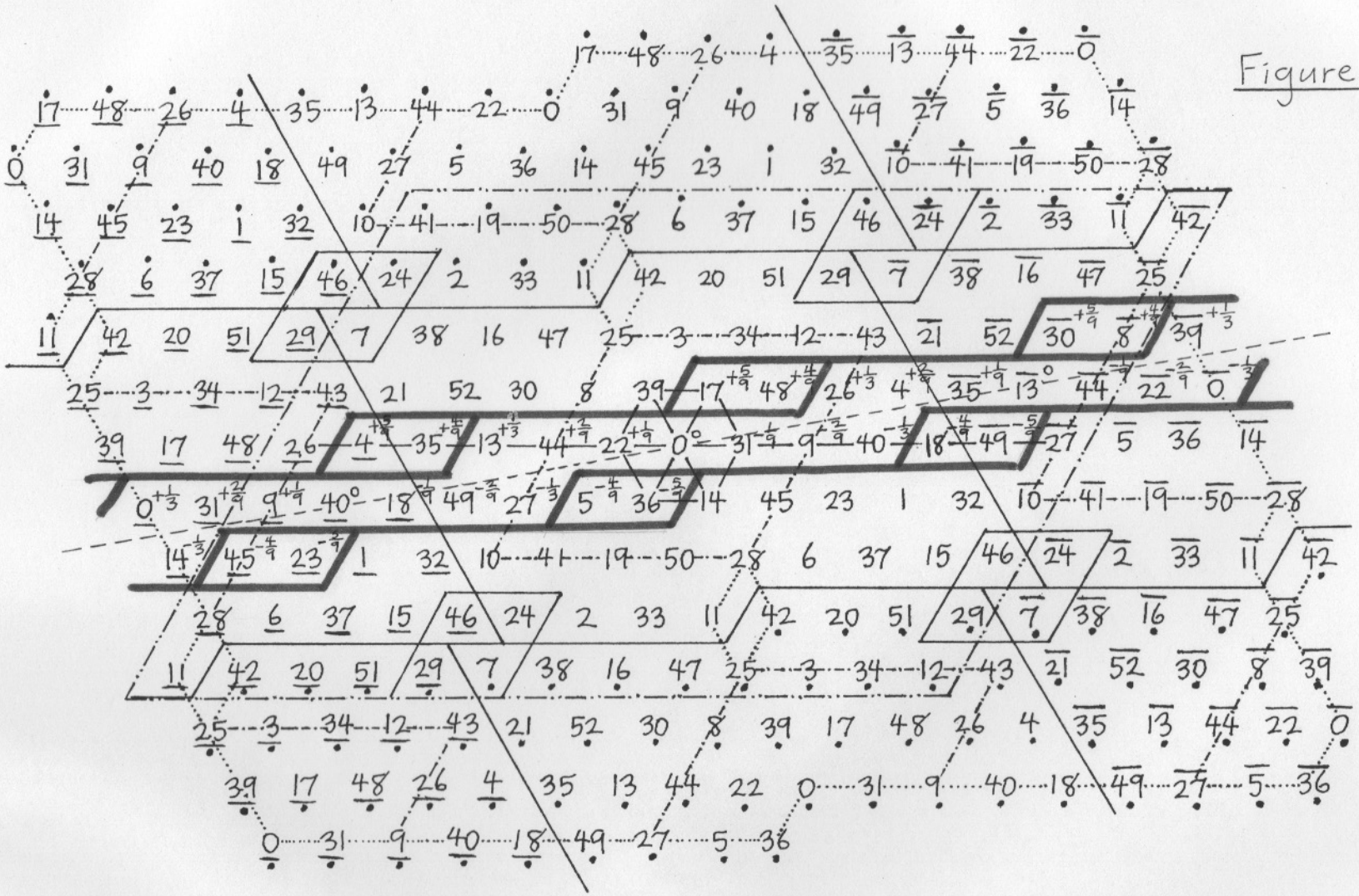
Figure 58:



247-E.T. (Quasi-One-Ninth Comma Meantone)

showing deviation from Just Intonation in fractional parts of a Comma

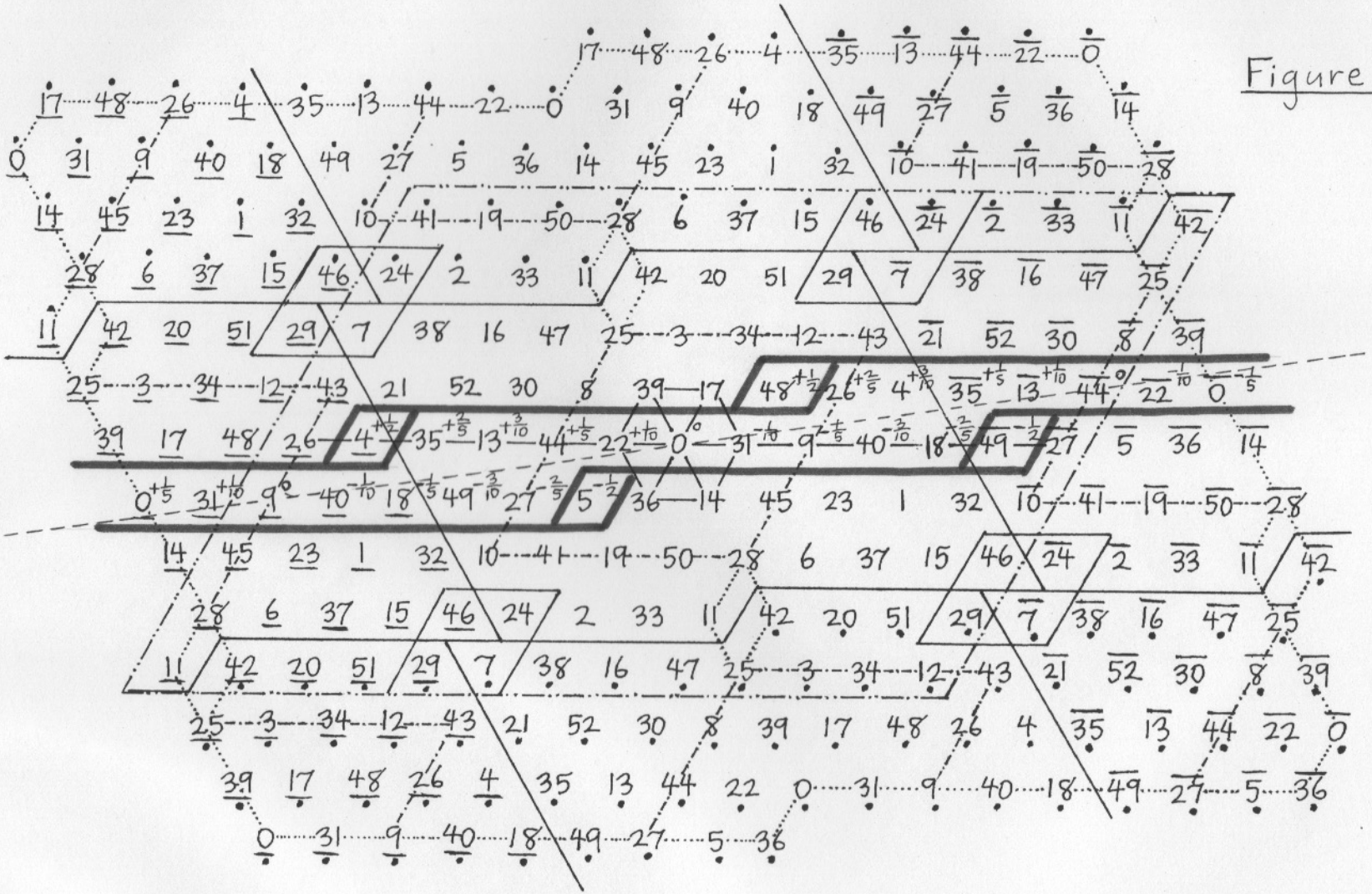
Figure 59:



499-E.T. (Quasi-One-Tenth Comma Meantone)

showing deviation from Just Intonation in fractional parts of a Comma

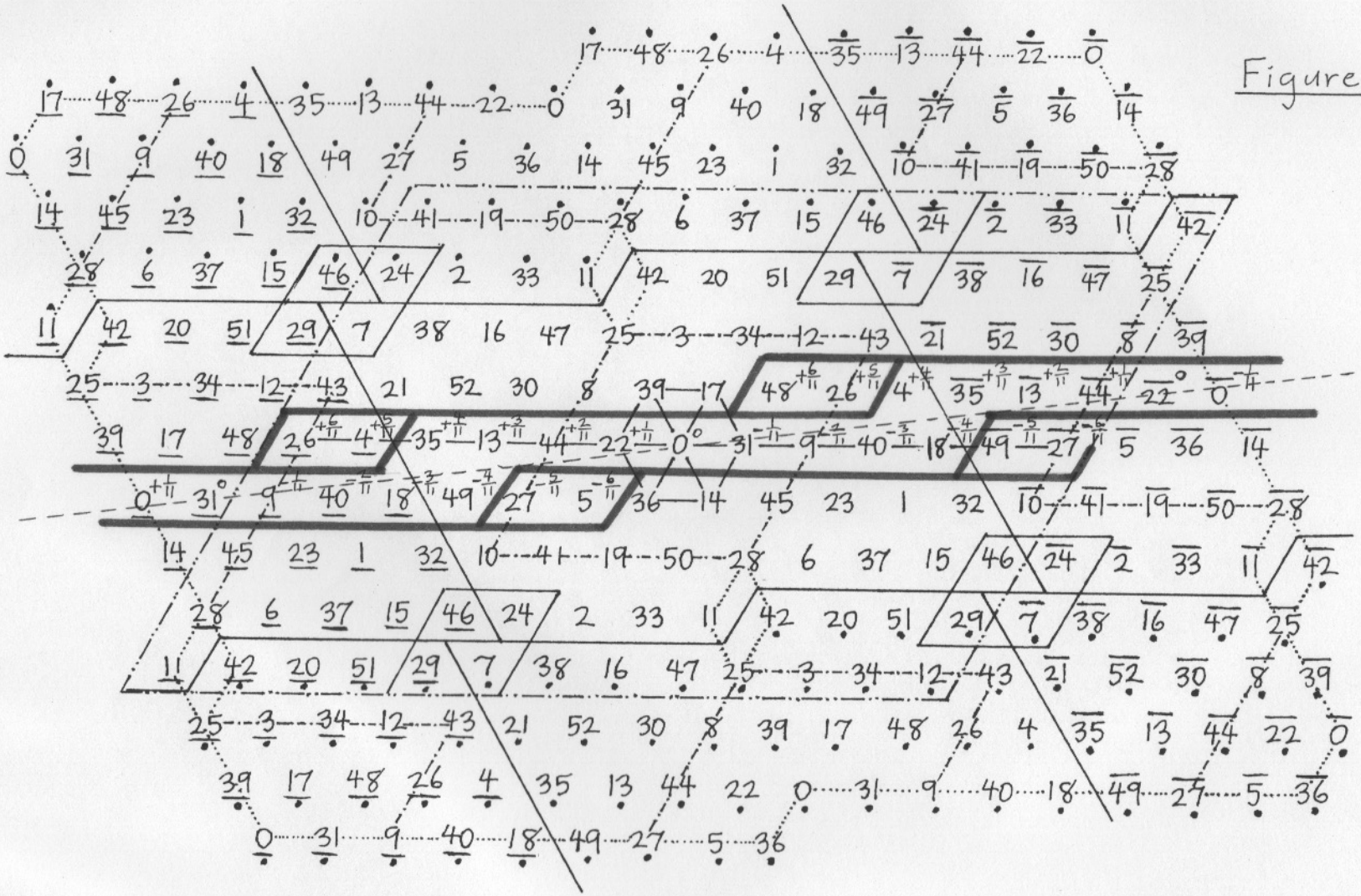
Figure 60:



559-E.T. (Quasi-One-Eleventh Comma Meantone)

Showing deviation from Just Intonation in fractional parts of a Comma

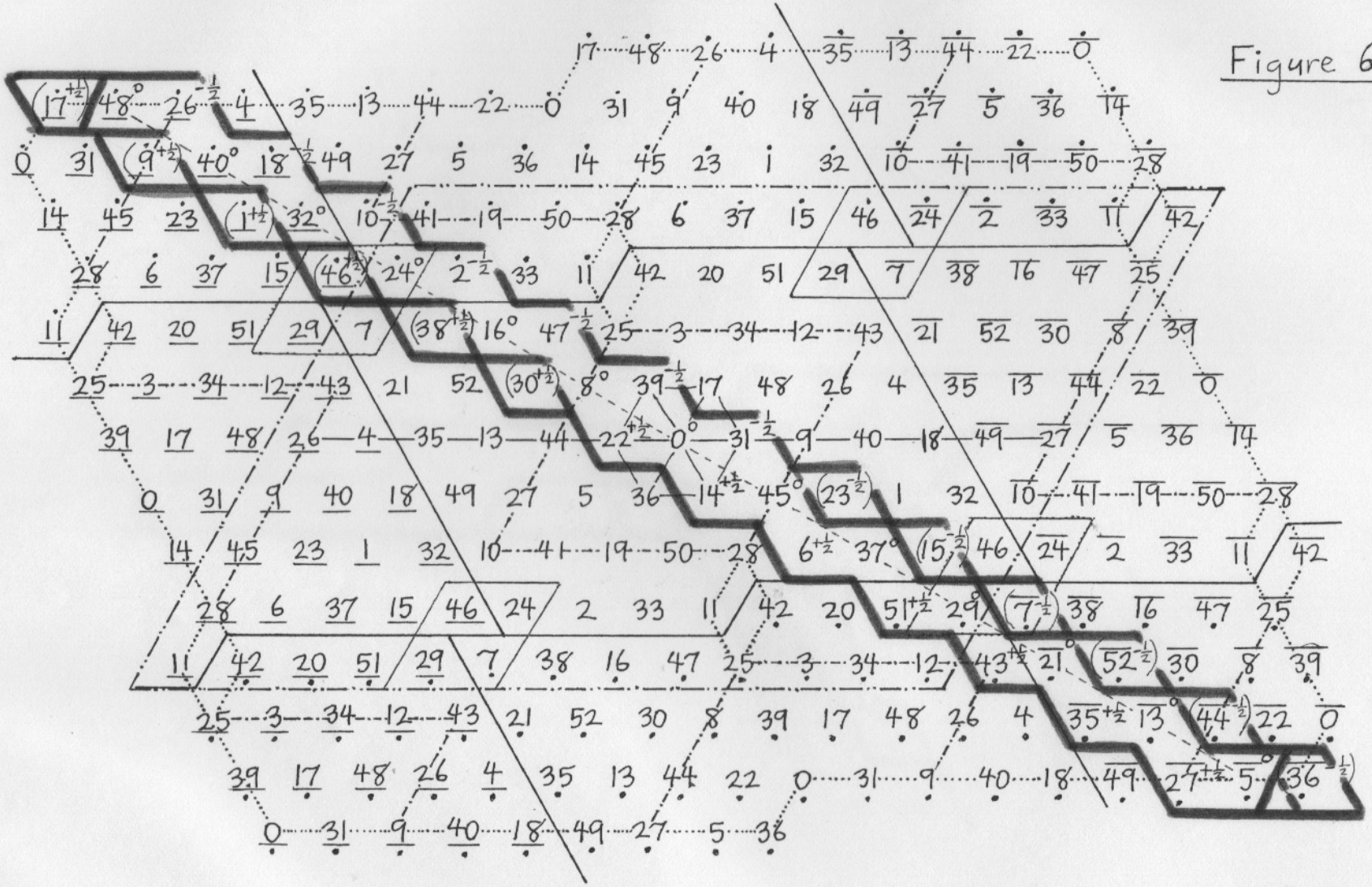
Figure 61:



26-E.T. (Quasi- 1/2 Comma Meantone)

Showing the deviation from Just Intonation in fractional parts of a Comma :

Figure 62 :



45-E.T. (Quasi-Two-Fifths Comma Meantone)

Showing deviation from Just Intonation in Fractional Parts of a Comma

45 = 26 + 19

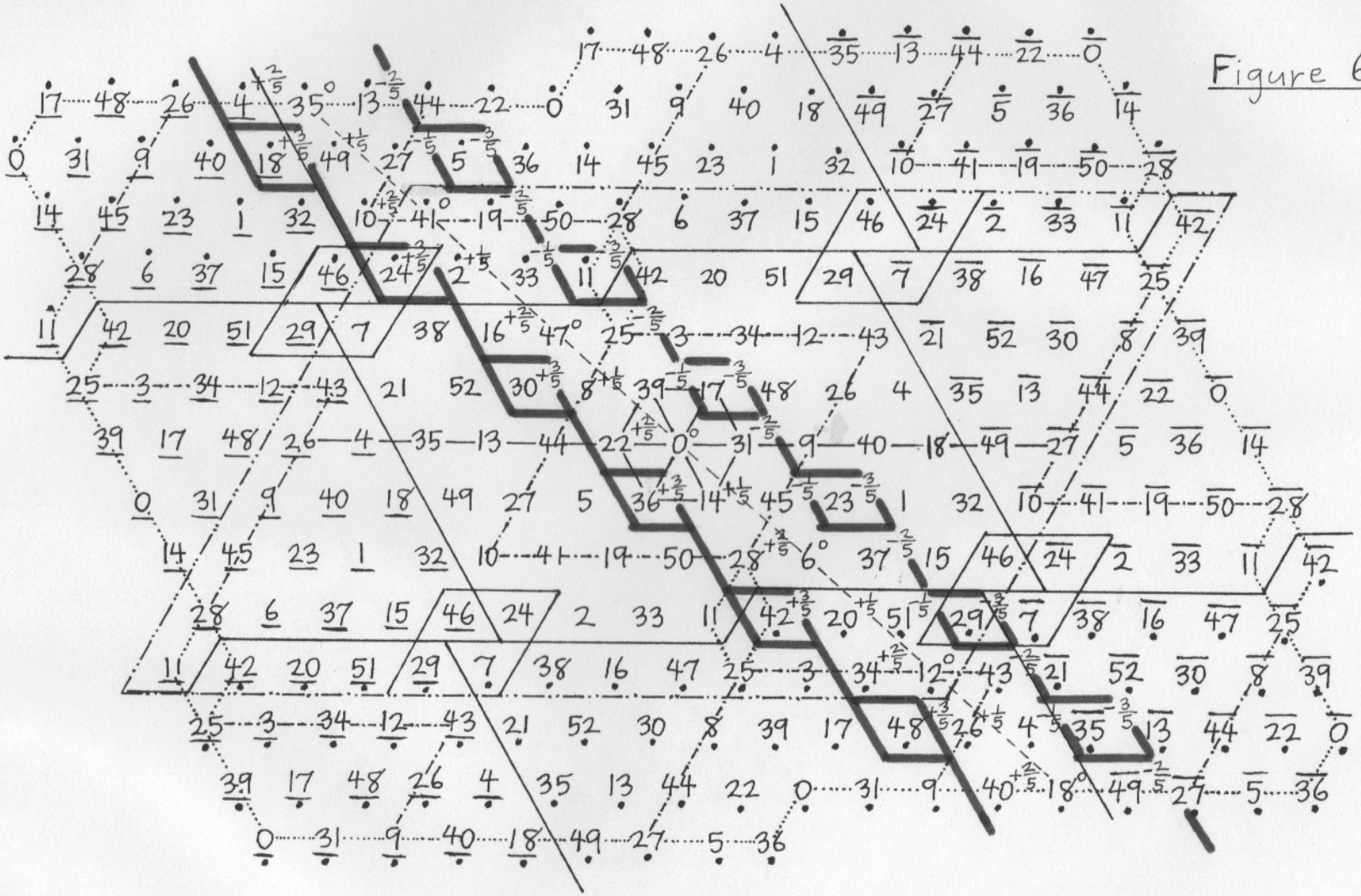


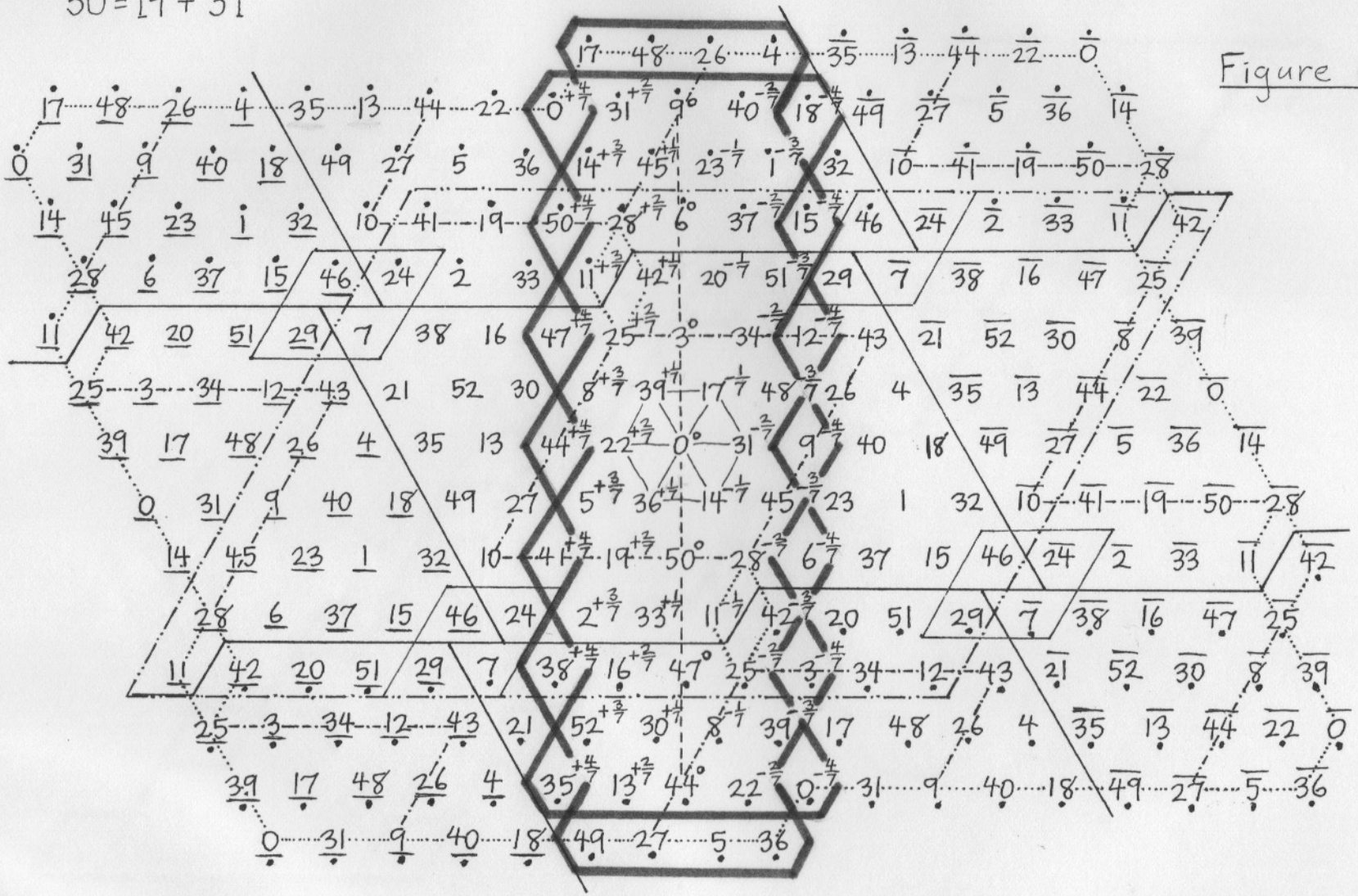
Figure 63:

50-E.T. (Quasi-Two-Sevenths Comma Meantone) Field:

showing deviation from Just Intonation expressed in fractional parts of a comma

50 = 19 + 31

Figure 64:



74-E.T. (Quasi-Two-Ninths Comma Meantone)

showing deviation from Just Intonation in fractional parts of a Comma

74 = 31 + 43

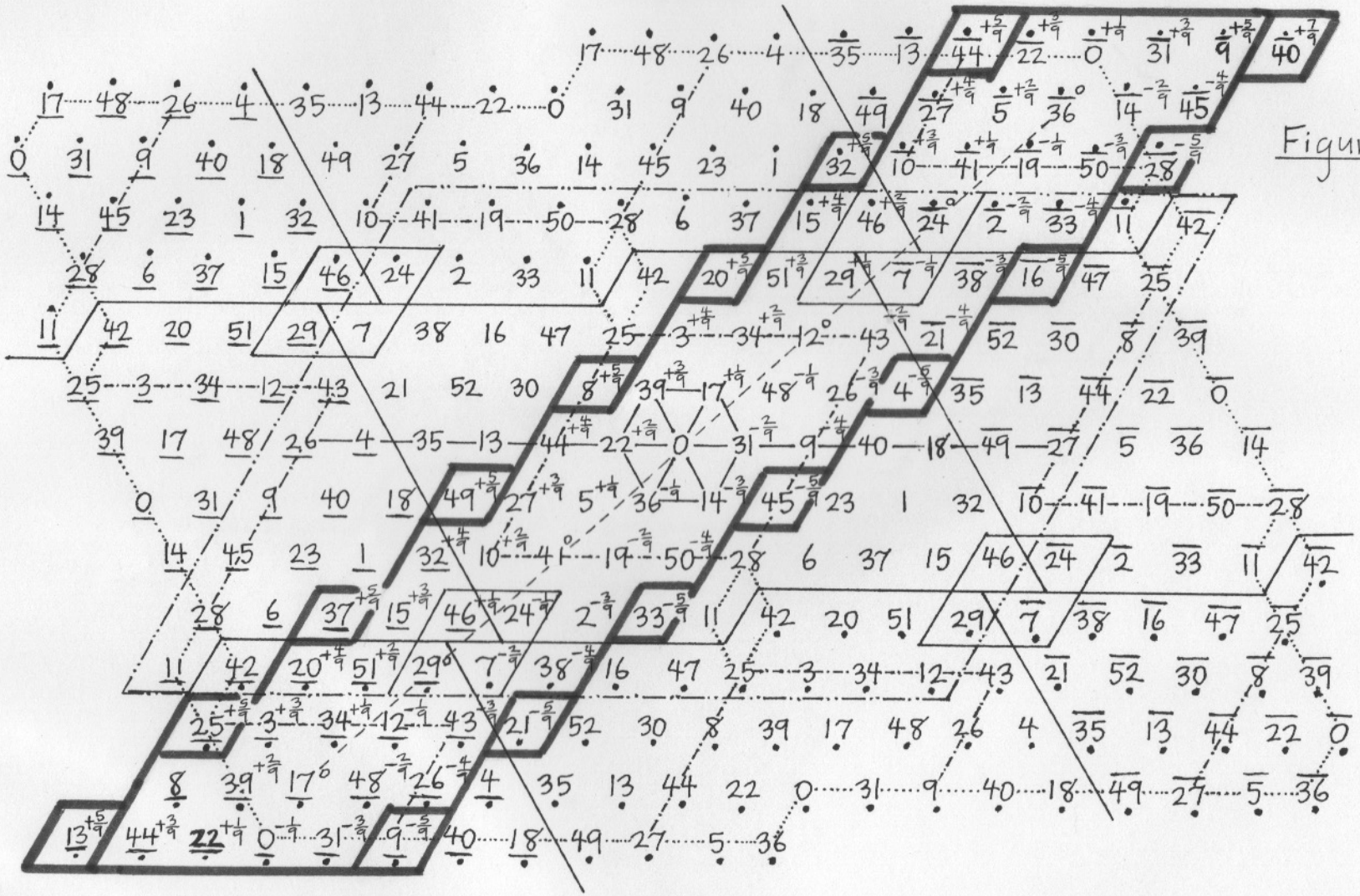


Figure 65:

98-E.T. (Quasi-Two-Elevenths Comma Meantone)

showing deviation from Just Intonation in fractional parts of a Comma

98 = 43 + 55

Figure 66:

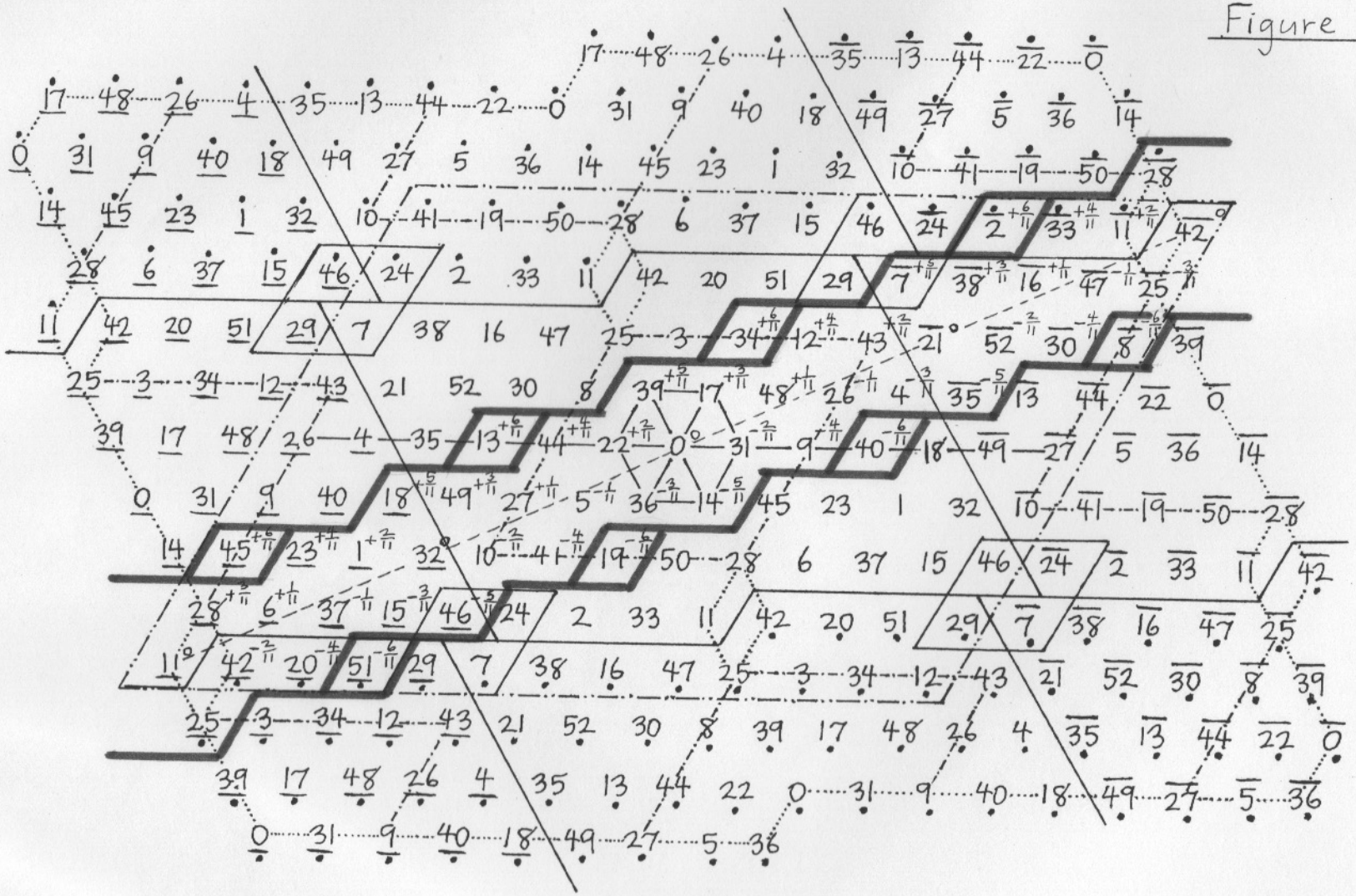
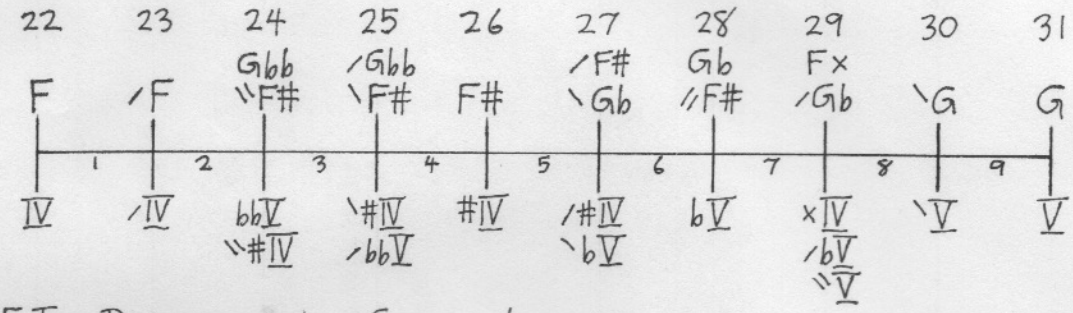


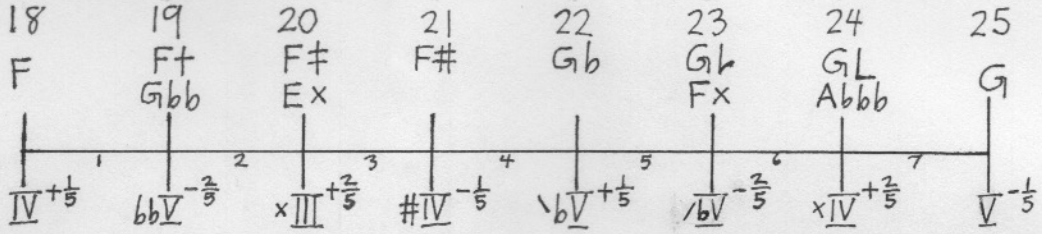
Figure 67:

The Division of the Wholestep between pitches F and G (Function IV and V) displayed logarithmically in fractional parts of a Comma, for the principal cyclical temperaments 12, 19, 31, 43, 53.

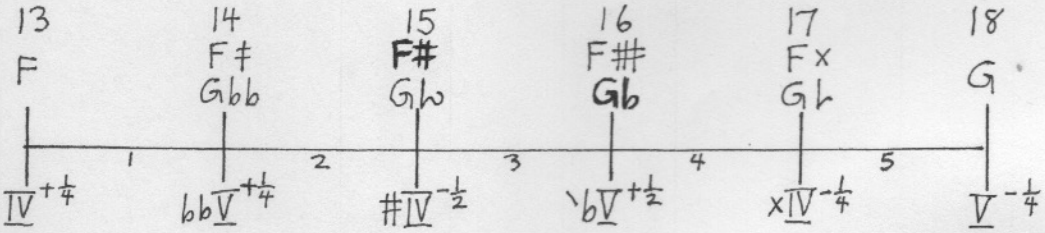
53-E.T. – Division into Nine steps (Commas)



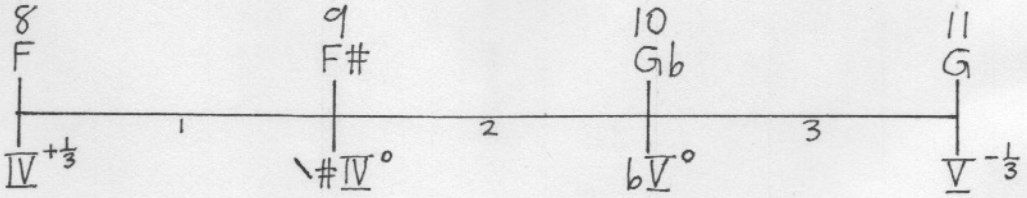
43-E.T. – Division into Seven steps



31-E.T. – Division into Five Steps (Diesis)



19-E.T. – Division into Three Steps (Chromatic Semitones)



12-E.T. – Division into Two Steps (Equal Semitones)

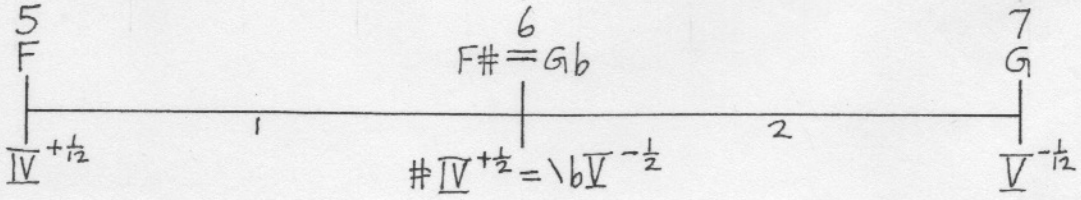


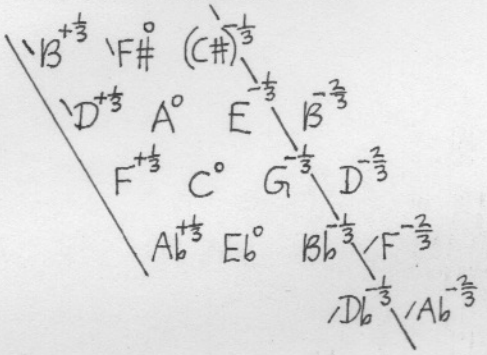
Figure 68: The size of the "Wolf" Fifth in the principal Cyclical Temperaments:

The "Wolf" Fifth occurs at the end of a chromatic line of Fifths when the beginning of the line must be substituted for the next natural Fifth. I shall use as the example the line of Fifths:

Db—Ab—Eb—Bb—F—C—G—D—A—E—B—F#···(C#)

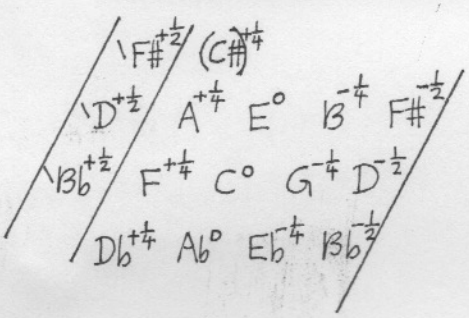
Since the line of Fifths is not allowed to continue beyond F#, the "Fifth" from F# to Db is substituted – a "Wolf" Fifth which is in reality a Diminished Sixth. This dissonance does not occur in 12-E.T., but is a feature of the other cyclical temperaments.

$\frac{1}{3}$ Comma (19-E.T.)



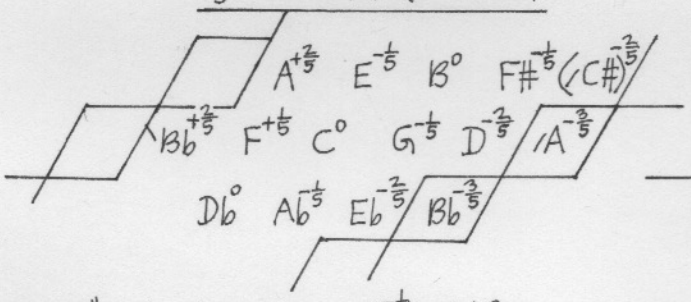
"Wolf" Fifth $F\#^{0}-Db^{-\frac{1}{3}}$ is about $2\frac{1}{3}$ commas sharp, ie. 56 Cents

$\frac{1}{4}$ Comma (31-E.T.)



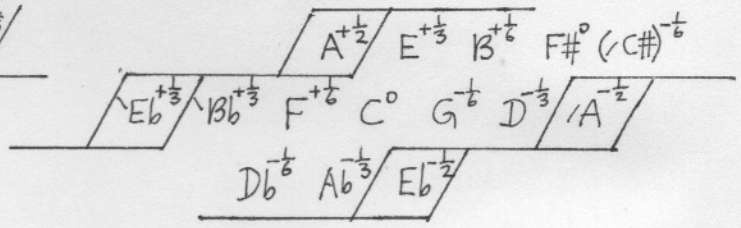
"Wolf" Fifth $F\#^{-\frac{1}{2}}-Db^{+\frac{1}{4}}$ is about 2 commas (a Diesis) sharp, ie. 33.58 Cents

$\frac{1}{5}$ Comma (43-E.T.)



"Wolf" Fifth $F\#^{-\frac{1}{5}}-Db^{0}$ is about $1\frac{1}{5}$ Commas sharp ie. about 23.67 Cents

$\frac{1}{6}$ Comma (55-E.T.)



"Wolf" Fifth $F\#^{0}-Db^{-\frac{1}{6}}$ is about $\frac{5}{6}$ Comma sharp – ie. about 17.92 Cents

It can be seen that the size of the "Wolf" Fifth diminishes from a maximum in $\frac{1}{3}$ Comma, through $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ etc to practically nothing in $\frac{1}{11}$ Comma – dissappearing in $\frac{1}{12}$th Comma (12-E.T.).

Figure 69: Interval Classes of the Major Cyclical Temperaments (Cycle of Fifths order):

12-E.T.		Step#
① Unison-Octave	0	12
② Fifth-Fourth	7	5
③ 2nd-Minor 7th	2	10
④ M6th-m3rd	9	3
⑤ M3rd-m6th	4	8
⑥ M7th-m2nd	11	1
⑦ Tritone	6	6

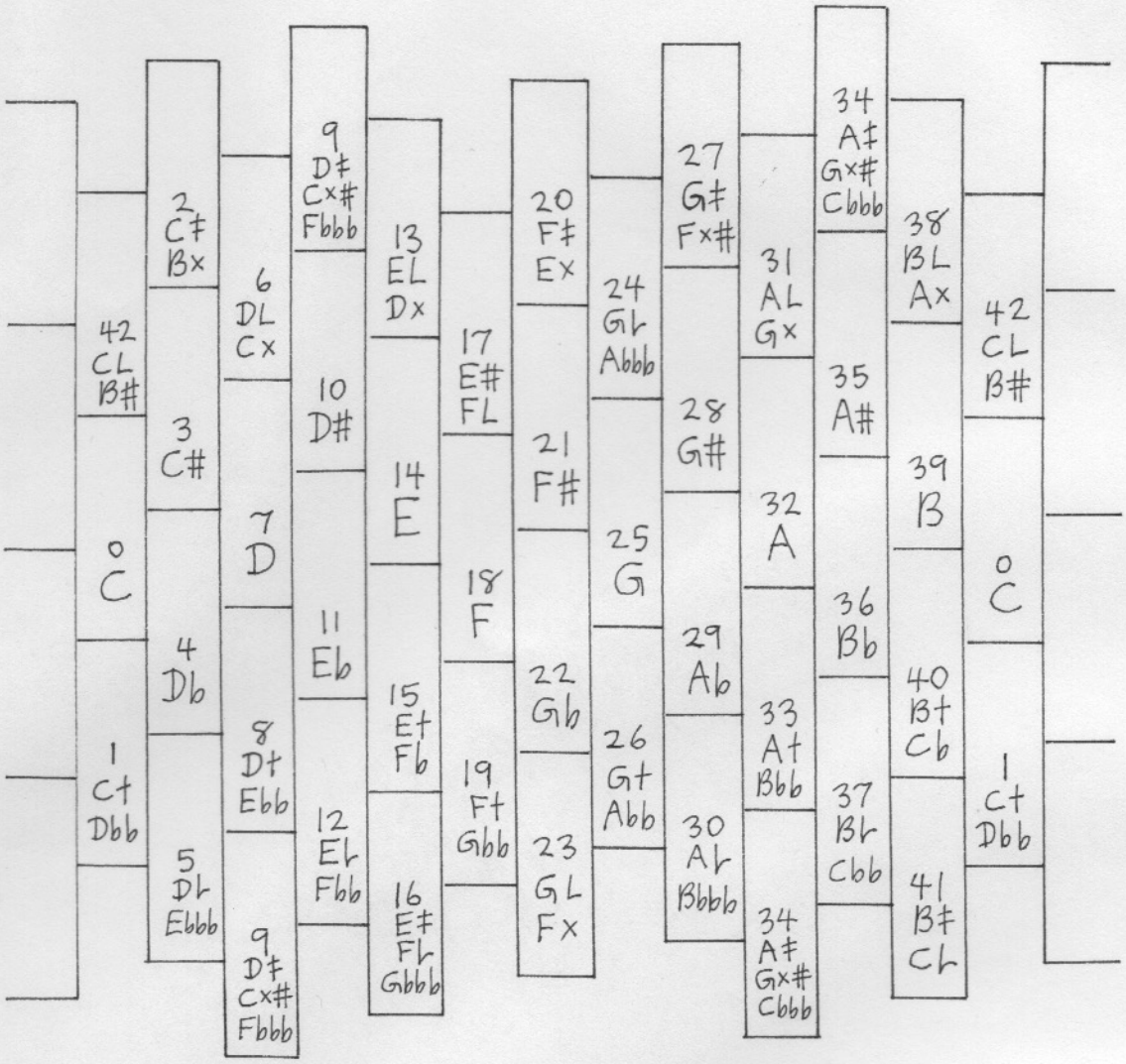
19-E.T.		Step#
① 1:1-2:1	0	19
② 3:2-4:3	11	8
③ "Just" form	3	16
④ " "	14	5
⑤ " "	6	13
⑥ "Stretched Diatonic"	17	2
⑦ Sharp 4th-Flat 5th	9	10
⑧ Chrom. m2-M7	1	18
⑨ Diss. #5th-b4th	12	7
⑩ Enharmonic G.	4	15

31-E.T.		Step#
① 1:1-2:1	0	31
② 3:2-4:3	18	13
③ Just M2-m7	5	26
④ Just M6-m3	23	8
⑤ Just M3-m6	10	21
⑥ Large "Diat." m2-M7	3	28
⑦ #4th-b5th	15	16
⑧ Small m2-M7	2	29
⑨ #5th-b4th	20	11
⑩ #2nd(7:6)-bb7th(12:7)	7	24
⑪ #6th(7:4)-bb3rd(8:7)	25	6
⑫ Wolf 4th-5th	12	19
⑬ Diesis	30	1
⑭ x4th-bb5th	17	14
⑮ Neutral 2nd-7th	4	27
⑯ Neutral 3rd-6th	9	22

43-E.T.		Step#
① 1:1-2:1	0	43
② 3:2-4:3	25	18
③ Just M2-m7	7	36
④ Just M6-m3	32	11
⑤ Just M3-m6	14	29
⑥ M7-Diatonic m2	39	4
⑦ #4th-b5th	21	22
⑧ Chrom. m2-M7th	3	40
⑨ #5th-b4th	28	15
⑩ #2nd-bb7th	10	33
⑪ #6th-bb3rd	35	8
⑫ #3rd-bb6th	17	26
⑬ #7th-"Comma"	42	1
⑭ x4th-bb5th	23	19
⑮ x1st-bb1st	6	37
⑯ x5th-bb4th	31	12
⑰ x2nd-bbb7th	13	30
⑱ x6th-bbb3rd	38	5
⑲ x3rd-bbb6th	20	24
⑳ x7th-bbb2nd	2	41
㉑ x#4th-bbb5th	27	16
㉒ x#1st bbb4th - x#5th bbb1st	9	34

53-E.T.		Comma
① 1:1-2:1	0	53
② 3:2-4:3	31	22
③ 9:8-16:9	9	44
④ /6th-\m3rd	40	13
⑤ /M3-\m6	18	35
⑥ /M7-\b2	49	4
⑦ \b5-#4	27	26
⑧ b2-M7th	5	48
⑨ b6th-M3rd	36	17
⑩ m3rd-M6th	14	39
⑪ b7th-\2nd	45	8
⑫ /4th-\5th	23	30
⑬ Comma	1	52
⑭ /5th-\4th	32	21
⑮ 8:7-7:4	10	43
⑯ 12:7-7:6	41	12
⑰ b4th-#5th	19	34
⑱ b1st-#1 (chrom)	50	3
⑲ b5th-\#4th	28	25
⑳ /b2nd-\M7th	6	47
㉑ /b6th-\M3rd	37	16
㉒ /m3rd-\M6th	15	38
㉓ Neutral 7th-2nd	46	7
㉔ 11:8-16:11	24	29
㉕ Diesis	2	51
㉖ Wolf 5th-4th	33	20
㉗ Enharmonic G.	11	42

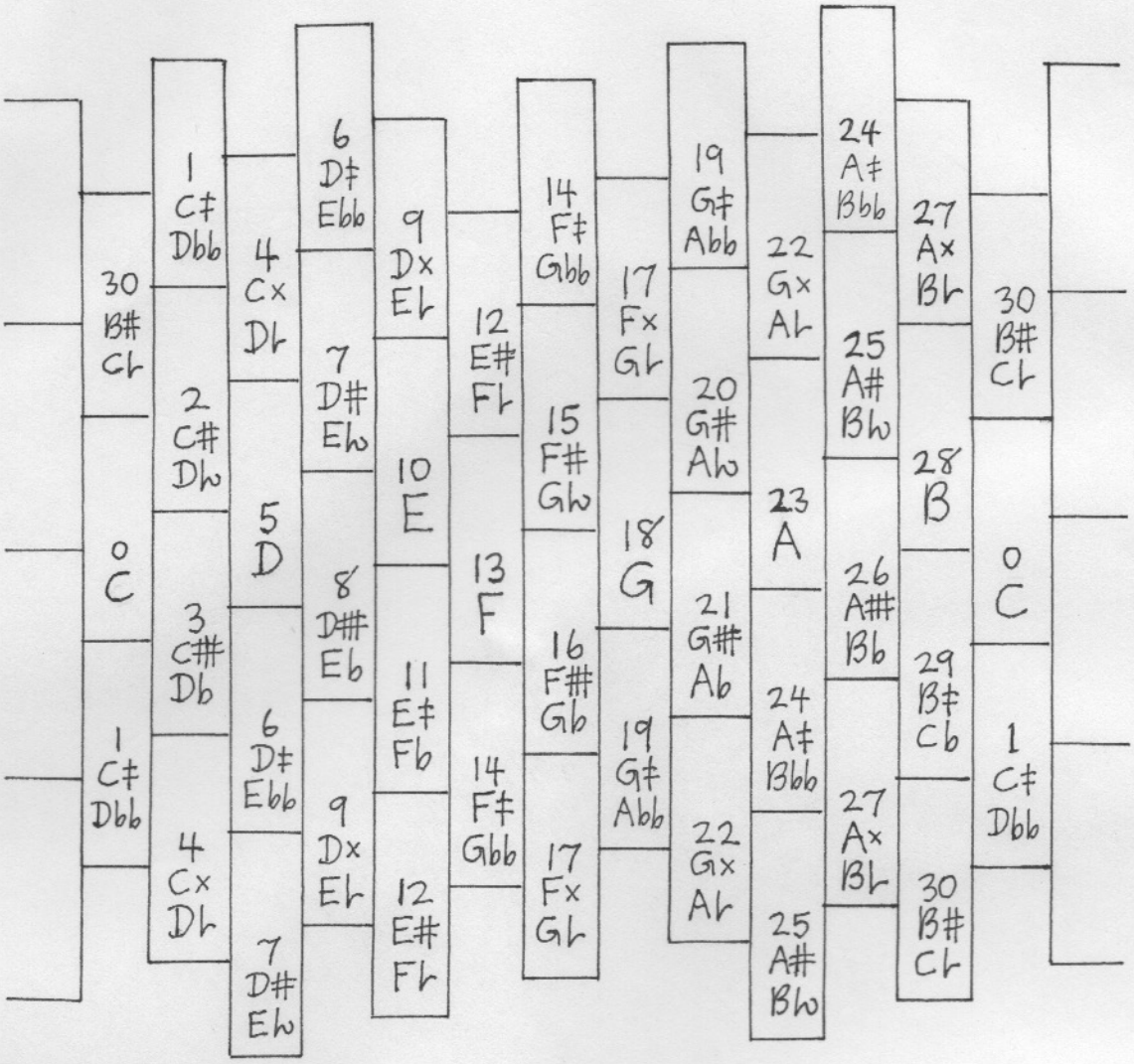
Figure 70: A Bosanquet Generalized Keyboard adapted for 43-E.T.



In this particular version of the keyboard, I have limited the number of digitals to 45 per octave, so that the systems 12, 19, 31, and 43 may be played. Ironically, the keyboard was invented for the 53 system; yet it is not well suited to that system. However, it is very good for these cyclical meantone temperaments.

In 43-E.T., there are two redundant digitals per octave. They are pitches 9 and 34 – the two enharmonics in the system.

Figure 71: A Bosanquet Generalized Keyboard adapted for 31-E.T.

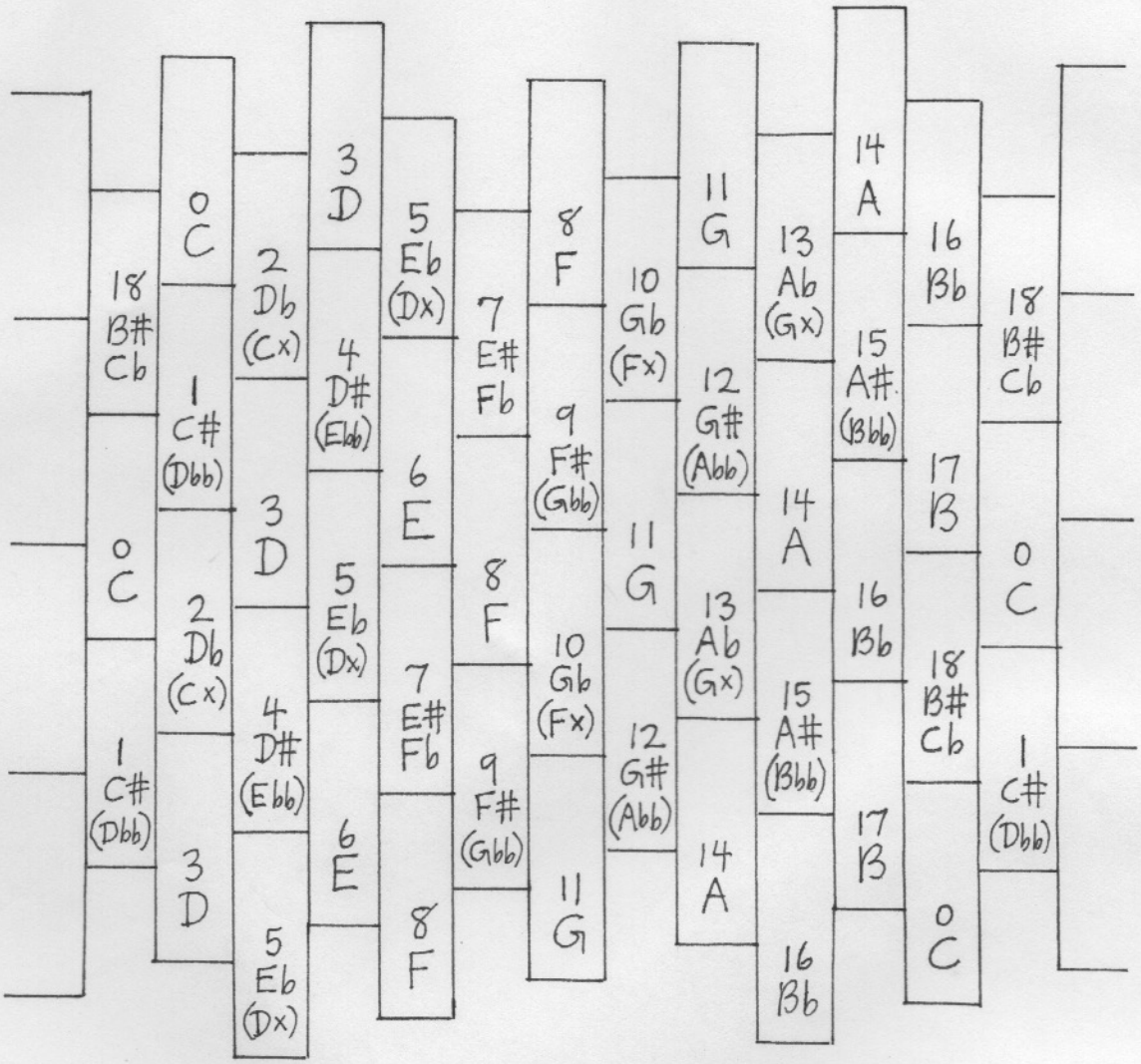


When the keyboard is tuned in 31-E.T., there are 14 redundant digitals per octave (since the keyboard has 45). These pitches span the antipodal region of the circle of Fifths. In Circle of Fifths order they are:

7, 25, 12, 30, 17, 4, 22, 9, 27, 14, 1, 19, 6, and 24.

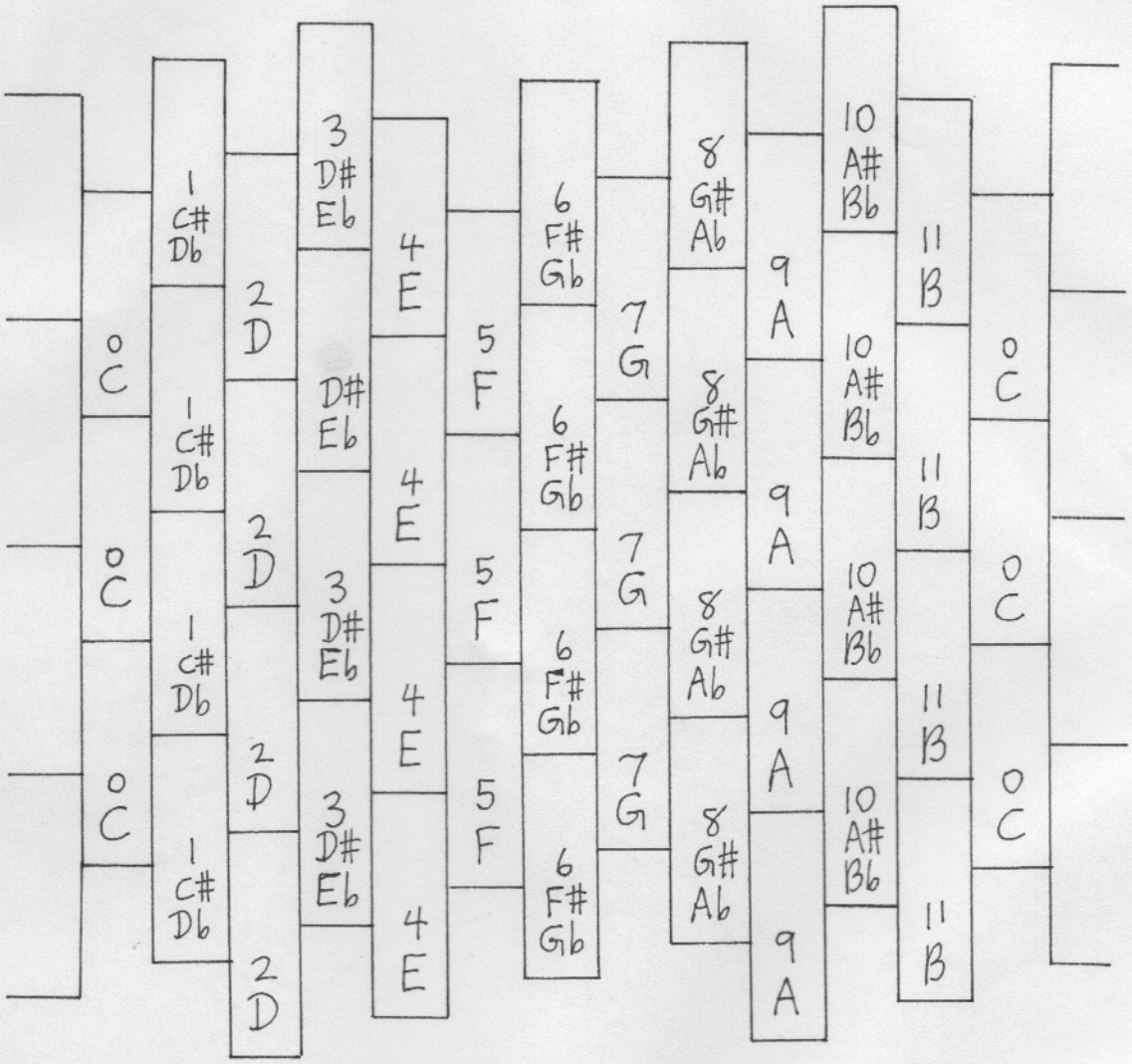
These redundancies introduce alternative fingerings and are thus desireable for rapid playing. This keyboard is ideal for the 31 system.

Figure 72: A Bosanquet Generalized Keyboard adapted for 19-E.T.



When the keyboard is tuned into 19-E.T., there are 26 redundancies per octave. Thus the entire scale is repeated more than once. The result is a very "fast" keyboard with lots of alternative fingerings.

Figure 73: A Bosanquet Generalized Keyboard adapted for 12-E.T.



When the keyboard is reduced to the 12-E.T. system, it becomes a type of "6-6" keyboard, with alternating rows of the wholetone scale peculiar to this intonation system. There are, of course, many redundancies. The fingering is much faster, more fluent, and easier than the standard keyboard. It is planimetric, so that "positions" are easier to memorize. Note that the fingering is the same for all intonation systems. The keyboard is "inter-system."