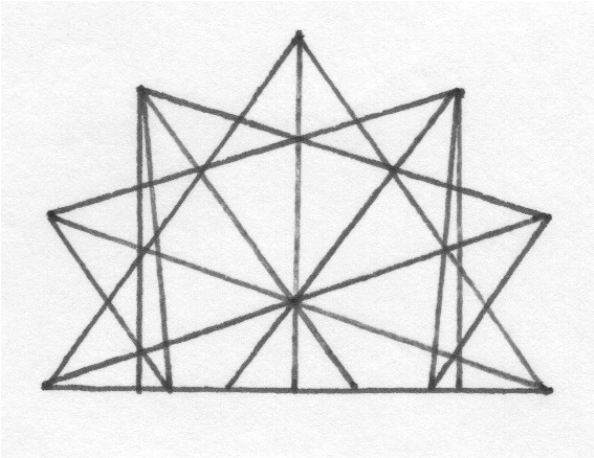
The Law of the Octave and Natural Resonances



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THE LAW OF THE OCTAVE AND NATURAL RESONANCES

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Those of us who own tuneable synthesizers possess an amazing freedom. For not only may we quit Equal Temperament and explore Just Intonation, but also we may use any frequency as our Pitch Standard. We are no longer tied to "A"=440 Hz. as our base frequency. But why choose one base frequency rather than another? Is the choice of a tuning Standard arbitrary? This article represents my search for a Standard which is based upon resonances found in the natural world. The key to unlocking these natural resonances is the Law of the Octave.

A vibration is a periodic phenomenon. It possesses a Period (t) which is its duration in seconds, and a frequency (v) which is the number of cycles of vibration in one second. Period and Frequency stand in inverse relation to each other; that is, t = 1 / v and v = 1 / t.

An octave is simply a vibrational double. To form an octave is to double or halve the frequency of a vibration. For example, to raise a vibration by five octaves is to multiply the frequency by 2*2*2*2*2 or 2^5 (two to the fifth power). In general terms, the Nth octave of a vibration is found by multiplying the frequency number by 2 to the Nth power. The pitch name for the Nth octave of a tone is the same as the original tone--a princple we call Octave Equivalence.

With this basic background we are ready for the Octave Law itself. It states that any periodic phenomenon may be converted by octave doublings into the sonic (hearing) range, producing sounds which are analogous to the periodic phenomenon in question. The formula which gives the tone frequency is $(1 / t) * 2^n$ or $v * 2^n$, where n is the appropriate number of doublings to reach sonic range. The frequency limits of human sonic range are approximately 2^5 Hz. to 2^{15} Hz. or about 10 octaves. About 8 octaves below average audible frequency are those frequencies which we perceive as tempo, rhythm, meter, and time--normally expressed by Metronome Markings (M.M.) in beats per minute. We shall focus here upon acoustical waves, disregarding the electromagnetic (light) sprectrum which lies within a frequency range of about 40 octaves above audible range.

Armed with the Octave Law let us examine the periodic phenomena around us and convert them into musical tones.

Firstly, we convert the Tropical Year, which is the Earth's rotation period around our Sun--more specifically, the period of time between two successive passages of our Sun through the Vernal Equinox. The period is 365.24219879 days = 525948.76624

minutes = 31556925.9747 seconds. The 25th octave of this period corresponds to 1 / 525948.76624 * 33554432 = M.M. 63.798 beats / minute. The 33rd octave vibrates at 1 / 31556925.9747 * 8.58993459E+09 = 272.204 Hz. This vibration is a "C#" somewhat below the standard C# of Equal Temperament. It is interesting to note here that Ravi Shankar in his book states that the sitar is tuned to a standard pitch slightly below the European C# (277.18 Hz.). Thus the "standard tuning" of Indian Classical Music is intuitively in tune with this natural cycle. Also note that the natural resonance of the body of a violin generally lies between 270 and 274 Hz. If we build our Just Intonation scales from this frequency standard we will be in resonance with the Earth's orbital cycle.

The Sidereal Year is the period of time between the sun's passage of a certain star. Its duration is 365.25636042 days, about 20 minutes longer than the Tropical year. Its tone is 272.1938 Hz. which is slightly flat of the Tropical Year, but the difference is practically imperceptible.

Let us now look at a long cycle--the Platonic Year. As the Earth revolves around its axis, the axis exhibits a cyroscopic motion (like the spinning of a child's top) caused by external gravitational influences, namely the Sun and Moon. This motion is called Precession. Consequently, the Equinoctal Points (reference points of Astronomical Coordinate Systems) move along the Ecliptic in a westerly direction at the rate of about 50 arc seconds per year, the total period being 25,827 years. The 40th octave of this period beats at M.M. 80.94 beats per minute. The 48th octave corresponds to frequency 345.36 Hz, an "F".

By the way, these calculations involve such huge numbers that I would have given up this project long ago if I didn't have my trusty computer to do the dirty work. Technology to the rescue!

Let's now examine a relatively short cycle--the Solar Day (the period between the successive culminations of the Sun). This period of 24 hours corresponds at the 17th octave to a M.M. of 91.02 beats per minute and at the 25th octave to a tone of 388.36 Hz. Thus it is a "G" flat of the Equal Tempered G (392.00 Hz.). The closely related period of the Sidereal Day measures the daily rotation of the firmament. Due to the annual motion of the Sun from west to east, compared to the fixed stars, the Sidereal Day is about 4 minutes shorter than the average Solar Day; ie., compared to the fixed stars, the Sun seems to fall behind by about 1 degree of arc daily in an easterly direction on the sphere. The meter is thus slightly faster at 91.27 beats per minute and the tone vibrates at 389.425 Hz., also sounding a "G".

Not only the Earth but also the other planets possess orbital periods and therefore corresponding musical tones. Here is a table summarizing the Planetary frequencies:

Planet	8ve #	v (Hz.)	Pitch name
Mercury	31	282.54	"D"
Venus	33	442.46	"A"
Mars	34	289.44	"D"
Jupiter	37	367.16	"F#"
Saturn	38	295.70	"D"
Uranus	40	414.66	"G#"
Neptune	41	422.90	"A"
Pluto	41	280.50	"C#"
Moon (Synodic)	30	420.84	"G#"
(Sidereal)	30	454.86	"A#"

Another interesting set of vibrations is known as the Schumann Resonances. The space between the Earth's surface and the ionosphere is a concave spherical resonator. When wavelengths close in length to the Earth's circumference are set up (by thunderstorms, lightning strikes, and earthquakes) the atmosphere vibrates at natural frequencies as follows:

7.8 Hz.	6th octave	499.2 Hz.	"B"
14.1 Hz	5th octave	451.2 Hz	"A#"
20.3 Hz	4th octave	324.8 Hz	"E"
26.4 Hz	4th octave	422.4 Hz	"Ab"
32.5 Hz	4th octave	520.0 Hz	"C"

We have now amassed a lot of data. Fortunately we are almost finished. I have left two very important cycles till last--the Sunspot cycle and the Long Solar Cycle. The Sunspot cycle of Magnetic storms is an average of 11.1 years in length. The 37th octave produces the pitch 392.37 Hz , a "G". The Long Solar Cycle of 179 years harmonizes the planetary orbits and the displacement of mass in our Solar System. Its 41st octave resonates at 389.30 Hz., a "G".

Note the preponderance of "G" resonances in our data. It appears that all the planets have a harmonic relationship with the two most obvious solar cycles. For example, 46 orbits of Mercury equal 11.079 years, 18 orbits of Venus equal 11.074 years, 11 orbits of the Earth equal 11.0 years, 137 synodic revolutions of the Moon equal 11.077 years, and 6 orbits of Mars equal 11.286 years. Looking at the massive, outer planets, 15 orbits of Jupiter equal 177.933 years, and 6 orbits of Saturn equal 176.748 years, both close to the Long Solar Cycle. Some other near commensurabilities are these:

Saturn / Jupiter = Pluto / 100
Saturn * Uranus = 2476 years = Pluto * 10 (error 3.2%)
Jupiter * Uranus = 997.07 years = Pluto * 4 (error 1.4%)
Mars * Saturn / 5 = 11.08 years
Mars * Jupiter / 2 = 11.15 years
Mars * Jupiter * 8 = 178.37 years

Thus a good argument could be made that the most predominant natural frequency corresponds to a "G". Interestingly

question is difficult to answer. Here I risk opinions which, I admit, are strictly subjective.

I first began tuning strings in the early sixties when I took up the guitar. But it was not till the early seventies that I became interested in "alternative" tunings. I had become a student of north Indian classical music, and quickly found that my sitar "sang" much better when I changed the intonation from 12-E.T. to Just intonation. The sitar has moveable frets so that the intonation is flexible. My personal "discovery" of alternative tunings was somewhat of a revelation to me since I had been accustomed to 12-E.T. for years through the guitar. I began a long time study of Intonation in general. My university training was in Philosophy, and I became fascinated by the influence of tuning lore, and its related number theory, on various world cultures. Meanwhile I expanded my practical experience by tuning harpsichords, harps, and zithers.

The move to Just intonation certainly effected my feeling for melodic nuance. The sitar (and the human voice) naturally uses Just in an almost purely melodic, modal manner. However, recently I have acquired the means to explore its "vertical" or harmonic potential. I received a Canada Council Research Grant to develop a computer music system playable in 53-E.T. (or any other division!) in real time. This software, which is MIDI compatible, is called the MODULAR KEYBOARD. It lets me access any wide pitch field through the intermediary of a TUNING MODULE. This chromatic module may be shifted at will anywhere in the field by pressing the appropriate command on my computer. This command retunes the entire keyboard instantly to the desired region. Thus I have free modulation anywhere within the tuning system.

Whether the move to a "natural" tuning standard effects the quality of the music, I don't know. But I have noticed that my plants do grow better in my music studio. Perhaps there is a connection, but then again, maybe its just the quality of the light in that room. It is definitely too early to make firm conclusions concerning these matters. However, my intuition tells me that tapping into natural resonances may have implications beyond my conscious conceptions. Nature works in mysterious ways!

Since writing this paper, more information on the 'bell tones' of the Earth have come to me, thanks to musicologist Mark Rankin. These resonances have 'surfaced' through the study of earth-quake waves. These periodicities relate to the Schumann Resonances, and should be added to the chart (I've added an asterisk). There are two 'deep' bell tones at 54.7 minutes and 53.1 minutes; In addition, a 'surface' bell tone of 42 minutes.

Period	Pitch	step		53-E.T	Difference	
54.7 min →	319.48 Hz	15	/Eb	318.33 Hz	−1.15 Hz	−6.2 Cents
53.1 min →	329.12 Hz	18	/E	331.07 Hz	+1.95 Hz	+10.2 Cents
42 min →	416.1 Hz	36	Ab	418.95 Hz	+2.85 Hz	+11.8 Cents

Mark 'splits' the difference between the first two cycles to get a compromise tone which is almost exactly a Schumann Resonance:

53.9 min →	324.24	16	\E	322.52 Hz	−1.72 Hz	−9.2 Cents

Frequency Relations between the visible spectrum and musical sound:

a - colour frequency ($\times 10^{12}$)
b - colour description
c - colour frequency down 40 octaves
d + e - Pitch using reference 12-E.T. and 53-E.T.
f - Natural Resonances

a	b	c	d	e	f
			349.2 -	F - 348.9	- 345.36 - Platonic Year
380	↑	345.6		/F - 353.5	
390		354.7		Gbb - 358.1	
				\F# - 362.8	
400	Infra-red	363.8		F# - 367.6	- 367.16 - Jupiter
410		379.9	370.0 -	\Gb - 372.4	- 372.83 - Circadian Cycle
				Gb - 377.3	
420		382.0		Fx - 382.3	
				\G - 387.3	~~388.36 - Solar Day~~
430	Very dark red	391.1			= 389.30 - Long Solar Cycle
440	} dark red	400.2	392.0 -	G - 392.4	= 389.425 - Sidereal Day
				/G - 397.6	392.37 - Sunspot Cycle
450		409.3		Abb - 402.8	
460	Darkish red	418.4		G# - 408.1	
470	Red	427.5	415.3 -	\Ab - 413.5	- 414.66 - Uranus
				Ab - 418.9	
480	Red-orange	436.6		Gx - 424.5	= 420.84 - Moon (Synodic)
				\A - 430.1	= 422.4 - Schuman Resonance
490	Orangish red	445.7		A - 435.7	422.9 - Neptune
500	Reddish Orange	454.8	440.0 -	/A - 441.5	- 442.46 - Venus
510	Light orange	463.8		Bbb - 447.3	
520	Yellow	472.9		/Bbb - 453.2	= 451.2 - Schuman Resonance
				A# - 459.1	= 454.86 - Moon (Sidereal)
530	Greenish Yellow	482.0	466.2 -	\Bb - 465.2	
540	} Yellow Green	491.1		Bb - 471.3	
				Ax - 477.5	
550		500.2		\B - 483.8	
560	Light Green	509.3	493.8 -	B - 490.1	
570	}	518.4		/B - 496.6	- 499.2 - Schuman Resonance
580	} Green	527.5		Cb - 503.1	
				B# - 509.8	
590		536.6		\C - 516.5	
600	Bluish Green	545.7	523.3 -	C - 523.3	- 520.0 - Schuman Resonance
610	Blue Green	554.8		/C - 530.1	
				Dbb - 537.1	
620	Sky blue	563.9		C# - 544.2	= 544.38 - Sidereal Year
				\Db - 551.4	= 544.40 - Tropical Year
630	Bluish Indigo	573.0	554.4 -	Db - 558.6	561.00 - Pluto
640	}	582.1		/Db - 566.0	= 565.08 - Mercury
650	} Indigo	591.2		Cx - 573.4	
				\D - 580.9	- 578.88 - Mars
660		600.3	587.3 -	D - 588.6	
670	} Indigo violet	609.4		/D - 596.4	- 591.4 - Saturn
680		618.5		Ebb - 604.2	
				D# - 612.2	
690	Light violet	627.6		\Eb - 620.2	
700	} Violet	636.7	622.2 -	Eb - 628.4	
710		645.7		Dx - 636.7	
				\E - 645.1	
720	}	654.8		E - 653.5	- 649.6 - Schuman Resonance
730	} Dark violet	663.9	659.3 -	/E - 662.1	
				Fb - 670.9	
740		673.0		E# - 679.7	
750	Very dark violet	682.1		\F - 688.6	- 690.72 - Platonic Year
760	↓ Ultra-violet	691.2	698.5 -	F - 697.7	
770		700.3		/F - 706.9	