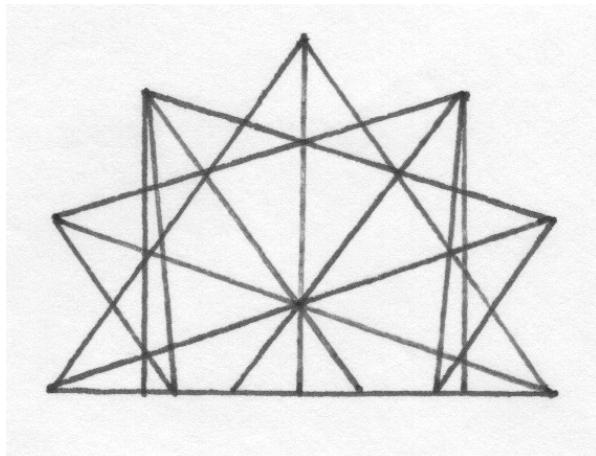


The Natyasastra and Vedic Harmonics diagrams



Siemen Terpstra
www.siementerpstra.com

The Vīṇā Experiment

Figure 1

Unchanged Vīṇā:

(Pitch numbers above traditional names)

(C)	(D)	(E _b)	(E)	(F)	(F#)	(G)	(A)	(B _b)	(B)	(C)
0 5a	1 Ri	2 Ga	3 5	6 Ma	7 Pa	8 10	11 12 13 14 15 16 Dha	17 Ni	18 19 20 21 22=0 5a	

Changed Vīṇā

First lowering:

21 5a		2 Ri	4 Ga		8 Ma		12 Pa	15 Dha	17 Ni		21 5a
----------	--	---------	---------	--	---------	--	----------	-----------	----------	--	----------

Second lowering:

20 5a		1 Ri	3 Ga		7 Ma		11 Pa	14 Dha	16 Ni		20 5a
----------	--	---------	---------	--	---------	--	----------	-----------	----------	--	----------

Third lowering:

19 5a		0 Ri	2 Ga		6 Ma		10 Pa	13 Dha	15 Ni		19 5a
----------	--	---------	---------	--	---------	--	----------	-----------	----------	--	----------

Fourth lowering:

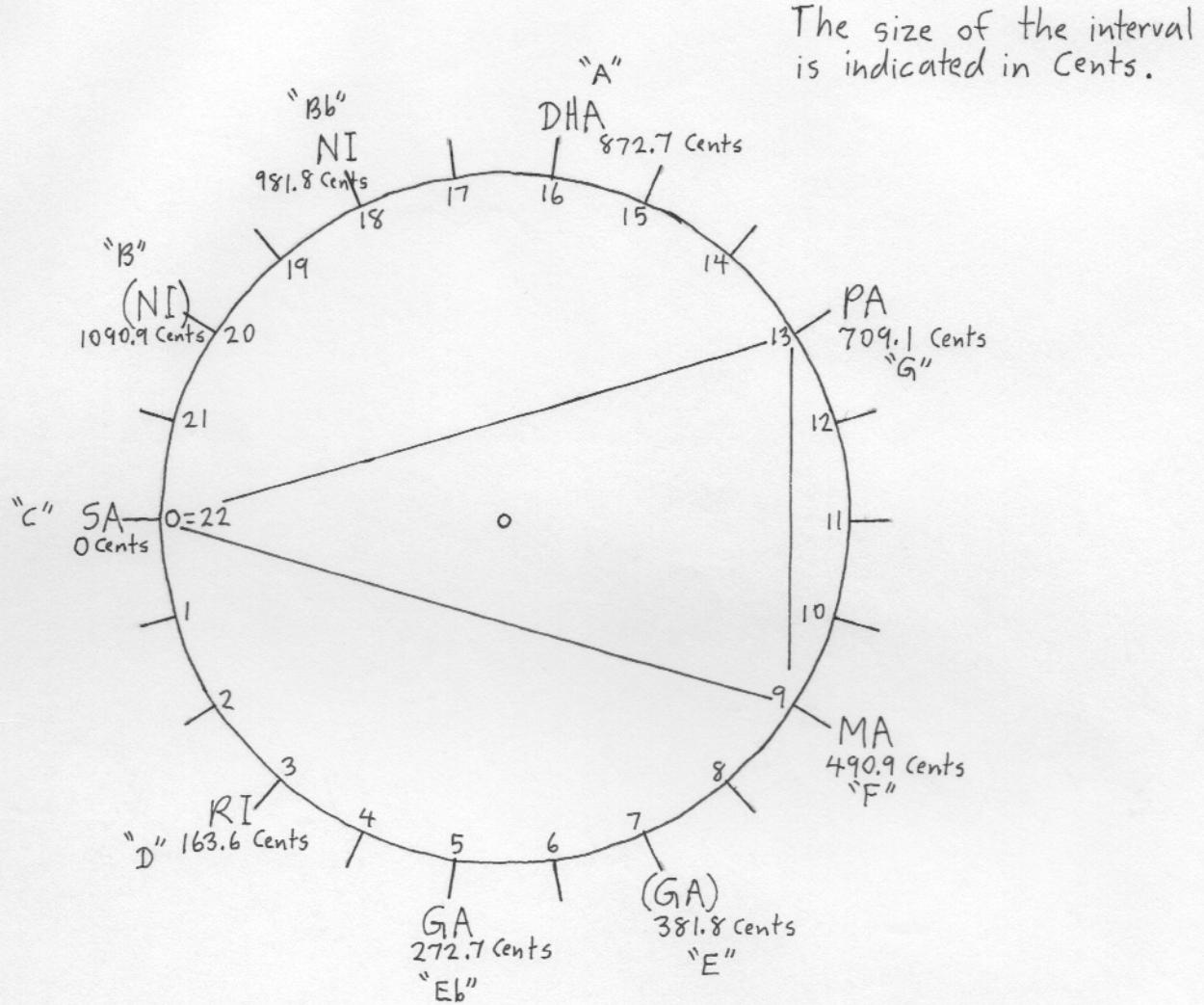
18 5a		21 Ri	1 Ga		5 Ma		9 Pa	12 Dha	14 Ni		18 5a
----------	--	----------	---------	--	---------	--	---------	-----------	----------	--	----------

The śrutis are assumed to be the same size as one another. The text says "lower again, in exactly this manner, (punarapi tadvadēvapakarṣat)." Indeed, the experiment would make little sense if the śrutis were different sizes.

The implication is thus that the śruti system is 22-Equal-Temperament. It is therefore convenient to number these pitches from 0 to 21, with 22=0. The number gives the number of steps in the tuning. For example, the musical fourth (Ma) is 9 steps in size. The musical Fourth thus can be given the number 9 as an alternative pitch name.

Figure 2

The ancient Śruti system interpreted as 22-Equal-Temperament:
The intervals are drawn on the circular graph, which defines
an octave as one round, and counter-clockwise movement
as an increase in pitch:



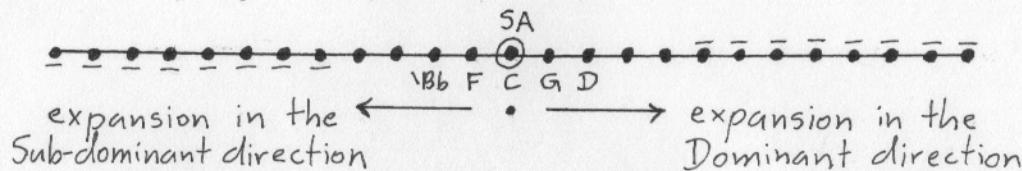
Comparison with Just Intonation ratios (showing the considerable inaccuracy in 22-E.T. tuning):

Just Ratio	+ $\frac{9}{8}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{5}{3}$	
In Cents:	0	203.9	315.6	386.3	498.1	701.9	813.7	884.4
22-E.T. approximation:	0	163.6	272.7	381.8	490.9	709.1	818.2	872.7
difference (all in Cents):	0	-40.3	-42.9	-4.5	-7.2	+7.2	+4.5	-11.7

One can see that the traditional consonances are unacceptably altered: the Fifth ($\frac{3}{2}$) is sharp by 7.2 cents and the Fourth ($\frac{4}{3}$) flat by the same amount—an amount easily audible.

Figure 3 The Śruti system as originating in the "cyclical" expansion of the line of Fifths-Fourths:

Abstract presentation :



The two directions of expansion are separated in order to present acoustical data on the sizes of the intervals:

Expansion in the Dominant direction

Cents	0 = 1200	701.9	203.9	905.9	407.8	1109.8	611.7	113.7	815.7	317.6	1019.6	521.6	23.5
Schismas	0 = 612	358	104	462	208	566	312	58	416	162	520	266	12
Commas	0 = 53	31	9	40	18	49	27	5	36	14	45	23	1
Pitch name	C	G	D	A	E	B	F#	D _b	A _b	E _b	B _b	F	C

Expansion in the Sub-dominant direction

	1176.5	678.4	180.4	882.4	384.3	1086.3	588.3	90.2	792.2	294.1	996.1	498.1	0 = 1200	Cents
600	346	92	450	196	554	300	46	404	150	508	254	0 = 612	Schismas
52	30	8	39	17	48	26	4	35	13	44	22	0 = 53	Commas

←

C	G	D	A	E	B	F#	D _b	A _b	E _b	B _b	F	C	Pitch name
---	---	---	---	---	---	----	----------------	----------------	----------------	----------------	---	---	------------

Figure 4 The Śruti system as originating in the "divisive" principle of Just Intonation :

The nucleus of Just Intonation tuning is the Just Major Triad - tuned using pure Fifths, pure Major Thirds, and pure Minor Thirds. The ratios are derived from Monochord procedures and may be "mapped" as a triangular grid pattern around the line of pure Fifths-Fourths. The nucleus is represented below in ratios, along with a partial expansion. The full expansion is not shown in ratios, since our main interest is in the metrical properties of the inherent intervals :

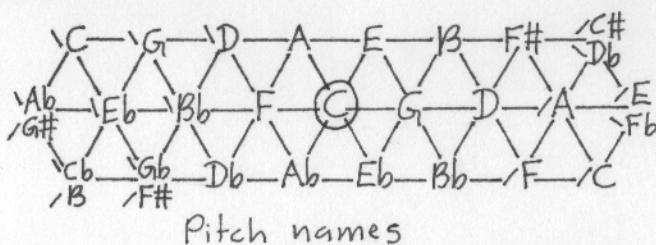
$$\begin{array}{c} E \\ \diagdown \quad \diagup \\ C \text{---} G \\ \diagup \quad \diagdown \\ A \end{array} = \frac{5}{4} / \frac{3}{2}$$

$$\begin{array}{c} A \text{---} E \\ \diagdown \quad \diagup \\ F \text{---} C \text{---} G \\ \diagup \quad \diagdown \\ Ab \text{---} Eb \\ \diagdown \quad \diagup \\ A \end{array} = \frac{5}{3} / \frac{5}{4} / \frac{2}{1} / \frac{3}{2}$$

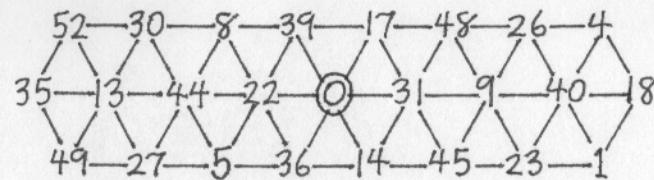
$$\begin{array}{c} D \text{---} A \text{---} E \text{---} B \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ Bb \text{---} F \text{---} C \text{---} G \text{---} D \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ Db \text{---} Ab \text{---} Eb \text{---} Bb \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ A \end{array} = \frac{10}{9} / \frac{5}{3} / \frac{5}{4} / \frac{15}{8}$$

$$= \frac{16}{15} / \frac{4}{3} / \frac{2}{1} / \frac{3}{2} / \frac{9}{8}$$

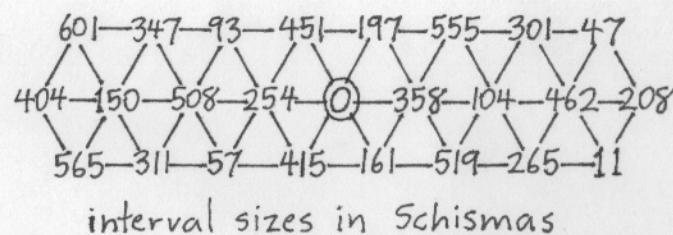
Below, the Fully expanded Field is shown in pitch names, Commas, Schismas, and Cents :



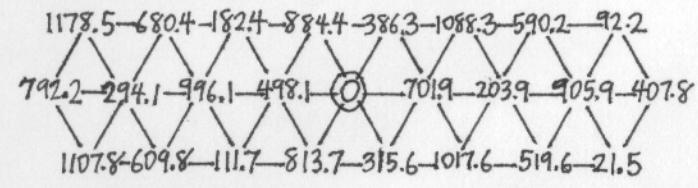
Pitch names



Comma sizes of intervals



interval sizes in Schismas



interval sizes in Cents

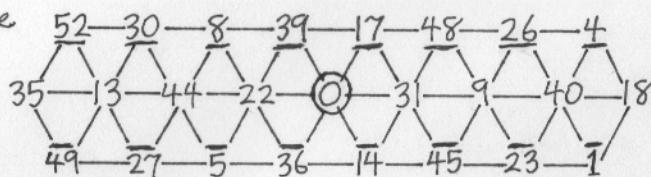
Figure 5 showing the practical identity of the two fields (cyclical and divisive), the intervals are expressed in their comma sizes and schisma sizes:

The "cyclical" Field: (Schisma measure shown above, comma measure shown below):

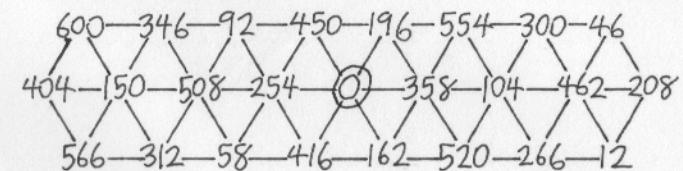
600	346	92	450	196	554	300	46	404	150	508	254	0	358	104	462	208	566	312	58	416	162	520	266	12
52	30	8	39	17	48	26	4	35	13	44	22	0	31	9	40	18	49	27	5	36	14	45	23	1

These pitches may be
redrawn as :

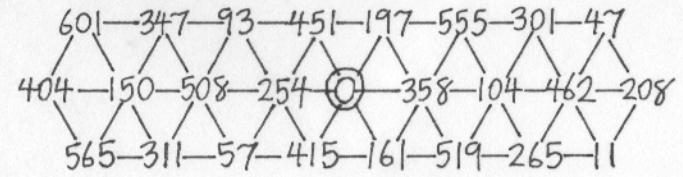
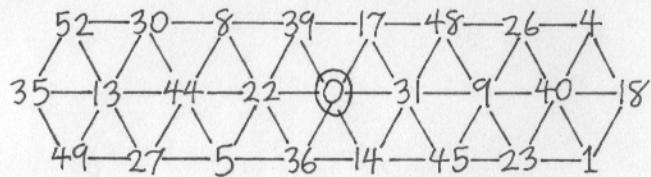
in commas:



in schismas:



Note the practical identity to the "divisive" Field shown below:



Differences in pitch amount to only one schisma, which is generally agreed to be subliminal or at least at the very edge of our pitch discrimination abilities! Hence the Sruti Field may be modelled as a scale in which the smallest interval is a comma (and, therefore, the tuning system is practically identical to 53-Equal-Temperament).

All scales are subsets of the above pitches. The Sruti pitches are shown on the circular graph as Figure 6.

Figure 6 The *srutis* shown on the circular graph which displays their metrical properties:

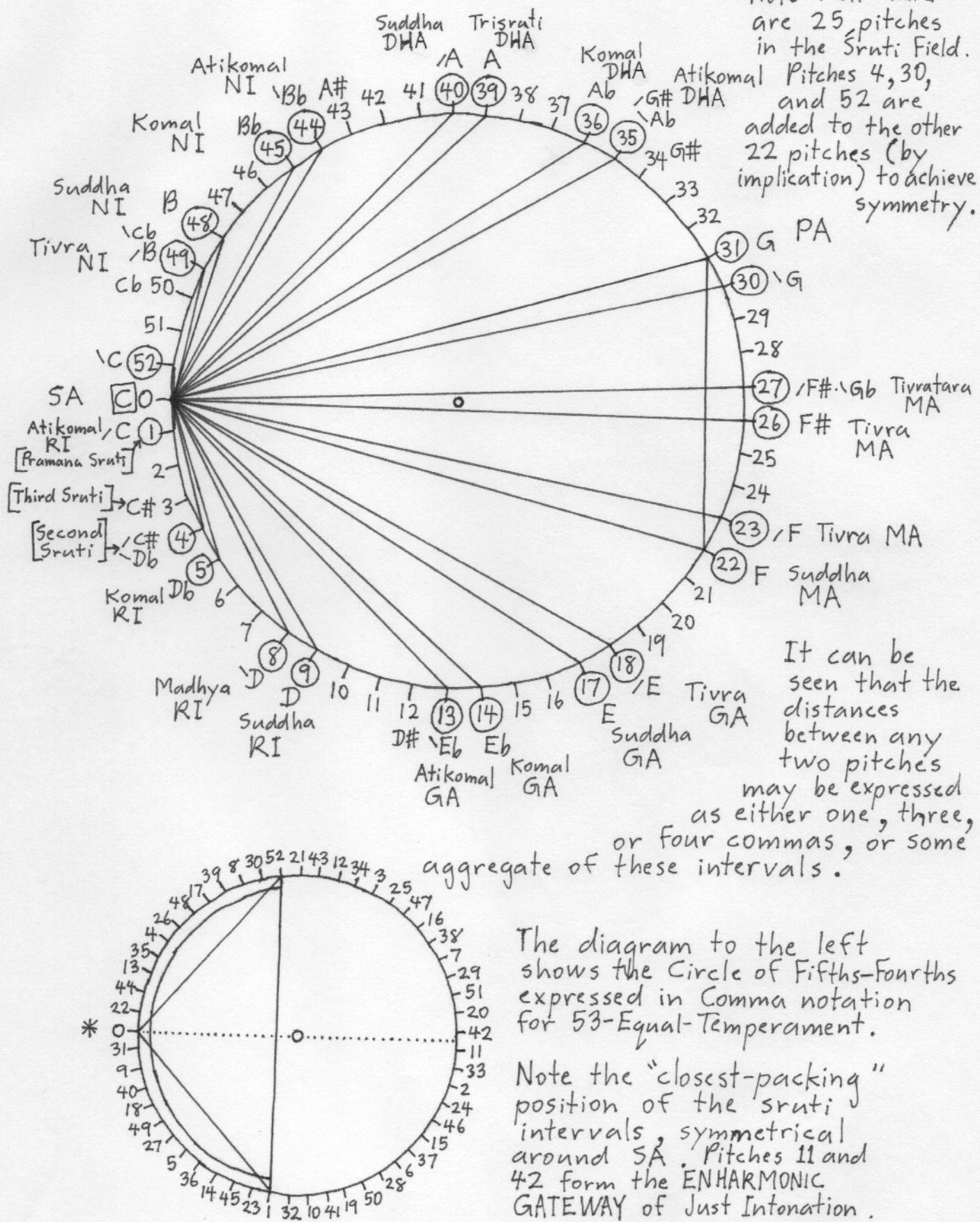


Figure 7 Quasi-Just Intonation (53-E.T.)
Pitch Field

Showing the special position of the Sruti Field subset.

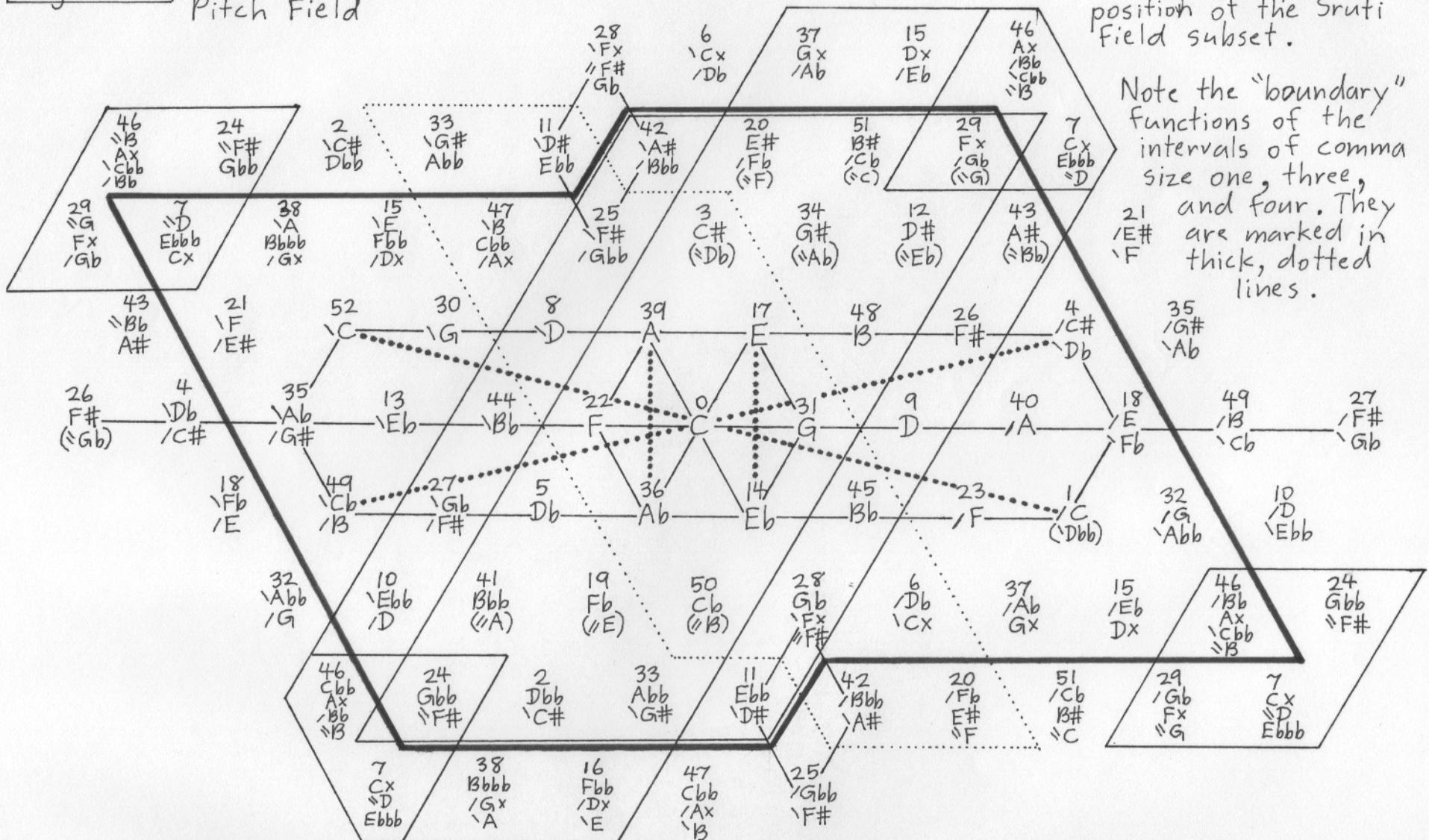
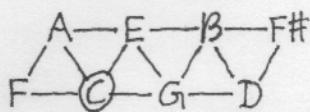
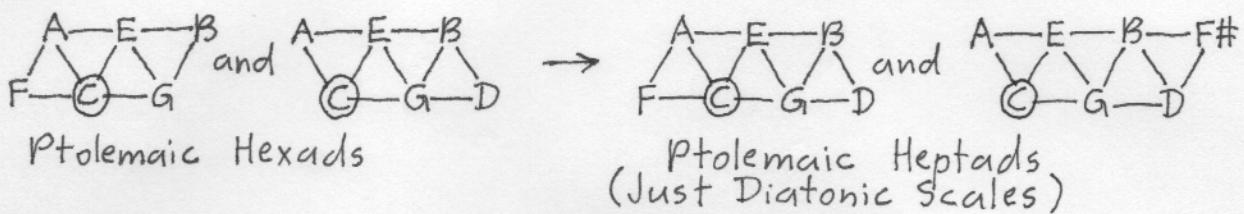
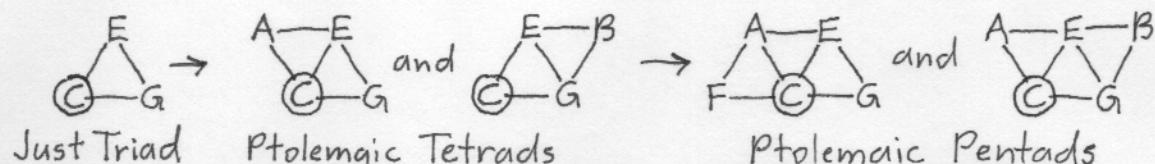


Figure 8 The Ptolemaic Ogdoad

Ptolemaic expansion :

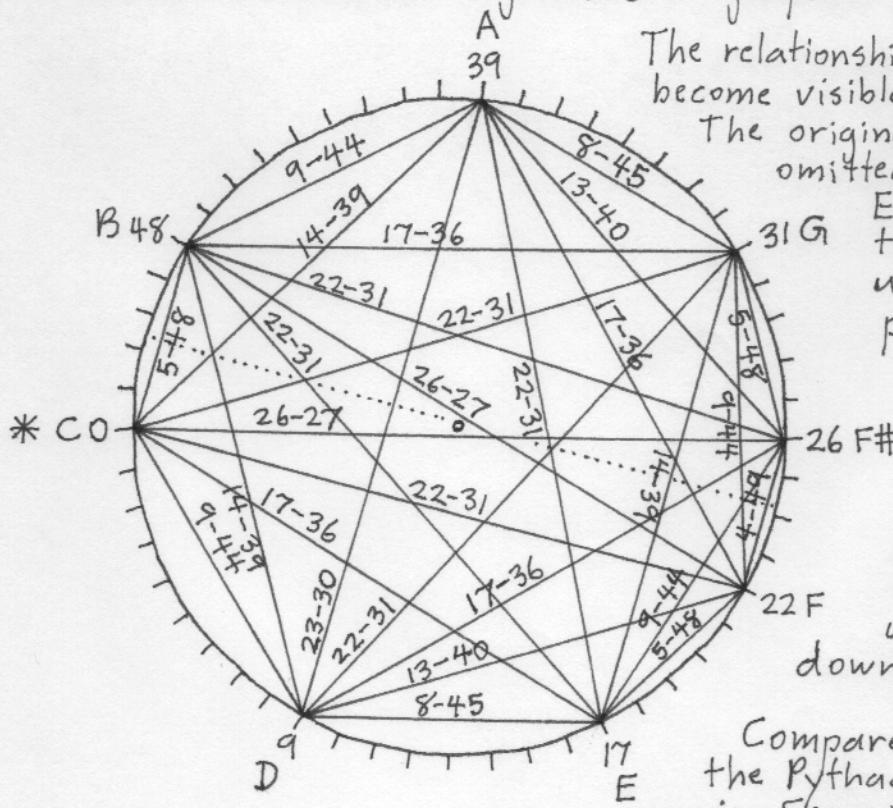


Maximum expansion without incurring a comma juxtaposition



Note the comma juxtaposition. Any further expansion of the Ptolemaic sequence leads to further comma relations.

Below the Ptolemaic Ogdoad ① is graphed on the circular graph:



The relationships between the pitches become visible as lines on the graph. The original colour codes are omitted for this article.

Every harmonic pattern thus forms its own unique "mandala" pattern on the graph.

In place of the colour code, the interval sizes are marked in commas; eg. between C and D is marked 9-44, that is, 9 commas up in pitch or 44 commas down in pitch.

Compare this pattern with the Pythagorean Ogdoad shown in Figure 9.

Figure 9 The Pythagorean Ogdoad

Pythagorean expansion :

F—C—G Pythagorean "Suspended" Triad

F—C—G—D Pythagorean "Suspended" Tetrad

\Bb—F—C—G—D Pythagorean Pentad (Chinese scale)

\Bb—F—C—G—D—\A Pythagorean Hexad

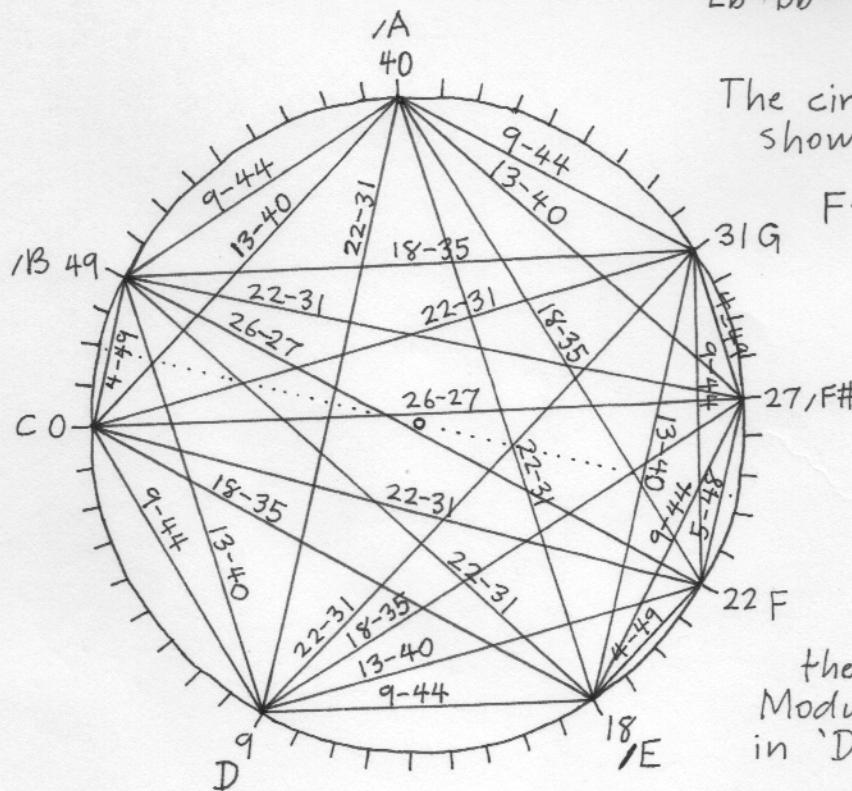
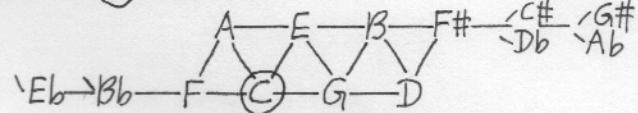
\Eb—\Bb—F—C—G—D—\A Pythagorean Heptad ('Dorian' Scale)

\Eb—\Bb—F—C—G—D—\A—\E Pythagorean Ogdoad

\Ab—\Eb—\Bb—F—C—G—D—\A—\E Pythagorean Ennead. Note that the interval between \E(18) and \Ab(35) is 17 commas—a just Major Third.

Any long Pythagorean sequence (longer than 8 tones) results in a Ptolemaic structure. An example below shows a chromatic scale which, rewritten, is seen to embody the Ptolemaic Ogdoad:

A—E—B—F#—\C#—\Ab—\Eb—\Bb—F—C—G—D



The circular graph at left shows the Pythagorean Ogdoad

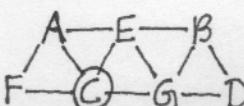
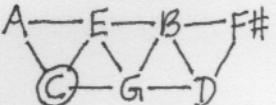
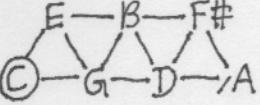
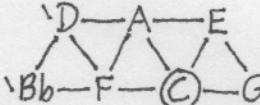
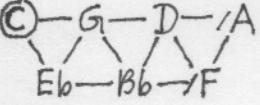
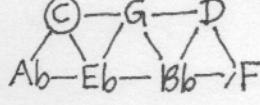
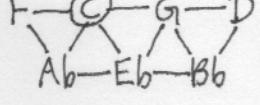
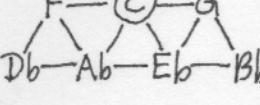
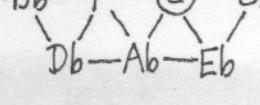
F—C—G—D—\A—\E—\B—\F#

Compare this tonal structure with the Ptolemaic Ogdoad shown in Figure 8.

The two structures are similar but have very divergent harmonic characteristics.

If one omits the \F#, then the resultant scale is the Modus Primus 'Dorian' scale in 'D.'

Figure 10 A sampling of some of the prominent Heptatonic modes (scales) in the Indian classical music tradition. Note that all scales have C-G (SA-PA) since the drone is ever present in the music.

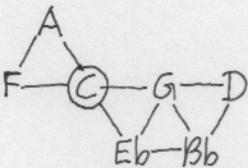
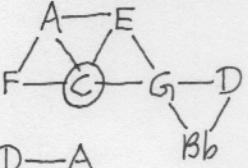
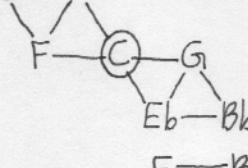
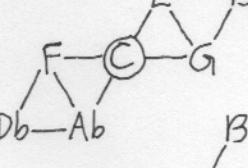
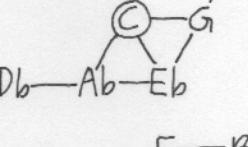
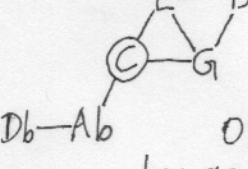
<u>Pattern on the Field Diagram</u>	<u>Scalar order</u>	<u>Name</u>
'E _b →B _b →F→C→G→D→A Pythagorean 'Dorian'	C 0 9 13 17 22 31 40 44 0 C D E _b F G A B B _b C	
	C 0 9 17 22 31 39 48 0 C D E F G A B C	Bilaval ('Ionian') Suddha Major Scale
	C 0 9 17 26 31 39 48 0 C D E F# G A B C	'Lydian' Kalyan
	C 0 9 17 26 31 40 48 0 C D E F# G A B C	'Lydian' Kalyan
	C 0 8 17 22 31 39 48 0 C D E F G A B C	'Ionian' Bilaval
	C 0 8 17 22 31 39 44 0 C D E F G A Bb C	'Mixolydian' Khamaj
	C 0 9 14 23 31 40 45 0 C D Eb F G A Bb C	'Dorian' Kaphi
	C 0 9 14 23 31 36 45 0 C D Eb F G Ab Bb C	'Aeolian' Ashawari
	C 0 9 14 22 31 36 45 0 C D Eb F G Ab Bb C	'Aeolian' Ashawari
	C 0 5 14 22 31 36 45 0 C Db Eb F G Ab Bb C	'Phrygian' Bhairavi
	C 0 5 14 22 31 36 44 0 C Db Eb F G Ab Bb C	'Phrygian' Bhairavi

-continued over page

Figure 10 - continued from previous page.

'Bb-F-(C)-G-D-A-E	C 9 18 22 31 40 44 0	'Mixolydian' Khamaj
'Ab-Eb-Bb-F-(C)-G-D	C 9 13 22 31 35 44 0	'Aeolian' Ashawari
F-(C)-G-D-A-E-B	C 9 18 22 31 40 49 0	'Ionian' Bilawal
'Db-Ab-Eb-Bb-F-(C)-G	C 4 13 22 31 35 44 0	'Phrygian' Bhairavi
(C)-G-D-A-E-B-F#	C 9 18 27 31 40 49 0	'Lydian' Kalyan

The above patterns may be classified as either Pythagorean or Ptolemaic. The patterns below have been labelled 'Chromatic', and all involve a "triple-strand" of Fifths.

	C 9 14 22 31 39 45 0	'Dorian'
	C 9 17 22 31 39 45 0	'Mixolydian'
	C 8 14 22 31 39 45 0	'Dorian'
	C 5 17 22 31 36 48 0	Bhairav
	C 5 14 26 31 36 48 0	Todi
	C 5 17 26 31 36 48 0	Purvi

Obviously, many more scales are possible, given the large pool of tones available. Tuning is only one component within the make-up of a raga, a component conducive to numerical classification.

Figure 11 A simple monochord:
More elaborate models could have
multiple strings and a hollow body.

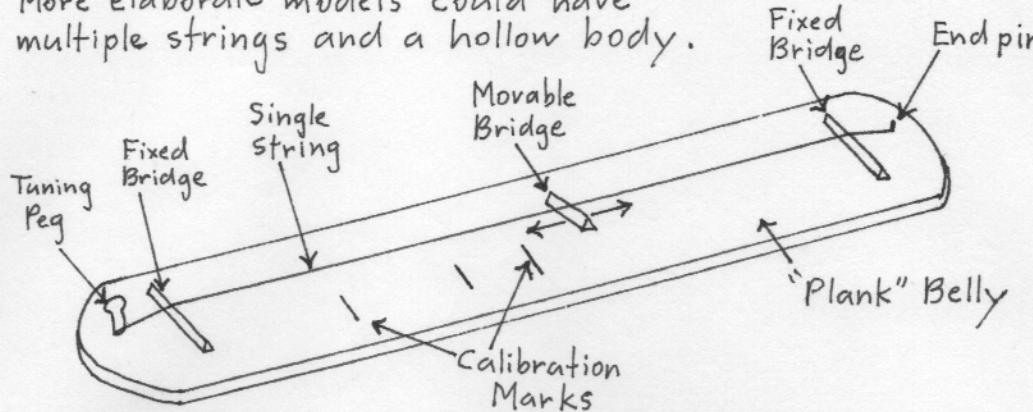
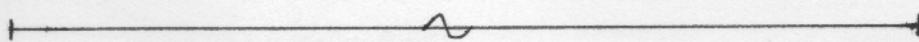


Figure 12

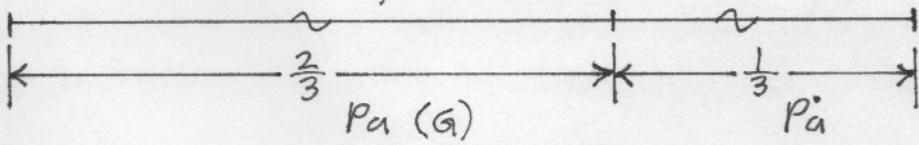
A simple demonstration of monochord divisions:
The open string vibrates at Sa (C)



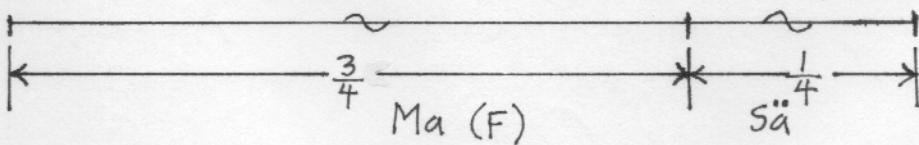
The placement of the movable bridge in the exact middle, that is, a division of the length by two, results in the upper Octave $S\ddot{a}$



The string now has two vibrating sides both sounding the octave.
The placement of the bridge at $\frac{1}{2}/3$ distance from either end results in the harmony of Pa and $\dot{P}a$.



The placement at $3/4$ distance results in Ma and $S\ddot{a}$



String divisions by simple ratios result in strong harmonies.
The ancient technique was to start from the middle (the octave) and tune the scale as a falling pattern to the fundamental.
Most importantly, they always avoided the use of fractions by clearing them through a least-common-multiple. For example:

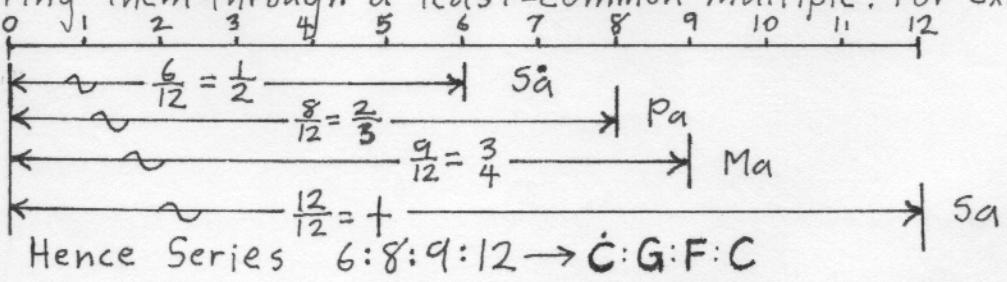
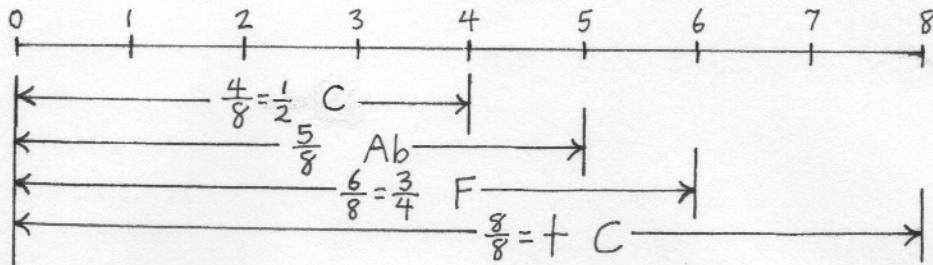
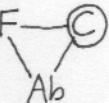


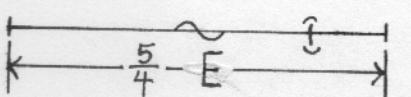
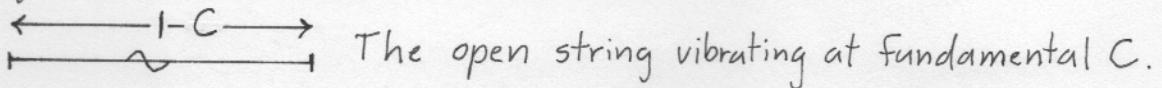
Figure 13 A division of the monochord string into the number series ruled by the double $\sqrt[4]{4:8}$ (4:5:6:8)



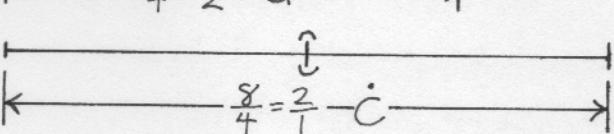
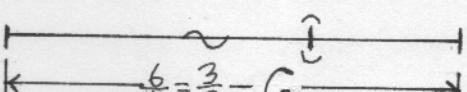
The resulting harmony is the Just Minor Triad associated with the Sub-harmonic Series :



Although not practical from an experimental point of view, the monochord string can theoretically be MULTIPLIED in addition to its division, for the two processes are reciprocal to each other. The diagram below shows the multiplication of the string following the above number series 4:5:6:8 :



We see that the multiplication of the string involves ratios which are identical to Frequency ratios and the Harmonic Series.



The resulting harmony here is the Just Major Triad associated with the Harmonic Series.

Whereas the monochord division produces harmonies in a characteristically falling scale from Sa to Sa, the monochord multiplication presents a rising scale from Sa to Sa.

Such reciprocation is understandable since the ratios may be used in a rising and a falling series. Thus

Series 4 : 5 : 6 : 8

C Ab F C - Falling series - monochord procedure
C E G C - rising series - harmonics procedure

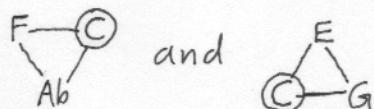
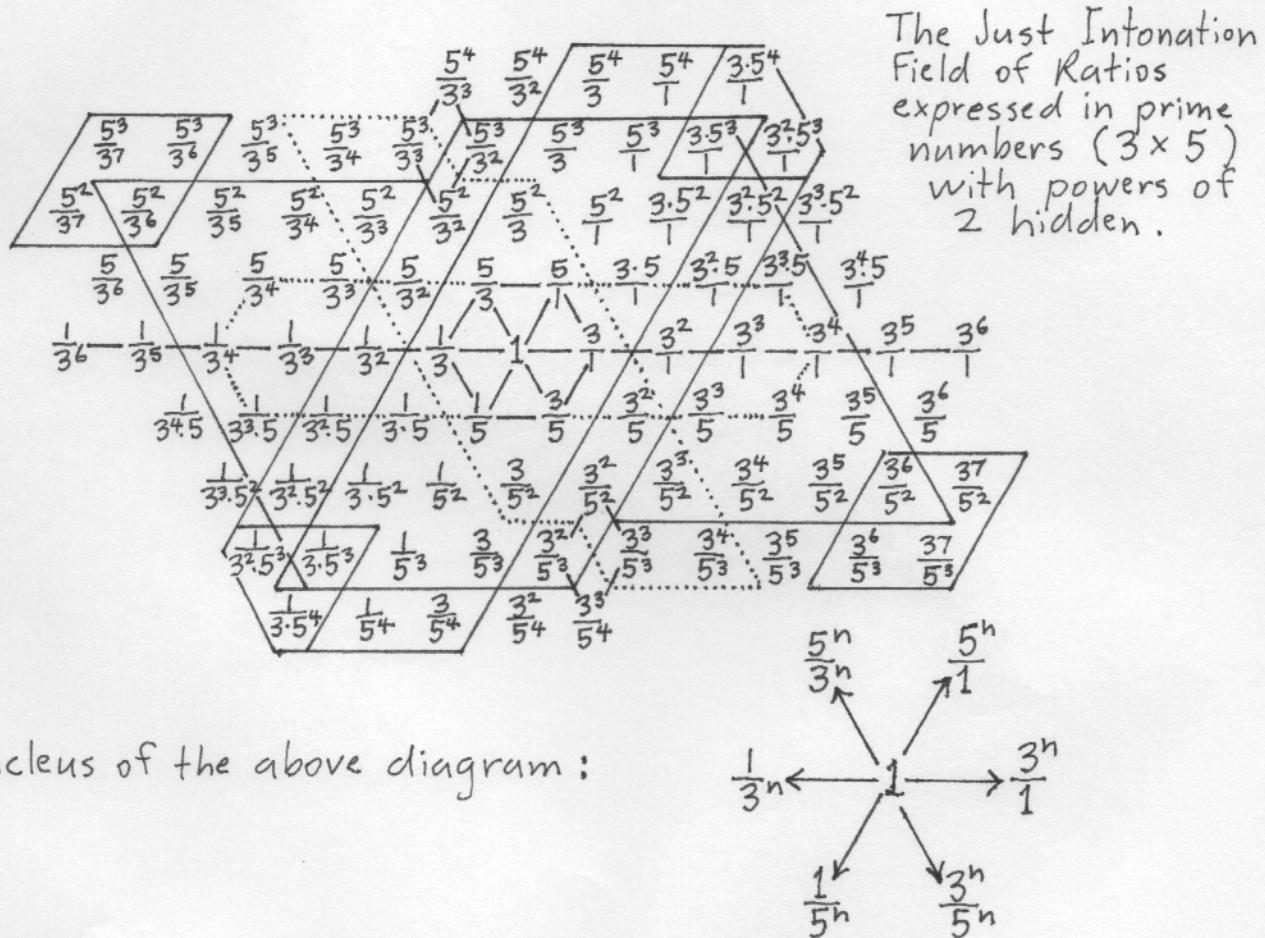


Figure 14 An important rule in exploring monochord divisions:

Only numbers divisible by 2, 3, or 5 are allowed in a series ruled by whatever double we choose. For example, the series 4:5:6:8 omits 7—a prime number strictly forbidden from taking part in the game.

In effect, we have again constructed our Field Diagram for Just Intonation, since that Field may be looked at in its most abstract sense as the 3×5 multiplication table. The powers of the number 2 are hidden since the octave may be considered a "floating" or "invisible" quantity :



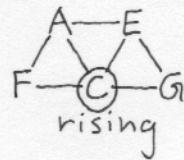
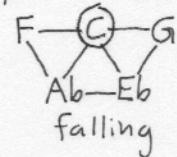
The prime numbers 2, 3, and 5 define Just Major Triads and their reciprocal Just Minor Triads. As the series double (the ruler of the series) gains greater "numerosity," it expresses a larger segment of the Just Intonation Field.

Of greatest interest are those "doubles" which occupy the same "territory" under reciprocation. The simplest example is the Series 6:8:9:12 from Figure 12 : 6 : 8 : 9 : 12

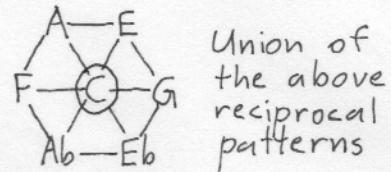
C	F	G	C-rising
C	G	F	C-falling

Figure 15 A small sampling of some prominent monochord series:

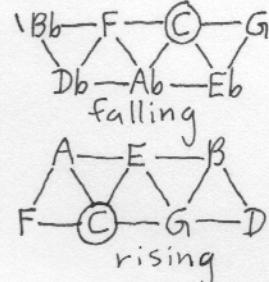
Double 12:24 12 : 15 : 16 : 18 : 20 : 24
 Falling - C Ab G F Eb C
 rising - C E F G A C



This series defines two important Ptolemaic Pentads. Note that C-F-G are invariant under reciprocation

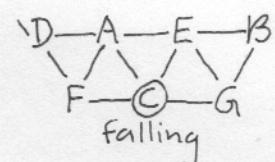


Double 24:48 24:(25): 27 : 30 : 32 : 36 : 40 : 45 : 48
 falling - C (Cb) \Bb Ab G F Eb Db C
 rising - C (C#) D E F G A B C

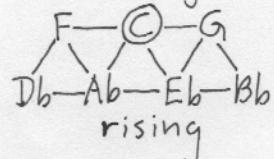


The series defines two fundamental Just Scales, the suddha bilaval (rising) and Bhairavi (falling). These scales are ancient among various Asian music cultures.

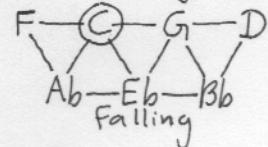
Double 30:60 30 : 32 : 36 : 40 : 45 : 48 : (50) : 54 : 60
 falling - C B A G F E (Eb) \D C
 rising - C Db Eb F G Ab (A) Bb C



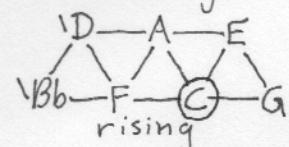
The series defines two more ancient and hallowed scales.



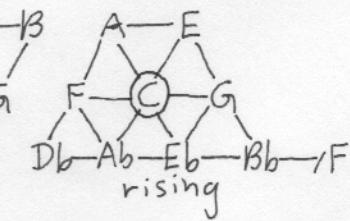
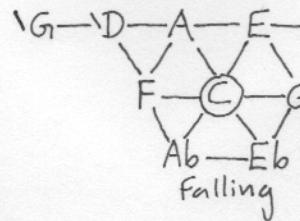
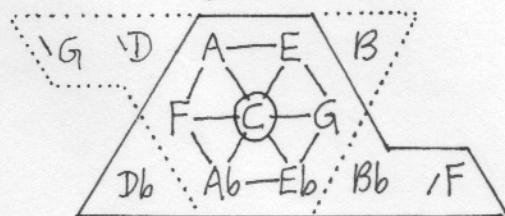
Double 36:72 36 : 40 : 45 : 48 : (50) : 54 : 60 : 64 : 72
 falling - C Bb Ab G (Gb) F Eb D C
 rising - C \D E F (F#) G A \Bb C



Two more important scales appear, Ashawari and Khamaj.



Double 60:120 60 : 64 : 72 : 75 : 80 : 81 : 90 : 96 : 100 : 108 : 120
 falling - C B A Ab G \G F E Eb \D C
 rising - C Db Eb E F /F G Ab A Bb C

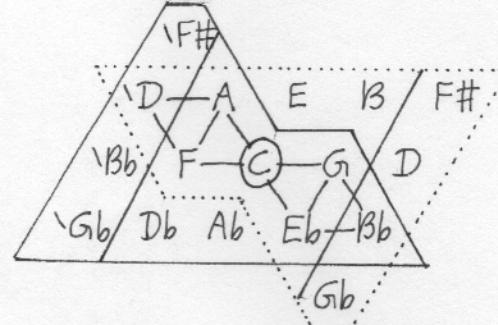


Note the region of invariance - the Harmonic Heptad, which is the nucleus of the Sruti field.

Figure 15 -continued from previous page

Double 90:180 90:96:100:108:120:125:128:135:144:150:160:162:180
 falling-C B Bb A G Gb F# F E Eb D D C
 rising-C Db D Eb F F# Gb G Ab A Bb Bb C

Notice that in the invariant zone is a prominent 'Dorian' Scale.



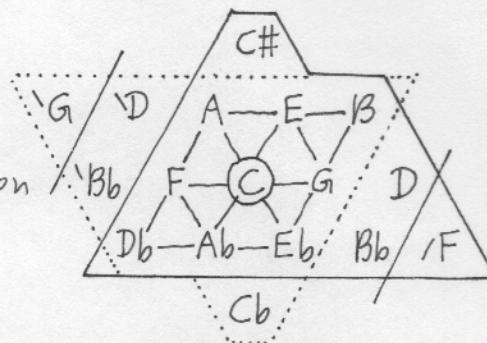
The union of the rising and falling forms.

Double 120:240

120:125:128:135:144:150:160:162:180:192:200:216:225:240
 falling-C Cb B Bb A Ab G Gb F F E Eb D Db C
 rising-C C# Db D Eb E F F G Ab A Bb B C

This double also embodies a sizeable invariant zone.

The interval between Db-B is the Just Intonation close variant of the 7:4 ratio.

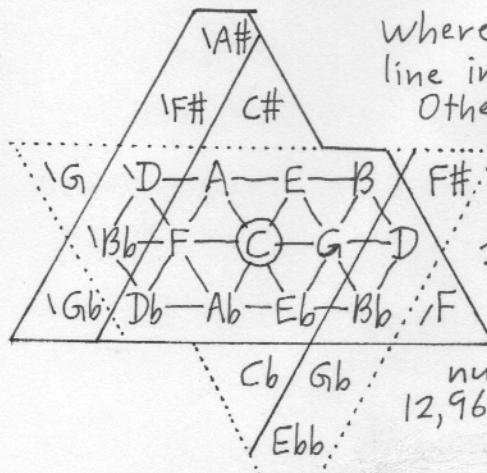


Other doubles with prominent zones of invariance are 180:360, 240:480, 270:540, and 300:600. In the interest of brevity, I present only one more series, but a prominent one:

Double 360:720

360:375:384:400:405:432:450:480:486:500:512:540:576:600:625:640:648:675:720
 C Cb B Bb Bb A Ab G Gb Gb F# F E Eb Ebb D D Db C
 C C# Db D Eb E F F Gb G Ab A A# Bb Bb B C

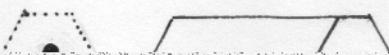
Note the large segment of the Sruti Field which is invariant. This series is associated with the calendar and various cosmologies of the ancient world.



Where do we draw the line in limiting numerosity? Other important doubles include 600:1200, 3600:7200, 108,000:216,000, 216,000:432,000, and Plato's "master cyclical number" 6,480,000: 12,960,000.

Figure 15 -continued from previous page

Illustration of the portion of the Harmonic Field ruled
by the double 108,000 : 216,000



Pattern "recognizes"
the Enharmonic
Gateway (creating

Figure 16 Babylonian standard Table of Reciprocals
 (Pairs whose products are $60 = 1$)
 Translated into Decimal equivalents :

God	Sexagesimal Reciprocals		Expressed in Ratios:		Musical Tones	
	2	x 30	$\frac{2}{60} \times \frac{30}{1}$	=	$\frac{1}{30} \times \frac{30}{1}$	B
	3	x 20	$\frac{3}{60} \times \frac{20}{1}$	=	$\frac{1}{20} \times \frac{20}{1}$	Eb
	4	x 15	$\frac{4}{60} \times \frac{15}{1}$	=	$\frac{1}{15} \times \frac{15}{1}$	B
	5	x 12	$\frac{5}{60} \times \frac{12}{1}$	=	$\frac{1}{12} \times \frac{12}{1}$	G
Adad	6	x 10	$\frac{6}{60} \times \frac{10}{1}$	=	$\frac{1}{10} \times \frac{10}{1}$	Ab
	8	x $7\frac{1}{2}$	$\frac{8}{60} \times \frac{15}{2}$	=	$\frac{2}{15} \times \frac{15}{2}$	B
	9	x $6\frac{2}{3}$	$\frac{9}{60} \times \frac{20}{3}$	=	$\frac{3}{20} \times \frac{20}{3}$	A
Adad	10	x 6	$\frac{10}{60} \times \frac{6}{1}$	=	$\frac{1}{6} \times \frac{6}{1}$	Eb
	12	x 5	$\frac{12}{60} \times \frac{5}{1}$	=	$\frac{1}{5} \times \frac{5}{1}$	F
Ishtar	15	x 4	$\frac{15}{60} \times \frac{4}{1}$	=	$\frac{1}{4} \times \frac{4}{1}$	C
	16	x $3\frac{3}{4}$	$\frac{16}{60} \times \frac{15}{4}$	=	$\frac{4}{15} \times \frac{15}{4}$	Db
	18	x $3\frac{1}{3}$	$\frac{18}{60} \times \frac{10}{3}$	=	$\frac{3}{10} \times \frac{10}{3}$	Eb
Shamash	20	x 3	$\frac{20}{60} \times \frac{3}{1}$	=	$\frac{1}{3} \times \frac{3}{1}$	F
	24	x $2\frac{1}{2}$	$\frac{24}{60} \times \frac{5}{2}$	=	$\frac{2}{5} \times \frac{5}{2}$	Ab
Marduk	25	x $2\frac{2}{5}$	$\frac{25}{60} \times \frac{12}{5}$	=	$\frac{5}{12} \times \frac{12}{5}$	Eb
	27	x $2\frac{2}{9}$	$\frac{27}{60} \times \frac{20}{9}$	=	$\frac{9}{20} \times \frac{20}{9}$	D
Sin	30	x 2	$\frac{30}{60} \times \frac{2}{1}$	=	$\frac{1}{2} \times \frac{2}{1}$	C
	32	x $1\frac{7}{8}$	$\frac{32}{60} \times \frac{15}{8}$	=	$\frac{8}{15} \times \frac{15}{8}$	Db
	36	x $1\frac{2}{3}$	$\frac{36}{60} \times \frac{5}{3}$	=	$\frac{3}{5} \times \frac{5}{3}$	Eb
Ea-Enki	40	x $1\frac{1}{2}$	$\frac{40}{60} \times \frac{3}{2}$	=	$\frac{2}{3} \times \frac{3}{2}$	A
	45	x $1\frac{1}{3}$	$\frac{45}{60} \times \frac{4}{3}$	=	$\frac{3}{4} \times \frac{4}{3}$	F
	48	x $1\frac{1}{4}$	$\frac{48}{60} \times \frac{5}{4}$	=	$\frac{4}{5} \times \frac{5}{4}$	G
Bel-Enlil	50	x $1\frac{1}{5}$	$\frac{50}{60} \times \frac{6}{5}$	=	$\frac{5}{6} \times \frac{6}{5}$	Ab
	54	x $1\frac{1}{9}$	$\frac{54}{60} \times \frac{10}{9}$	=	$\frac{9}{10} \times \frac{10}{9}$	Eb
Anu-An	$60=1$	x $1=60$	$\frac{60}{60} \times \frac{1}{1}$	=	$\frac{1}{1} \times \frac{1}{1}$	A
Not deified	64	x $15\frac{1}{16}$	$\frac{64}{60} \times \frac{15}{16}$	=	$\frac{16}{15} \times \frac{15}{16}$	Db
	72	x $5\frac{5}{6}$	$\frac{72}{60} \times \frac{5}{6}$	=	$\frac{6}{5} \times \frac{5}{6}$	Eb
	75	x $4\frac{4}{5}$	$\frac{75}{60} \times \frac{4}{5}$	=	$\frac{5}{4} \times \frac{4}{5}$	A
	80	x $3\frac{3}{4}$	$\frac{80}{60} \times \frac{3}{4}$	=	$\frac{4}{3} \times \frac{3}{4}$	E
	81	x $20\frac{20}{27}$	$\frac{81}{60} \times \frac{20}{27}$	=	$\frac{27}{20} \times \frac{20}{27}$	F
					$\frac{1}{G} \times \frac{1}{F}$	G

Thirty pairs of reciprocals - all of them "regular numbers" so that their reciprocals can be expressed by finite sexagesimal fractions. These tables employ three "places," thus $60^3 = 216,000$ is the equivalent decimal common denominator. Auxiliary tables had many more "places."

The three "greatest gods": Anu-An = $60/60 = \frac{1}{1}$, Ea-Enki = $40/60 = \frac{2}{3}$, and Bel-Enlil = $50/60 = \frac{5}{6}$ relate by ratios 4:5:6.