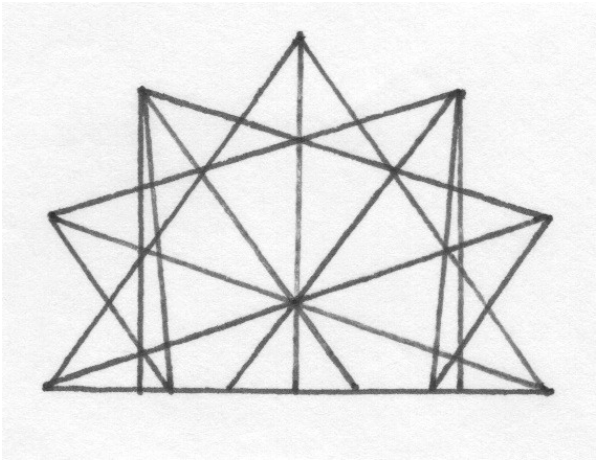
The Natyasastra and Vedic Harmonics



Siemen Terpstra

www.siementerpstra.com

The NĀṬYAŚĀSTRA and Vedic Harmonics

-co-discovered ?
-Egypt
-Sumer
-Indus

SIEMEN TERPSTRA

ABSTRACT

The author presents a new interpretation of the NĀṬYAŚĀSTRA (the ŚRUTI system) of ancient India, showing links with a much older system originating in the nuclear Near-East. The ancient text shows an awareness of the Comma scale (associated with pure intonation). Yet it hides the true structure of this scale in a net of confusing and contradictory information. The author concludes that the artificiality of the system was purposefully presented in order to preserve the truth from the uninitiated.

INTRODUCTION

The body of literature known as the Vedas (sacred "knowledge") is said to have been revealed by Brahmā to the ancient sages (ṛṣi) in the form of the intoned word. Thus this knowledge was, to a certain extent, musical in its essence. We shall see that musical concepts lie at the heart of the ancient Vedic cosmologies.

It is now generally believed that these writings were composed by the Aryan tribes who first invaded India from the North-West, around the middle of the second millenium B.C. The earliest of these texts is the Ṛgveda, a collection of religious verse which became the basis for the Yajurveda and the Sāmaveda. The Yajurveda rearranged the verses as ritual formulae for the priest officiating at sacrifices. The Sāmaveda is a manual for the "priest-singers" (sāmaga). Here the verses are presented in a more melismatic, musical form, with its attendent distortions and meaningless syllables (stobha) inserted into the verses. The lesser known Atharvaveda is a collection of magical spells. There were also auxiliary texts to each of these four Vedas--including the Brahmaṇas, the Āraṇyakas, and the Upaniṣads.

The proper recitation of the Vedas constitutes the "axis of the universe." Any mistake could have disastrous consequences, unless expiated by sacrifice and prayer. It is the duty of a special class of priests, the Brahmans, to preserve the Vedas absolutely unchanged to the end of this creation (sarga) and to perform the proper rituals.

This whole tradition has an oral basis. The vast Vedic literature was memorized by the Brahmans, requiring complete, lifelong dedication. As the Vedic language gradually evolved into classical Sanskrit, there was a danger and concern that the original meanings of the verses would be lost, so that eventually the texts were written down. Even so, the Vedic tradition is likely the longest continuous oral tradition of its kind. The

chants have been preserved with extraordinary accuracy for over three millenia.

The Aryans, being a conquering race, probably introduced the caste system to India. In this social structure, the Brahman priests had immence influence in the community, even controlling the political leaders and their armies. Their power came from their knowledge of the magic spells, rituals, and sacrifices contained in the Vedic writings which only they knew. It was in their interest to preserve the inner workings of this system from the masses.

The Nāṭyaśāstra is probably the earliest extant text dealing with the theory of Indian music. It is ascribed to the author Bharata, sometime during the first centuries A.D. Even though it is a latter writing than the Vedas themselves, it comes out of the same tradition. The author claims that the various aspects of drama (Samgīta--which includes music and dance) are derived from the Vedas. Recitative speech (pāṭhya) comes from the Ṛgveda, song (gīta) from the Sāmaveda, acting (abhinaya) from the Yajurveda, and the sentiments (rasa) from the Atharvaveda. He wishes to show that secular music comes from the same hallowed source as chant.

It would be wise, at this point, to briefly review the information presented in the Nāṭyaśāstra. Then we will look at various interpretations of the śruti system. Finally, we shall offer evidence for the equivalence of the śruti and the comma, and explore the implications of that equivalence.

THE TWO GRĀMAS

The intonation system is based upon two parent scales (grāmas), both of which contain seven notes (svaras) each. These scales are the Sadjagrāma and the Madhyamagrāma, abbreviated to SAGRĀMA and MAGRĀMA. The seven tones of the scale are also abbreviated to SA, RI, GA, MA, PA, DHA, and NI. The tones are further characterized in terms of consonance (saṃvādī), assonance (anuvādī), and dissonance (vivādī). Only the Fourth and Fifth are considered consonant, and we presume that they are 'perfect' in the modern acoustical sense. However, no hint of a 'monochord' connection to the ratios 2/3 or 3/4 are given. The minor second is dissonant, and the other tones are assonant. In addition, one tone is the vādī, considered the most important tone in the scale, and sometimes compared to a 'king.'

These various interval relationships are compared in terms of śrutis. The word śruti is derived from the root śru (to hear), so that the word may be translated as "that which is heard" by the ṛṣis. Tones having interval sizes of either nine or thirteen śrutis are consonant to each other. On the other hand, tones which have two śrutis between them are dissonant to each other.

The differences between the two grāmas are expressed in terms of śrutis. In the Sagrāma, Sa and Pa are consonant, and in the

Magrāma, Pa and Ri. Now this implies that in the Sagrama Pa and Ri are not consonant, while in the Magrāma Sa and Pa are not consonant. This last assumption is explicitely stated. The tones (svaras) of the two grāmas are then described in terms of their śruti values.

Here the wording of the text becomes somewhat confusing, since the śrutis are described as integral parts of the svaras. For example, in the Sagrāma, Sa is described as being 'four-śruti,' Ri as 'three-śruti,' Ga as 'two-śruti,' Ma as 'four-śruti,' Pa as 'four-śruti,' Dha as 'three-śruti,' and Ni as 'two-śruti.' In the Magrāma, there are two changes. Pa is 'three-śruti,' and Dha is 'four-śruti.'

From this description it is clear that the śruti is a very small interval (antara) or microtone. However, the size of the śruti is not specified in exact terms, such as a monochord ratio. Moreover, it is not immediately clear whether the intervals are meant to be read above the tones or below the tones.

If we interpret them as being above the tones, the resulting Sagrāma and Magrāma could be roughly compared to the Ionian ('C' mode) and the Lydian ('F' mode) scales respectively:

SAGRĀMA	SVARAS	Sa	Ri	Ga	Ma	Pa	Dha	Ni	(Sa)
	ŚRUTIS		4	3	2	4	4	3	2
	WESTERN NOTES	C	D	E	F	G	A	B	C

MAGRĀMA	SVARAS	Ma	Pa	Dha	Ni	Sa	Ri	Ga	(Ma)
	ŚRUTIS		4	3	4	2	4	3	2
	WESTERN NOTES	F	G	A	B	C	D	E	F

However, if the śrutis are interpreted as below the svaras, the two scales compare to the Dorian ('D' mode) and Mixolydian ('G' mode) scales:

SAGRĀMA	SVARAS	Sa	Ri	Ga	Ma	Pa	Dha	Ni	(Sa)
	ŚRUTIS	4	3	2	4	4	3	2	(4)
	WESTERN NOTES	C	D	Eb	F	G	A	Bb	C

MAGRĀMA	SVARAS	Ma	Pa	Dha	Ni	Sa	Ri	Ga	(Ma)
	ŚRUTIS	4	3	4	2	4	3	2	(4)
	WESTERN NOTES	F	G	A	Bb	C	D	Eb	F

The first interpretation is certainly consistent with modern Indian music practice. It also seems reasonable, since the scale is given in ascending form. But only the second interpretation is consistent with the given conditions regarding consonance laid down in the text itself. The interval from Ri to Pa in the Sagrāma, and Pa to Sa in the Magrāma, are both ten śrutis and thus not consonant.

More evidence that the śrutis are meant to be located below the tones comes from the famous Vīnā experiment described in the Nāṭyaśāstra. The experiment shows the interval distribution of the śrutis among the svaras, and supposedly proves that there are twenty-two śrutis in the octave.

THE TWO VĪNĀ EXPERIMENT

The experiment uses two vīnās tuned identically to the Sagrāma. One vīnā remains unchanged, serving as a frame of reference for the experiment. The other vīnā is lowered in pitch by four successive steps. At each stage its tones are compared with those on the unchanged vīnā. Each time, the vīnā is lowered one PRAMĀNA ŚRUTI, which is translated as 'measured' or 'standard' śruti. Subsequent lowerings are presumed to be by the same amount, since the text says "lower again, in exactly this manner," (punarapi tadvadevapakarṣāt). Hence the text hints that the steps are all the same size. After two, three, and four lowerings certain tones on the changed vīnā co-incide with their neighbours on the unchanged vīnā. Thus if this coincidence happens after three lowerings, the interval between the two tones is proven to be three śrutis. This experiment is best seen diagramatically: see Figure 1. At the first lowering, no tones co-incide. At the second, Ga and Ni of the changed vīnā co-incide with Ri and Dha of the unchanged vīnā. At the third lowering, Dha and Ri coincide with Pa and Sa; at the fourth lowering, Ma, Pa, and Sa connect with Ga, Ma, and Ni.

The text does not mention if the śrutis are of varying sizes, the same size, or what that size is. But the experiment strongly hints that the śrutis are all the same size, since the experiment would make little sense if it were otherwise. Thus the pramāna śruti would be a standard measure which can be applied to establish the sizes of the other śrutis (if the śrutis are thought to be of varying sizes).

The text gives a further clue: that the difference between the two grāmas lies in the position of the Pa, which is one śruti lower in the Magrāma than in the Sagrāma. The first lowering is by this amount, converting the Pa of the Sagrāma to the Pa of the Magrāma. This first śruti is the pramana sruti.

If these lowerings are all the same size, then the tuning system must be 22-Equal-Temperament, at least in theory. However, an acoustical examination of this tuning system reveals that it is quite artificial, noticably (and audibly) mis-tuning the fourths and fifths. See Figure 2. It is an acoustical fact that, given perfect musical fourths and fifths, the 22 śrutis cannot be the same sizes to each other. In spite of these contradictions, many eminant musicologists accepted the 22-E.T. conclusion, for example Pandit V. N. Bhatkhande, according to L. R. Patwardhan.[1]

The contradiction between acoustical theory and fact could arise from the conclusion that Bharata thought that the srutis were the same size, although in reality they are not. This interpretation would claim that the experiment was conducted only in theory and not in fact, or that the experiment was carried out in fact but that the differences were glossed over. In other words, he was sloppy in his tuning procedure, and did not measure his intervals accurately. Those putting forth this argument remind us that there were no experimental monochords with which to

accurately measure intervals in ancient India. Exactly how much tolerance should we allow our ancient tuner?

Although I have no proof, I find it highly unlikely that the ancient author was sloppy, given the high state of the ancient music culture in India. Just as the ancient Vedic oral chant tradition has been extraordinarily well preserved, so too the music culture has been faithfully handed down from generation to generation through the 'guru-shishya-parampara' system of instruction. There seems little reason to believe that the modern tuning system varies at all from the Vedic one. If we accept this argument, then an examination of the existing practice reveals that the musical fifths and fourths are still sung 'pure,' and were probably always sung pure (that is, acoustically correct as a 3/2 frequency ratio). Hence, in spite of the hints in the ancient text, the śrutis must be of varying sizes; or alternatively, the śruti is one size but there not 22 of them in the octave.

We also have doubts that the Sagrāma refers to the Dorian mode. Modern practice connects the Sagrāma with the Ionian mode. The śrutis could be interpreted as being above the svaras, if the second vīnā were tuned up in pitch rather than down in relation to the first vīnā. Yet the ancient text is explicit in stating the downward direction. We find ourselves being suspicious of the experiment. It seems highly improbable that the śruti system was 22-Equal-Temperament. Moreover, A. K. Coomaraswamy[2] has argued, on the strength of textual descriptions, that the ancient Indian vīnā was a bow harp--an idea corroborated by early Indian bas-reliefs. Yet the experiment makes little sense unless one is using a fretted or stopped stringed instrument of the monochord type. Thus the experiment is highly problematic.

MODERN INTERPRETATIONS OF THE ŚRUTI MUSICAL SYSTEM

Many scholars have commented on the mystery of the ancient śruti system. Modern musicological opinion generally falls into two camps. One camp is epitomized by Kolinski,[3] who derives the śruti from the 'cyclical' method of tuning pure fourths and fifths in a line. This 'up and down' method arrives at the Pythagorean Comma (or Ditonic Comma) which is about 23.5 Cents in size. This interval is the pramāna śruti. But the śrutis are of three different sizes, so that they can also be about 66 Cents and 90 Cents in size. All of the 22 notes which are variously used in Indian music may be derived from using aggregates of these three intervals.

3-Limit JI

The other musicological camp was first expounded by Fox Strangways,[4] who derives the śrutis from the 'divisive' principle, where tones are determined on the basis of simple fractions of string length. In support of this theory, it is true that modern practice supports this Just Intonation norm. The Hindu singer not only sings the fifths as a simple 3/2 frequency ratio, but also the thirds as a simple 5/4 ratio. Sophisticated measurement of the

5-Limit JI

modern tunings confirm the acoustical accuracy of the intervals. Moreover, there is no reason to suppose that it was different in ancient India. Modern practice does not differ from the pattern which we in the west call Just Intonation. The 'divisive' theory further postulates that the śrutis are of three different sizes: about 21.5 Cents (Syntonic Comma), about 70 Cents, and about 90 Cents. The other tones are derived from these fundamentals.

5-Limit JI

Although these two camps stand opposed to each other, it is an acoustical fact that they are not very different from each other at all. The difference in pitch between the two pramāna śrutis, 21.5 and 23.5 Cents, is only the Schisma interval of about 2 Cents, which is practically subliminal in our pitch discrimination. We can determine both camps to characterize the three śruti sizes to be: one comma, three commas, and four commas. In both theories, the pramāna śruti is the comma. All other intervals in the scale can be expressed as aggregates of the comma.

Critics of the 'divisive' principle contend that there is no evidence of the monochord measurement in the Indian musical literature until comparatively recent times, namely the 17th Century Sangītapārijāta; and even this reference is relatively crude. On top of this, the 'cyclical' theory reminds us that the consonances of the fourths and fifths were of fundamental importance in ancient India. Both modern theories diverge from the implication of the Nāṭyaśāstra that points to one size of śruti.

A RE-EXAMINATION OF THE COMMA FIELD

It is my contention that the two principle theories are 'splitting hairs,' with regard to their differences, and that the pramāna śruti is, indeed, the comma. An examination of the Field of extended Just Intonation will reveal a number of structural features which are all hinted at (but not directly stated) in the ancient text.

Let us look at the 'cyclical' division first. Figure 3 shows what happens when we tune by pure fourths and fifths in a line which expands symmetrically from unity or SA. The intervals are described in a number of ways to clarify their acoustical size. Their size is first described in terms of the number of Cents (an arbitrary division of the octave into 1200 units), and, under that, the number of Schismas (612 in the complete octave). Then the pitch is described in terms of its comma size (53 in the complete octave). Under that, the pitches are named according to the integrated pitch designation which I have worked out for Just Intonation and 53-E.T. Sa is defined as 'C.'

We expand the line of fourths and fifths by the 'up and down' method; that is, we tune up a fifth, down a fourth, up an fifth, and so on in either direction so that the pitches land in the same octave. The principle of octave equivalence is assumed, as it is in all high music cultures. Hence this field can be considered to

lie within one octave (any octave). Indian theory recognizes three octaves: low (Mandara Septaka), middle (Madhya Septaka), and high (Tara Septaka). Let us assume that our Field pattern lies in the middle octave.

There must be some natural boundary which limits the expansion of our line of fifths-fourths, although the line could be expanded theoretically to infinity. This natural boundary is, of course, the Pythagorean Comma up in pitch when we move to the right, and the same interval down in pitch as we move to the left. Note that this interval is 12 Schismas in size. The Syntonic Comma, on the other hand, is 11 Schismas in size.

Figure 4 shows a pitch Field which results from the application of Just intervals by the 'divisive' principle. Again the intervals are described by our various measures. In this diagram we witness a series of triangles around the familiar line of fifths-fourths. Just as horizontal lines indicate pure fourths and fifths, lines which move diagonally to the upper right indicate pure Major Thirds (ratio 5/4); and lines which move diagonally to the upper left indicate pure Minor Thirds (ratio 6/5). A segment of this Field diagram is also shown in ratios. All of these diagrams could be expressed as frequency ratios, but it is my desire to minimize the amount of 'numbers' on the diagrams.

The reason for presenting all of this technical information is to show the practical unity of the two approaches. It can be seen that there is virtually no audible difference in sound between the 'cyclical' approach and the 'divisive' approach. The differences amount to Schisma alterations, which can be considered as subliminal. These conclusions are shown on Figure 5 which shows the 'cyclical' Field, expressed in commas, as a virtual duplication of the 'divisive' Field. As a consequence, we can consider the field of Just Intonation tuning to be a scale of commas.

When one asks which comma, the Ditonic or the Syntonic, one is really splitting hairs. The former is 12 Schismas in size, the latter 11 Schismas. This acoustical fact is the basis of the 53-E.T. tuning system as such a good approximation of the Field of Just Intonation. The 53-E.T. system uses a 'mean' comma about halfway in size between the Ditonic and the Syntonic, or about 22.6 Cents (about 11 and 1/2 Schismas). As a result the octave becomes divisible into 53 equal sized steps. The Field of śrutis is best considered as a subset of this overall scheme.

Those interested in further structural features of the Quasi-Just Field (53-E.T.) should consult my article: REFLECTIONS ON AN IMPROVED NOTATION SYSTEM FOR 53-E.T. AND JUST INTONATION; in INTERFACE: Journal of New Music Reasearch, Vol. 14, number 1-2, 1985, Utrecht, Netherlands. The same tuning model may be used for traditional Western vocal intonation, as well as the Hindu system. This is not surprising, since the Indian system is also based on a vocal model. The same psycho-acoustical phenomena apply in both

-in Egypt
Sumer
Indus
China

cases. In this article, I shall only deal with the Hindu cultural matrix, omitting non-relevant aspects of 53-E.T. harmony.

The śrutis thus come in three sizes; one comma, three commas, and four commas. These intervals are not scale tones by themselves; rather, they are used to derive the scale tones employed as svaras in the music. These 22 scale tones are really a subset of the 53, and the 22 themselves are used as a 'pool' from which to make heptatonic (seven note) scales for practical use. How the srutis are combined to produce the 22 tones are shown in the table below:

SVARA	SRUTIS IN SVARA	COMMA NUMBER	CENTS NUMBER	WESTERN NAME
Sa		0 = 53	0 = 1200	C
Ri	atikomal Ri	1	22	/C
	komal Ri	5	112	Db
	madhya Ri	8	182	\D
	suddha Ri	9	204	D
Ga	atikomal Ga	13	294	\Eb
	komal Ga	14	316	Eb
	suddha Ga	17	386	E
	tivra Ga	18	408	/E
Ma	suddha Ma	22	498	F
	tivra Ma	23	520	/F
	tivra Ma	26	590	F#
	tivratara Ma	27	610	\Gb
Pa		31	702	G
Dha	atikomal Dha	35	792	\Ab
	komal Dha	36	814	Ab
	trisruti Dha	39	884	A
	suddha Dha	40	906	/A
Ni	atikomal Ni	44	996	\Bb
	komal Ni	45	1018	Bb
	suddha Ni	48	1088	B
	tivra Ni	49	1110	/B
Tara = high Sa		53 = 0	1200 = 0	C

Just as the pitches of 22-E.T. may be shown on the circular graph, so also the pitches of 53-E.T may be so indicated. On this graph are drawn the Hindu tones: see Figure 6. Note that this ŚRUTI FIELD is really comprised of 25 pitches, not 22. The added three pitches include the second śruti of four commas, and two others which result in the symmetrical structure identical to the expansion we examined earlier. On this same Figure 6 is shown the Circle of Fifths-Fourths for 53-E.T. Note the pattern for the Hindu tones. Figure 7 shows the unique and special position of the śruti tones within the complete Field of "Quasi-Just Intonation" or 53-E.T. For now, we will ignore the other system boundaries.

From Figure 6 it can be seen that the distances between any two adjacent pitches may be expressed as either one, three, or four commas, or some aggregate of these. It is interesting to observe the position of these interval sizes on the Field diagram (Figure 7). Commas one and four are truly boundary positions for the Field expansion. Comma three appears as the difference in pitch between the 'bottom' and the 'top' of the sruti Field, for

example between Eb and E, or between Ab and A. Therefore it also serves to define the Field boundaries.

We have come a long distance from the vīnā experiment described in the Nāṭyaśāstra to the relatively complex modern Field of Just Intonation. Yet there is no reason to believe that the ancients were not aware of the comma scale. Their ears were just as sensitive as ours (maybe more so since their environment was not inundated by machine noise as is ours). Orthodox Hindu tradition claims that the ancient musicians were more advanced than us moderns, since evolution in the Kali Yuga is toward degeneration (not progress as in the Western concept). Surely the ancient authors of the Nāṭyaśāstra knew more about the tuning Field than they revealed in the writings. My reasons for coming to this conclusion will become apparent as this article progresses.

THE TWO SCALAR PARADIGMS

An essential feature of Just Intonation is the choice of more than one way to tune a given scale type. For example, we could tune the 'Ionian' mode using pitches (designated by their comma numbers) 0-9-18-22-31-40-49, or alternatively 0-9-17-22-31-39-48. Although the differences amount to only comma alterations, the quality or 'mood' of the two scales is quite divergent. The first 'Major' mode is called Pythagorean, since it is made up of pitches from a compact portion of the line of Fifths-Fourths. The second tuning I have labelled as Ptolemaic (in honor of the Egyptian Ptolemy, who accurately described so many of the ancient tunings). These Ptolemaic tunings consist of a 'double strand,' or two lines of Fifths-Fourths connected by Just Major and Minor Thirds.

These heptatonic scales are both subsets of the Pythagorean Ogdoad, and the Ptolemaic Ogdoad. In order to understand the two tuning paradigms of Just Intonation, it is important to clarify the structure of these two limiting scales.

We will look at the Ptolemaic Ogdoad first. Figure 8 shows the essential expansion of the 'Ptolemaic Sequence' from its Just Major Triad nucleus, through the Ptolemaic Tetrads, Pentads, Hexads, Heptads, and Ogdoads. Note that any further expansion of the Ptolemaic Sequence always involves a comma juxtaposition, so that the Ptolemaic Ogdoad is the furthest expansion of this 'double-strand' to avoid such a dissonance. It is also interesting to note here that the interval between F and F# on this pattern is four commas in size. We associate this size of semitone to with Pythagorean sequences.

The Pythagorean sequence is more familiar to Western theorists. See Figure 9. An expansion along the line of Fifths-Fourths avoids the 'Just' Thirds and instead uses the more dissonant Pythagorean Thirds. However, after we reach eight tones, we again come to a limiting boundary. The reason is that, with the ninth tone, we encounter a 'Just' Third rather than a Pythagorean Third. For example, look at the central line of Fifths given in

comma numbers: 35-13-44-22-0-31-9-40-18. Now the difference in pitch between 18 and 35 is a Just Major Third, not a Pythagorean Major Third. As a result, any long line of Fifths (for instance a chromatic sequence of 12 Fifths), always results in a 'Pythagorean' segment, as well as a 'Ptolemaic' segment. Hence, if we wish to keep our scale as purely 'Pythagorean,' we must limit it to eight notes or less. These eight notes form an altered version of the Ptolemaic Ogdoad. Most scales have seven notes (or less). Hence we can always choose a Pythagorean version, or at least one Ptolemaic version. These two patterns form the scalar paradigms of Just Intonation. 3-Limit + 5-Limit

How does all this information relate to the Nāṭyaśāstra? The Pythagorean group of scales can be classified according to 'where' it sits on the line of Fifths. In a central, pivotal position is the 'Dorian' mode: 0-9-13-22-31-40-44, the only such scale to be symmetrical about the tonic. This special tuning was one of the two modes which Plato allowed for his ideal city in the Republic. (You can guess what the other mode was!). It is also the Modus Primus of medieval European music. It is the scale associated with the 'under' interpretation of the śrutis.

On the other hand, the 'over' interpretation of the śrutis led to the 'Ionian' mode and the 'Lydian' mode. But these two scales we associate with the Ptolemaic Ogdoad, since they form the two heptatonic subsets of this pattern. We are left with the feeling that the ancient authors were implying the existence of the two paradigms, even though it is not stated explicitly. Anyone who makes practical and consistent experiments in Just Intonation will be led to discover these two forms of the Diatonic scales.

Figure 10 shows some of the most used Just scales from the Indian classical tradition. It is interesting that these scales are also fundamental to classical Greek music and classical Persian music. It is my conclusion that tuning information was the common lore of all the ancient high civilizations, including China. This tuning lore originated from monochord procedures which were probably worked out in the earliest of civilizations (Sumer), then spread across Asia and Europe. It constituted a common spiritual heritage which influenced all of the high musical cultures.

It can be inferred from examining Figure 10 that any scalar tuning can be 'drawn' using the triangular Field grid to define its modal structure. The pattern is planimetric. Using this Field diagram, it is possible to classify all of the possible scales in the śruti system (or, for that matter, all of 53-E.T.). I have explored this territory in a related writing--the Book of Functional Tables for 53-E.T. The ancient Pythagoreans, who took information from the Near-East to Europe, used 'pebble arithmetic' to classify this information. Pebbles on a sand table were laid out in a triangular grid. This grid was considered sacred, witness the Tetractys. It is my contention that the triangular grid referred to is the Field diagram which I have mapped as Figure 7.

Many veiled references to it are found in the writings of Plato and other ancient authors.

There are many other close similarities between Hindu and classical Greek music theory. One notable example is the Anga, or Tetrachord. The word literally means 'limb' or 'body' in music. A scale consists of two angas: purvanga (Sa, Ri, Ga, Ma), and uttaranga (Pa, Dha, Ni, Sa). This awareness of the central 'spine' of the scale is identical in Hindu, Greek, Persian, Egyptian, and Chinese music cultures. It comes from an awareness of acoustical phenomena which could only have developed from practical tuning experience. Even though the monochord is not mentioned in the ancient Hindu texts, I conclude that it was nevertheless known. Indeed, the vīnā experiment indicates an awareness of fret placement, and hence of the monochord. Moreover, monochord procedures were the common heritage of all the ancient cultures. These procedures resulted in the system of musical harmony being represented as a hierarchy of the number field. Such 'musical numbers' surface in the cosmology, the mathematics, and the calendrics of India and the other Asian cultures.

MONOCHORD PROCEDURES

In order to show the significance of various musical numbers, it is necessary to explain some features of monochord procedures. This introduction is necessarily brief, and the subject is fairly complex. Those wishing to delve further into its intricacies should refer to my articles: MONOCHORD PROCEDURES and MONOCHORD NUMBERS.

Figure 11 is an illustration of a simple monochord, consisting of a single stretched string with a moveable bridge in the middle. Although this instrument may appear as 'primitive,' it proves to be the source of an intricate and wonderful mathematics which maps the tuning Field. The length of the string is measured, and the moveable bridge is calibrated in various positions. When the string is sounded, the resulting harmony can be accurately described in terms of number positions. Ancient monochords probably quickly evolved into multi-stringed instruments, so that comparisons between various tunings would be more convenient. All fretted instruments are distant 'cousins' of the monochord.

We modern musicologists realize why the monochord is so effective for mapping the Field of frequency ratios. A basic law of physics tells us that string length (wave length) is inversely proportional to frequency. Hence, even though the ancient tuners were apparently ignorant of frequency ratios, they could still roam freely about 'sonic space' through the use of the monochord.

As Figure 12 shows, the division of the monochord string by simple ratios results in strong consonant harmonies. Every tone has a corresponding number, so that quality and quantity become 'analogues' of each other. Neither can explain the other, but both are valid aspects of empirical experience. This 'proto-scientific'

experiment may be among the first truly scientific experiments to be systematically explored by the old tuner-mathematicians who were probably also astronomers and healers.

The simplest division is to put the moveable bridge in the exact middle of the string length. The octave of the fundamental (the open string) then sounds on both sides of the string. Thus we associate the octave interval with the 1/2 string length ratio, as well as the 2/1 frequency ratio. The fraction simply inverts, since one is reciprocal to the other. Octaves also become identified with powers of 2 (that is: 2^2 , 2^3 , ... 2^n), since the 4th Harmonic yields the next octave, the 8th Harmonic yields the 3rd octave up from the fundamental, and so on. The octave has been variously described as the 'miracle' in music, for it creates a 'microcosm' within the 'macrocosm' of the pitch continuum. We hear an infinity of tones in the slide from extremely low pitch to extremely high pitch. Yet the pattern repeats as a 'higher reflection' with each octave. Any one octave pattern contains all the information one needs for the creation of the scale. The miracle lies in the acoustical fact that the octave 'microcosm' preserves the 'infinity' already found in the macrososm. There is an infinite number of pitches within an octave. Hence the main musical problem is really the decision on what to omit from the scale, since much more must be deleted than retained.

The ancient convention was to tune the scale starting from this octave point and progressing in a falling pattern to the fundamental. Throughout ancient culture, the scale was seen as a falling pattern. In ancient Greece, the octave point was called the mese (the middle), and Persian practice also started the scale there. It is somewhat surprising that the Nāṭyaśāstra describes the scale as a rising pattern, as do we moderns. Of course, the ancients (as well as us moderns) must have been aware of the essential reciprocity of the pitch continuum. The scale has a rising and a falling aspect. Modern Hindu theory still pays homage to this basic acoustical truth in its concept of arohana (rising) and avarohana (falling) scales. We shall see that reciprocity is a fundamental principle in monochord procedures. Division and multiplication are reciprocal aspects of the same procedure.

One more essential ancient technique needs to be clarified. The simple divisions shown in Figure 12 are herein described as fractional ratios, assuming that the fundamental is measured as unity (that is: the string length is one unit). But the ancients took great pains to eliminate or 'clear' fractions by using a least-common-multiple. The example shown indicates that the scale C-G-F-C is properly represented by dividing the string into 12 units. Then the middle is at position 6 (since 6/12 = 1/2), and the other tones can also be described in terms of whole numbers. The tuning becomes a number series ruled by a 'double,' in this case the number series 6:8:9:12 ruled by the double 6:12.

This business of clearing fractions by a least-common-multiple is the element of complexity in the mathematics. We are being asked to look at the same 'tonal

information' from different numerical standpoints within an octave modulus. With every increase in 'numerosity' we include a larger 'patch' of the Just Field. Compare the 'patch' ruled by the double 60:120 and the 'patch' ruled by the double 360:720, shown on Figure 15. The procedure sets up a hierarchy in the number field, so that certain doubles are considered as musically more significant than others. We will look at only a few representative examples of divisions which result in prominant scales, and a few which relate to cosmological numbers.

Figure 13 shows a very basic division into the number series 4:5:6:8, ruled by the double 4:8. The falling scale which results is C-Ab-F-C, the Just Minor Triad associated with the Sub-Harmonic Series. By simple reciprocation, the number series could also refer to the rising pattern C-E-G-C. This pattern is the natural Just Major Triad associated with the Harmonic Series and frequency ratios. Even if the ancient tuners were not aware of the Harmonics on an open string (which I highly doubt), they could have derived the rising pattern from a theoretical multiplication of the monochord string, as shown in Figure 13. Hence the number series must always be written as a falling and a rising sequence.

One more interesting point surfaces from an examination of the monochord division in Figure 13. Notice that the 7 is omitted. Only numbers which are divisible by 2, 3, and 5 are allowed to partake in the series. For example, the number $60 = 2^2 * 3 * 5$, or again $64 = 2^6$. Our multiples must be derived from powers of the three prime numbers 2, 3, and 5. This special class of numbers was called by Neugebauer[5] the "regular numbers." The prime number 7 is not allowed. I will not speculate on the reasons for this old restriction, but it is an acoustical fact that the frequency ratio 7/4 is fairly closely approximated by the Just ratio 225/128, being 7.7 Cents sharp. This ratio becomes even closer when one tempers the Just Field into 53-E.T. Hence the 7th Harmonic is present within the Just fabric, but in an 'indirect' sense. The number 7 has a special, esoteric significance for monochord theory, but I will not delve into it here. It is the number of mystery.

The restriction to number sets having the form $2^X * 3^Y * 5^Z$ means that the information is amenable to a layout involving a triangular grid pattern. This pattern is the Field of Just Intonation. In an abstract sense, this Field can be regarded simply as the multiplication table of 3 * 5. Since powers of 2 are hidden by the principle of octave equivalence, powers of 3 and 5 radiate out in six directions to form the Field--see Figure 14. We can take these number relations out to whatever numerosity we want, but eventually we reach the Schisma interval and natural sonic boundaries to the Field. However, the game could be carried out mathematically far beyond the practicalities of setting up the musical scale. Given a fairly large monochord with a string length of, say, that of a sitar, then the comma interval can easily be demonstrated and sounded. The Schisma, on the other hand, is doubtful. This does not necessarily mean that the mathematicians were not aware of the Schisma, at least as a mathematical entity.

5-Limit

There is much evidence that this mathematical game was carried quite far, definitely beyond the necessities of setting up a seven note scale, and possibly beyond the frontiers of our pitch discrimination perception.

The double acts as a kind of index, limiting the amount of information to some finite set. Obviously, a double such as 216,000:432,000 will contain many more members and cover a wider portion of the Field than a double like 36:72. Any number and its double can theoretically serve as the limiting index for a number series, but some doubles are more interesting than others. Those doubles which are the most interesting and significant from a musical and cosmological view are those which are invariant under reciprocation. In other words, they contain the same scale as a rising and a falling pattern. The simplest example of such a sequence is the musical series 6:8:9:12. Figure 15 shows some other patterns which embody relatively large regions of invariance. Once the numerosity is great enough, the whole sruti Field sits in the region of invariance. Interestingly enough, the double 12,960,000:25,920,000 harmonizes not only the sruti Field but also the wider Just Field shown in Figure 7. The first number has been called Plato's master cyclical number. The latter of the two numbers has a geo-physical connection, since 25,920 is the traditional measure of the natural cycle of precession. All of these numbers are divisible only by multiples of 2, 3, and 5.

-I now have more doubts, possibly Indus

BABYLONIAN-SUMERIAN ORIGINS

There can be little doubt that this mathematical-musical game originated in Sumer, probably between 4000 and 2000 B.C. They employed a number system which used number base 60 (as well as 10). Now base 60 is peculiarly suited to this investigation, since it becomes easy to clear fractions. The types of calculations involved in this musical work had evolved to utmost facility, since the Babylonian art of calculation in the 3rd millenium was quite advanced--in many ways comparable to the mathematics of the early renaissance. They possessed tables of reciprocals, multiplications, squares and square roots, cubes and cube roots, exponential functions, coefficients giving numbers for practical computations, and numerous metrological calculations giving the areas of rectangles, circles, and so on. They also used the Pythagorean Theorem a 1000 years before Pythagoras. They were accustomed with the treatment of the ratio of two numbers as an entity.

In addition to this technical know-how, organologists point out the advanced state of Babylonian (and Egyptian) stringed and wind instruments, and the generally high position afforded to music in their culture. -Contemporary Indus maybe just as high.

The standard Babylonian Table of recoprocals is shown as Figure 16. The first 60 numbers are gods. Notice that the number series is familiar since we have already seen it as a monochord series double. The Diatonic scale is found between the double

30:60. In this diagram I have translated the numbers into their base 10 equivalents, but it is important to realize just how convenient for musical arithmetic is base 60. The sexagesimal arithmetic makes it possible to erect reciprocal tables in which all fractions are finite.

The standard Table of reciprocals stops at the number 81. This is also significant, since the ratio 80:81 is the Syntonic comma. The harmony of numbers resolves to the comma scale. This ratio first appears between the ratios 9/8 and 10/9, the two alternative forms of the Just wholestep intervals. The comma is the basic step interval for the entire Field.

In the Babylonian-Sumerian tradition, the three greatest gods are Anu-an (60), Ea-Enki (40), and Bel-Enlil (50). Since these numbers can also be interpreted as ratios, Anu-An is 60/60, which is the unison and octave, the magical interval which makes the whole Just Field possible. Ea-Enki is 40/60 = 2/3, representing the horizontal axis on the Field. This axis creates the Schisma interval, and hence the 'eastern and westwern' boundaries of the Just Field. Ea-Enki is traditionally called "The Divine Patron of Music," a fitting name for the intervals of the Fourth and Fifth. The god Bel-Enlil is 50/60 = 5/6, representing the Minor Third--Major Sixth axis. An extension of this axis results in the

song, representing prayer, is supreme through all the divisions of the world." Those divisions number 30.

The 360 day year seems absurd to modern astronomers, who point out that very basic observations would soon teach them of the 365 and 1/4 day solar year. Yet the Vedic peoples, as well as the Egyptians, stuck to the numbers which were musically more significant. The excess of the true solar year over the calendar year forms a kind of 'cosmic comma,' being a ratio of about 24.8 Cents (compared to a Ditonic comma of 23.5 Cents). Therefore the comma also appears in the calendar.

The number of days in a week is also interesting. The Sumerians had a 5 day week, 72 weeks in the year. They also invented the 6 day week, with 60 weeks in the year. Finally, at some early point in history, the 7 day week was introduced, and took hold, sooner for the Hindus and Hebrews than for the Egyptians and Greeks. At first it seems surprising that an incommensurate number would be introduced into the calendar; but again there is a musical connection. Most musical scales have 7 tones. In addition, there are 7 planets or 'luminaries' visible in the sky. The ancients attempted to harmonize music and astronomy, since both reflect the orderly patterns of the universe. It is also an interesting synchronicity that, given a 7 day week, then the solar year consists of 52+ weeks--a possible allusion to the 53 note comma scale.

In the Ṛgveda, "the moon is that which shapes the years." Now 12 lunations create a 354 day period. This period is deficient of the true solar year by a ratio of about 53.9 Cents--a somewhat over-sized diesis (double comma). We see this 'deficiency' on the Field diagram between pitches C and B# (follow the line of thirds from C-E-G#-B#). Hence the moon harmonizes with the sun in a musical manner which highlights the Major Third--Minor Sixth axis. This axis is prominant in the 31-E.T. tempering of the Field, with its double-comma step interval. The 31-E.T. temperament is very significant since it harmonizes the 7th Harmonic as well as introduces ratios relating to the Golden Mean into the tuning fabric. Although this tuning is important to the Pythagorean School, there is no indication of its recognition in Vedic India.

-no temperament in India – temperament is "esoteric"

The Ṛgveda is replete with a number of long 'cosmic' cycles, which also prove to be significant monochord numbers. The Bṛhaspati cycle is 60^2 = 3,600 years. The Prajāpati cycle is 60^3 = 216,000 years. The latter cycle harmonizes a significant portion of the Just Field. These numbers again suggest a close connection with Babylonian sexagesimal arithmetic. The relations between Hindu and Babylonian astronomy are still largely unclear, but their joint musical origins cannot be denied.

Hindu cosmology recognizes four Yugas, or "ages of the world." These numbers are also only divisible by 2, 3, and 5, forming a "ten-ness" in which our own Kali Yuga "dark age" is the unit. Notice that the multiplication of monochord numbers by powers of 10 is a 'fast' way of increasing their 'numerosity.' The

number 10 was considered sacred for various reasons which we will not pursue here. Here are the Yuga numbers:

	Kali Yuga	= 1	= 432,000
	Dvāpara Yuga	= 2	= 864,000
	Tretā Yuga	= 3	= 1,296,000
	Kṛta Yuga	= 4	= 1,728,000
total:	Mahā Yuga	= 10	= 4,320,000

The number 432,000 is also an important Babylonian cosmic number. 1000 Mahā Yugas constitute a Kalpa of 4,320,000,000 years, and its double is Brahmā--the immense being--a cycle of 8,640,000,000 years. The use of such huge numbers as monochord divisions becomes quite impractical from an experimental standpoint, yet they have theoretical significance in illustrating the Schisma. Amazingly enough, this is not the largest number to surface in the Ṛgveda. The Kalpa is called "one day of Brahmā." Hence one year of Brahma is 4,320,000,000 * 360 = 1,555,200,000,000 years. Brahmā's life lasts for 100 years. After this time, Brahmā ceases to exist and the universe dissolves. This time span, which is called the "life of Brahmā" is 155,520,000,000,000 years. This number is doubled to form the Puruṣa number 311,040,000,000,000. Such inconceivably huge numbers must have been generated as a mirror to the infinite. Although they go far beyond the practical bounds of monochord procedures, still they have the proper prime factors, and could be mapped on the Field grid. For example, the number 155,520,000,000,000 is equal to $2^{16} * 3^5 * 5^{10}$. Perhaps such numbers refer to the scale which is the ultimate resolution of Just Intonation in a closed cycle: the Schisma scale of 612 steps to the octave.

Even if we deny these lofty assumptions, we must not forget that the song-poems of the Ṛgveda are saturated in mysterious arithmetical and geometrical details. The universe is seen as a victory of gods over demonic forces which can be defeated but never eradicated. And <u>everything</u> is counted. These numbers seldom make any sense, especially in translation. Yet they present a certain attitude which is a perfect companion for our monochord procedures. The poems are to be understood allegorically, and their 'logic' involves a constant sacrifice of meaning to numerical context. The quasi-scientific empirical procedures which had set up the Field of musical relations are extrapolated to help explain the movements of the cosmos.

In FOUR-DIMENSIONAL MAN[6] the author, attempting to retrieve meaning from the Ṛgveda by internal comparison and reconstruction, concludes that the sacred book embodies a "lattice logic" consisting of four "languages:" the language of Non-existence (Asat), of Existence (Sat), of Image and Sacrifice (Yajña), and of Embodied (Ṛta) Vision (Dhīḥ). These philosophical languages all have their origin in music. The language of Non-existence is exemplified by the pitch continuum or chaos (Vṛtra-the dragon). The language of Existence is exemplified by any and every tone, every number which defines a scale or metrical scheme (a mask of God). The language of Images and Sacrifice comes from the various

alternative number sets which can conflict with each other, creating the need for some limitation and compromise in the scale. Finally, the language of Embodied Vision is required to maintain the validity of the alternative number sets so that no one set becomes dominant. We must constantly be willing to change our viewpoint with regard to the numerical layout of the scale. Indra slays the dragon by chopping up the pitch continuum, but the conflict never ends, since the powers of prime numbers are incommensurate.

EPISTEMOLOGY

These philosophical orientations stem from an attitude which judges nature primarily on a model of sound. In view of the inconstancy of the world of form, the ancients questioned the reality of static, spatial phenomena, believing rather that transient, temporal dynamic rhythms are a better guide to understanding the inner substance of nature. Hence tuning procedures and models are central in understanding the universe.

Our knowledge of the external world is conditioned by our possibilities of perception. The phenomenal aspects of things (which we tend to confuse with reality) are an effect of the limits of our sense perceptions. Mankind is limited to the five spheres of sensory perception, but other worlds may exist without our knowing it since we cannot perceive it. We thus perceive five aspects of the external world, and they do not necessarily co-incide. With these perceptions, we construct our ideas about the world, and confuse them with the world itself.

Indian thought tells us that we can escape the limitations of the perceptual field by orienting the perceptual apparatus inward by meditation, perceiving deeper and more profound aspects of the world. The inner world and the outer world are one. The outer world is a sea of vibrations with various levels of complexity. Through hearing, however, we have a direct perception of the vibratory state of the universe. The other senses produce perceptions of vibratory states which are more complex and difficult to analyse. Thus the vibrations of sound are the forms of perception closest to the primary state of cosmic manifestation. The creation itself is a mental vibration, the thought of the universal being-ness. The world is thus a manifestation of mental energy, a thought which appears as a reality to us because of the limits of our perceptions. Sound is thus an image of the world's intrinsic nature. It is also, therefore, the vehicle of revelation and all spiritual development. Ravi Shankar puts it well in his book[7], "Sound is God--Nada Brahma. That is, musical sound and the musical experience are steps to the realization of the self."

Pure vibration, as an organized expression and vehicle of thought, is aurally perceived by us through two forms: the spoken language and the musical language. These two forms are closely linked. Certain sound relations and certain syllabic units are

vibratory expressions of certain concepts, and sounding them brings them into existence. The more music and language approach these forms of the true language, the more profound is their effect on conciousness. Deviation from these norms results in weakness and artificiality, as our minds try to reconstruct the true norms through symbols which may become arbitrary. If we use formulas articulated to correspond to certain cosmic entities, we immediately come into contact with them. Hence the use of mantras in yoga practice.

The musical language is somewhat more abstract than the verbal language, since here we perceive sound manifestations of numerical relationships, which correspond to sentiments or ideas. It is a direct perception of the mathematical nature of the universe. Hence the use of a natural tuning system rather than an artificial one is of great importance for the communication of the true subtlety of the world. Each tuning system has its own peculiar character or 'melos,' and the accurate tuning of each scale sets up the corresponding 'mood' only if the tuning is accurate.

According to tradition, the divine art of music was created by the holy Hindu trinity: Brahmā the Creator, Vishnu the Preserver, and Shiva the Destroyer. Shiva's cosmic dance represents the rhythms of the universe. Art, in turn, is threefold: vocal music, intrumental music, and the dance. We already saw this 'threeness' deified by the Babylonians through their three chief gods who relate by the ratio 4:5:6, and through the inherent 'threeness' of a number field restricted to powers of the numbers 2, 3, and 5. It is tempting to relate the Hindu trinity to the three powers, but exact correspondances are not important. What is important is the cultural preservation of the wonderful intonations which have served by elevate human kind for milleniums.

It is due to the above epistemology that musical information was held in such high esteem in ancient times. The audial sense is a direct response to the proportional laws of the universe. It is thus the basis for science and philosophy. It is to be held in the highest regard as a vehicle for enlightenment. We see this attitude in the writings of Plato and many other ancient writers. Music is ultimately a pathway to God. The typically Western attitude toward music, that it is merely a pastime or an entertainment for personal pleasure, may stem from the prolonged use of a crude and artificial tuning system (12-E.T.) so that the verities are hidden; on the other hand, the influences could also work the other way. The debasement of the function of music to entertainment, and the denial of its ancient spiritual role, may have led to an attitude where it is considered as unimportant that the intonation be accurate. The saturation of modern culture in excess noise could also be a factor. Potentially, degenerate music could do great harm to the individual and the society. Already in classical Greece Plato warned of the degrading effect of certain kinds of music on the populace.

It could be possible that the veiled and confusing elements in the Nāṭyaśastra stem from the desire to preserve the sacred monochord knowledge from defilement by the populace. That modern Indian musicians seldom use the monochord ratios is a moot point. They have sung the ratios accurately for so long, that the knowledge is "in the ear." They had learned their lessons from the ṛṣis, and learned them well. To our endless spiritual delight, the Hindu musicians have preserved the essense and the spirit of the Vedic teachings.

NOTES:

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2) Coomaraswamy, A.K.: THE PARTS OF A VINA; Journal of the American Oriental Society, New Haven, 1930.

3) Kolinski, M.: THE ORIGIN OF THE INDIAN 22-TONE SYSTEM; Studies in Ethnomusicology, Volume 1, New York, 1961.

4) Fox Strangways, A.H.: THE MUSIC OF HINDUSTAN; Oxford, 1914, reprinted 1966.

5) Neugebauer, Otto: THE EXACT SCIENCES IN ANTIQUITY; New York, Dover Publications, 1969.

6) Antonio T. de Nicolas: FOUR-DIMENSIONAL MAN: THE PHILOSOPHICAL METHODOLOGY OF THE RGVEDA; Bangalore, Dharmaram College Studies, No. 6, 1971. Quoted in Mclain, E.G.: THE MYTH OF INVARIANCE: THE ORIGIN OF THE GODS, MATHEMATICS, AND MUSIC FROM THE RGVEDA TO PLATO; Shambhala, Boulder and London, 1978, p. 2.

This book is an excellant introduction to the topic of musical mathematics in the ancient world. Unfortunately, it has some major flaws. The author uses an obsolete, 19th Century notation for Just Intonation, a notation which confuses the true structure of the Field. Moreover, the author equates temperament with 12-Equal-Temperament, and uses the ancient data to 'prove' that the ancients conceived and approved of 12-E.T. Now while I would agree that it was quite likely that the ancients understood the notion of temperament, it seems highly doubtful that they would choose an inferior form of temperament over the vastly superior forms of 53-E.T. or 31-E.T. The author is blind to any other form of temperament besides 12-E.T. In this he displays a typically western bias. The book inspired me to re-interpret the data along a less ethno-centric line.

7) Ravi Shankar: MY MUSIC, MY LIFE; Simon and Schuster, New York, 1968, p.17.