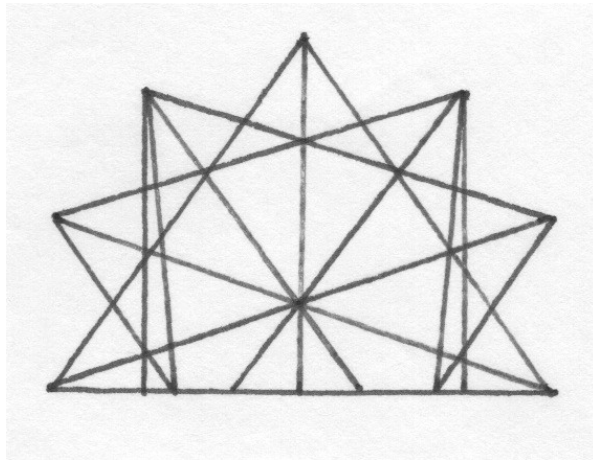# The 'ENTANGLED RELATIONS'
between the Diatonic Ogdoad and its superset – the Chromatic Dodecad

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The 'ENTANGLED RELATIONS' between the Diatonic Ogdoad and its superset – the Chromatic Dodecad

Both the single-strand 3-Limit archetype and the 5-Limit (double-strand and triple-strand) archetypes are examined

Showing the efficacy of the Comma boundaries

Also showing the primacy of the four controlling micro-intervals within the matrix of 5-Limit Just Intonation:
① the Schisma (s) – 1 s in size
② the klisma (k) – 4 s in size
③ the Diaschisma-Comma (ic) – 10 s in size
④ the Syntonic Comma (sc) – 11 s in size

All measurements are in schisma-steps (octave = 612 s) rather than cents-steps (octave = 1200 cents) in order to make the relations more transparent.
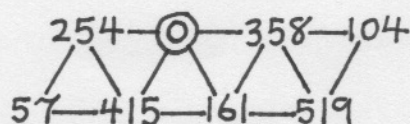
The chromatic (12-note) harmonies are shown together after their Diatonic subsets are presented. The aim is to show how the Diatonic Ogdoad sits within the several archetypes of the Chromatic Dodecad.

Consider the 5-Limit double-strand Diatonic Ogdoad in both of its principal modes: Major and Minor

Major

451—197—555—301

254—◎—358—104

Minor

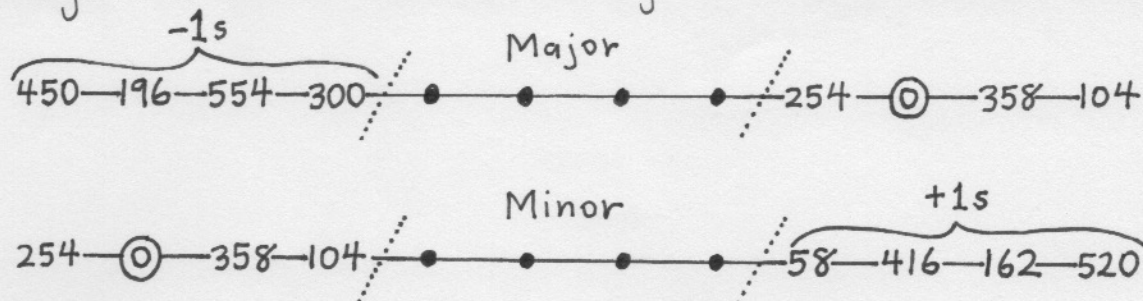254—◎—358—104

57—415—161—519
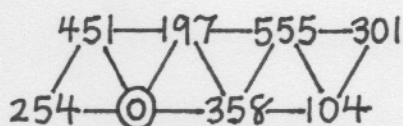
This most consonant of Ogdoad harmonies still has some dissonant elements, namely the Commatic Fifth (347 s) between 104-451 in the major and 519—254 in the minor. A pure fifth is 358 s; hence it is flat by 11 s or one sc. Also note that, besides the usual Diatonic Semitones (57 s), there is also the 5-Limit Medial Semitone (47 s) between 254-301 and 57-104. This semitone is flat of the diatonic archetype by 10 s (ic).

This Ogdoad harmony forms subsets of both the two double-strand Chromatic harmonies and the triple-strand chromatic harmony shown below.

We will now modify this Ogdoad harmony, altering one of the strands, but always keeping the 3-Limit Tetrad (254—◎—358-104) as the invariant core of the scale.

Even though the above Ogdoad is considered the epitome of 5-Limit harmony, it has a sonic 'near identity' as a subset of the 3-Limit single-strand Dodecad. The mutable line is shifted by only one schisma from the 5-Limit paradigm. Both the modified Major and Minor modes are shown:
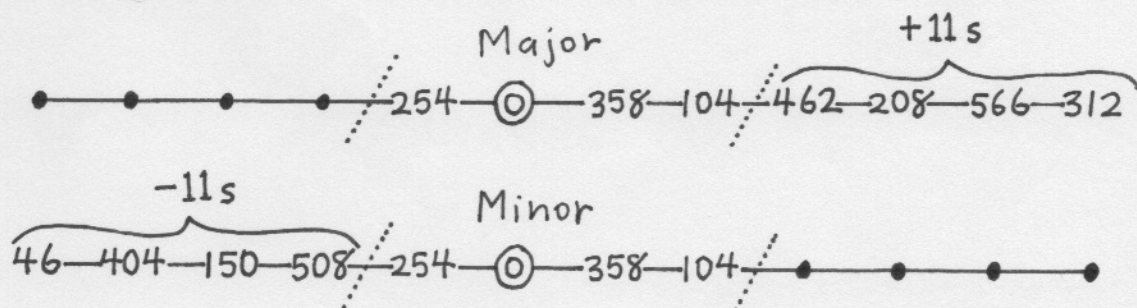
Major

$$\overbrace{450—196—554—300}^{-1s} \Big/ \bullet \quad \bullet \quad \bullet \quad \bullet \Big/ 254—\odot—358—104$$

Minor

$$254—\odot—358—104 \Big/ \bullet \quad \bullet \quad \bullet \quad \bullet \Big/ \overbrace{58—416—162—520}^{+1s}$$

Therefore, wide 3-Limit expansions result in intervals which are 'quasi-5-Limit.' For example, the Schismatic Major Third (196 s) is but one s flat of the 5-Limit Just Major Third; hence the appropriateness of the notation. Note that the dissonant fifth between 104—450 and between 520—254 is now flat by 12 s — the Ditonic Comma (dc). This interval can be derived from the primary controllers, since 12 s = 1 + 11. Also, the 5-Limit Medial Semitone has become the 3-Limit Medial Semitone (46 s) between the pitches 254—300 and 58—104. The near-identity of this schisma shift has also affected the Schismatic Diatonic Semitone (also called Apotome) of 58 s and the thirds and sixths of this harmony.

This harmony shows the futility of the strict conceptual separation of 3-Limit and 5-Limit harmony.

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We now alter the double-strand Diatonic Ogdoad in a more radical manner, using a shift of 11 s (one sc), to yield <u>the 3-Limit Diatonic Ogdoad</u>. This famous harmony, shown both in its Major and Minor modes, is a subset of the 3-Limit Chromatic Dodecad as well as the two 5-Limit double-strand Dodecads.

Major

$$\bullet \quad \bullet \quad \bullet \quad \bullet \Big/ 254—\odot—358—104 \Big/ \overbrace{462—208—566—312}^{+11\,s}$$

Minor

$$\overbrace{46—404—150—508}^{-11\,s} \Big/ 254—\odot—358—104 \Big/ \bullet \quad \bullet \quad \bullet \quad \bullet$$

This single-strand, also called the Pythagorean Ogdoad, is a dissonant 'cousin' of the double-strand

Ogdoad. For example, the consonant Major Third (197s) is replaced by the dissonant sc-raised Third (208 s). However, the initial dissonant Commatic Fifth of the double-strand paradigm is now converted to purity in the fifths 104-462 and 508-254. All fifths are now pure, but the thirds and sixths have been compromised in the process. Hence we have the comma boundary and the slash notation for sc-alteration.

Just as the initial double-strand Ogdoad has a 3-Limit 'near identity,' our single-strand Ogdoad also has a schisma-shifted 'near identity' within the 5-Limit. Instead of altering our initial mutable line by 11s, we alter it by 10s. Again we have a scale which is sonically very close to the single-strand Ogdoad, even though it is 5-Limit. It forms a subset of the two double-strand chromatic harmonies. The usual two modes are given, but each one is potentially a subset of either double-strand chromatic Dodecad.

In the Major mode:
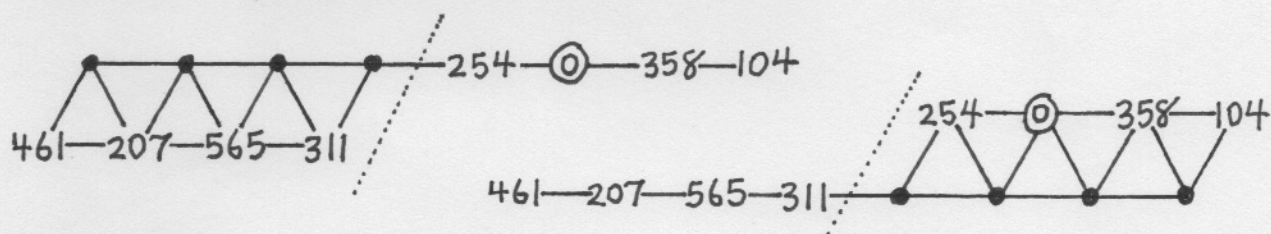
In the Minor mode:
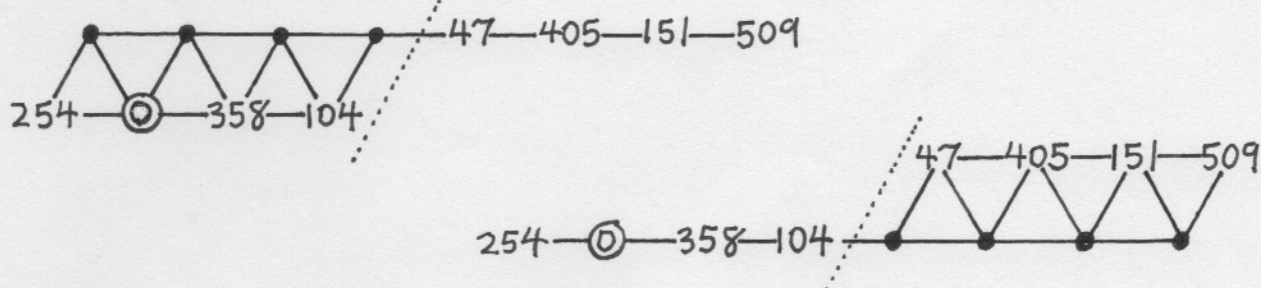
This harmony shares the dissonant characteristics of its Pythagorean 'near identity.' However, the harmony has been shifted in a subtle manner. For example, all the fifths but one share purity; but the fifths on pitches 104-461 and 509-254 are schisma-lowered. This is the Schismatic Fifth, a 5-Limit interval. The Major Third is slightly improved, since it is now 10s sharp rather than 11s. The overall effect is that of a slightly modified 3-Limit (Pythagorean) scale.

One more version of the Ogdoad belongs in this 'family.'
We have altered our initial mutable line by 1s, 10s,
and 11s. Now we alter it by a Diesis (21s), yielding
the 5-Limit triple-strand Ogdoad. The resulting harmony
is a subset of the triple-strand Dodecad shown below.
We shall continue to use the terms Major and Minor,
although this harmony has a high degree of dissonance.
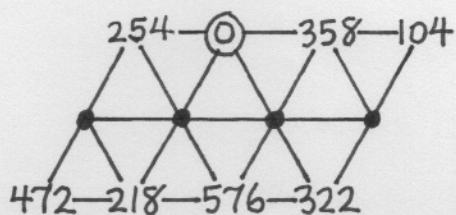
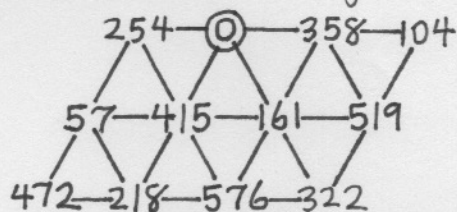Dissonant Major   Dissonant Minor
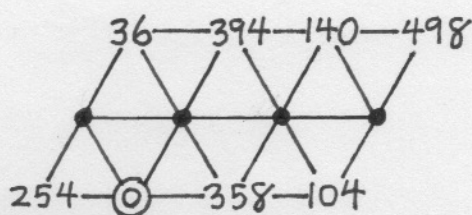
The Major Third is now sharp by a Diesis, so that it is
more properly a diminished Fourth. Also, the Diatonic
Semitone (57s) is flattened a Diesis to form the
Chromatic Semitone (36s). The Diesis can be derived
from the primary controllers, since 21s = 10 + 11.
Since the difference between the large semitone and
the small semitone is a Diesis, we see the logic in
naming the two mid-sized semitones (46s and 47s)
the Medial Semitones.

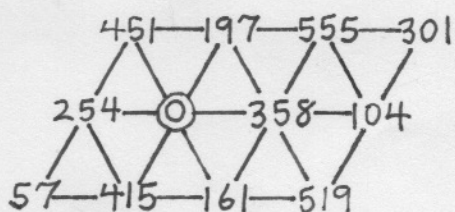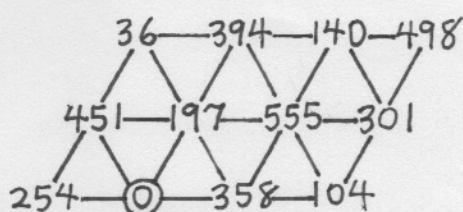...

The Chromatic Archetypes of relevance here:

The 5-Limit triple-strand Chromatic Dodecad
shown in its "major" and "minor" modes relevant to
the above ogdoad, also shown in a most consonant
"middle" mode which is historically called The Duodene.
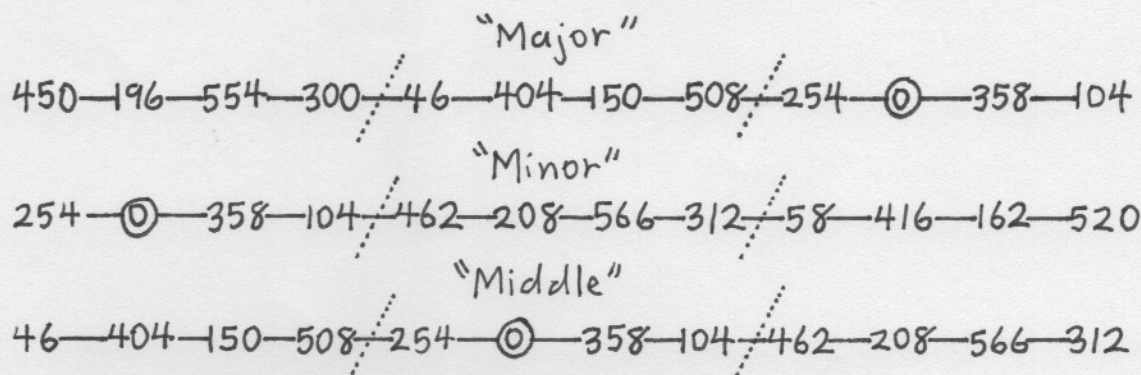
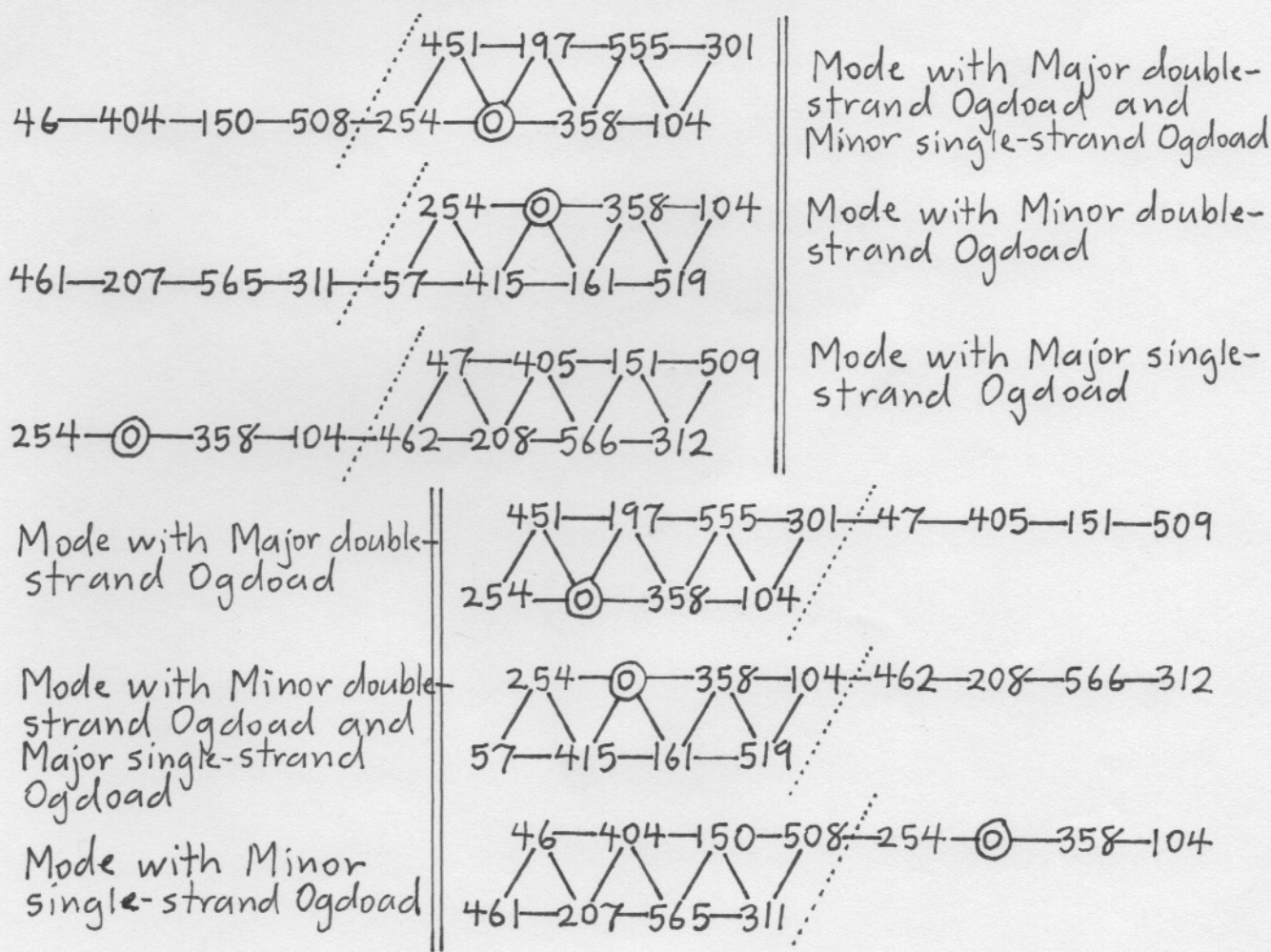"Dissonant Major"   "Dissonant Minor"

"middle"
(Duodene)

<u>The 3-Limit single-strand Chromatic Dodecad</u>, also called Pythagorean Dodecad, shown in the three relevant modes of this investigation :

"Major"

450—196—554—300—46—404—150—508—254—◎—358—104

"Minor"

254—◎—358—104—462—208—566—312—58—416—162—520

"Middle"

46—404—150—508—254—◎—358—104—462—208—566—312

<u>The 5-Limit double-strand Chromatic Dodecad</u>. This is the only harmony presented in this collection which is not symmetrical; Hence there are two complimentary double-strand dodecads. Each is given in the three modes relevant to this investigation:

451—197—555—301

46—404—150—508—254—◎—358—104

Mode with Major double-strand Ogdoad and Minor single-strand Ogdoad

254—◎—358—104

461—207—565—311—57—415—161—519

Mode with Minor double-strand Ogdoad

47—405—151—509

254—◎—358—104—462—208—566—312

Mode with Major single-strand Ogdoad

Mode with Major double-strand Ogdoad

451—197—555—301—47—405—151—509

254—◎—358—104

Mode with Minor double-strand Ogdoad and Major single-strand Ogdoad

254—◎—358—104—462—208—566—312

57—415—161—519

Mode with Minor single-strand Ogdoad

46—404—150—508—254—◎—358—104

461—207—565—311

Composite Matrix with key to the notation:

Matrix in schisma-step numbers

(1)

```
                              36—394—140—498
 ·    ·    ·    ·    ·    ·    451—197—555—301—47—405—151—509   ·
450—196—554—300—46—404—150—508—254—◎—358—104—462—208—566—312—58—416—162—520
(611) ·    ·   461—207—565—311—57—415—161—519   ·    ·    ·    ·    ·    ·
                    472—218—576—322
```
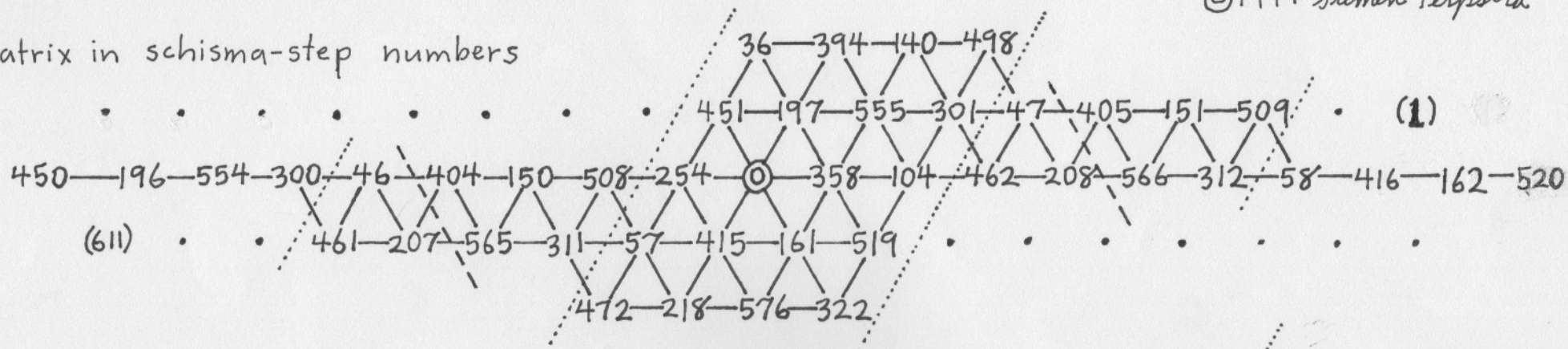
Matrix in Pitch Notation   ∧ – sc alteration
                           ± – s alteration

Syntonic Comma Boundary

Schisma Boundary

```
                                        C#—G#—D#—A#
  ·    ·    ·    ·    ·    ·    A—E—B—F#—C#/Db—G#⁺/Ab⁺—Eb⁺—Bb⁺   ·   (c⁺)
A⁻—E⁻—B⁻—F#⁻—Db⁻/C#⁻—Ab/G#—Eb—Bb—F—©—G—D—A—E/Fb—B⁺/Cb⁺—F#⁺—Db⁺—Ab⁺—Eb⁺—Bb⁺
(c⁻) ·    ·  A/E⁻—Fb/B—Cb—Gb—Db—Ab—Eb—Bb   ·    ·    ·    ·    ·    ·
                    Bbb—Fb—Cb—Gb
```
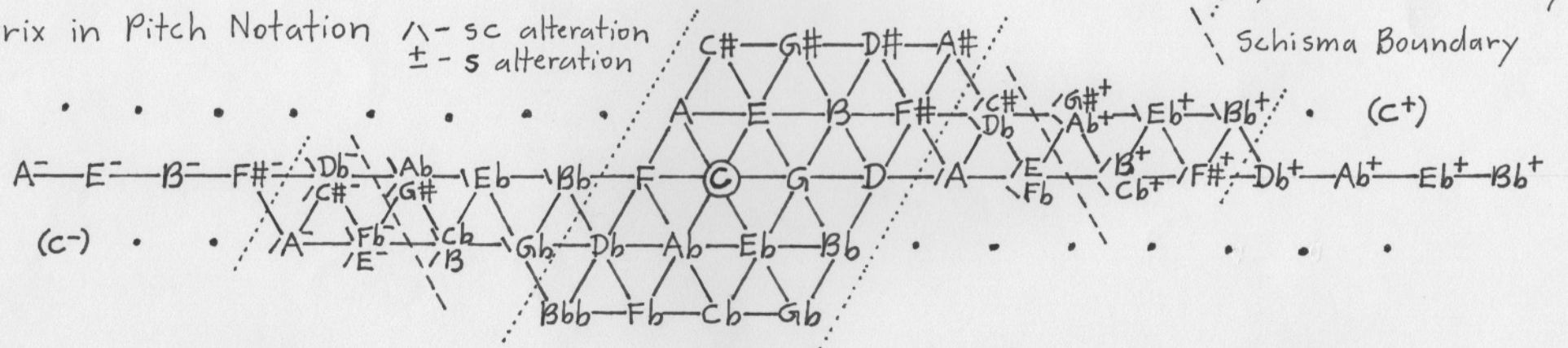
Conclusion: We have seen many instances of harmonic relations defined as **s**, **ic**, and **sc** (1, 10, 11) and their derivatives **dc** (12s) and Diesis (21s). These occur in all of the variant paradigms of the Diatonic Ogdoad and Chromatic Dodecad. The **k** (4s) makes its appearance in two contexts: 1. further 5-Limit vertical expansion which is not explored here, and 2. The relation between this matrix and a 7-Limit 'close cousin.'