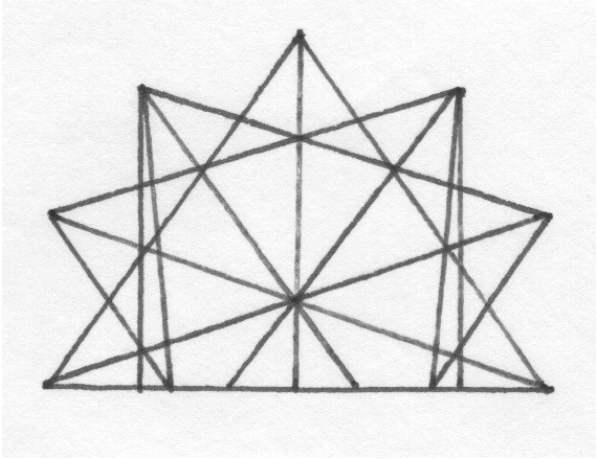
The Laws of Harmonics

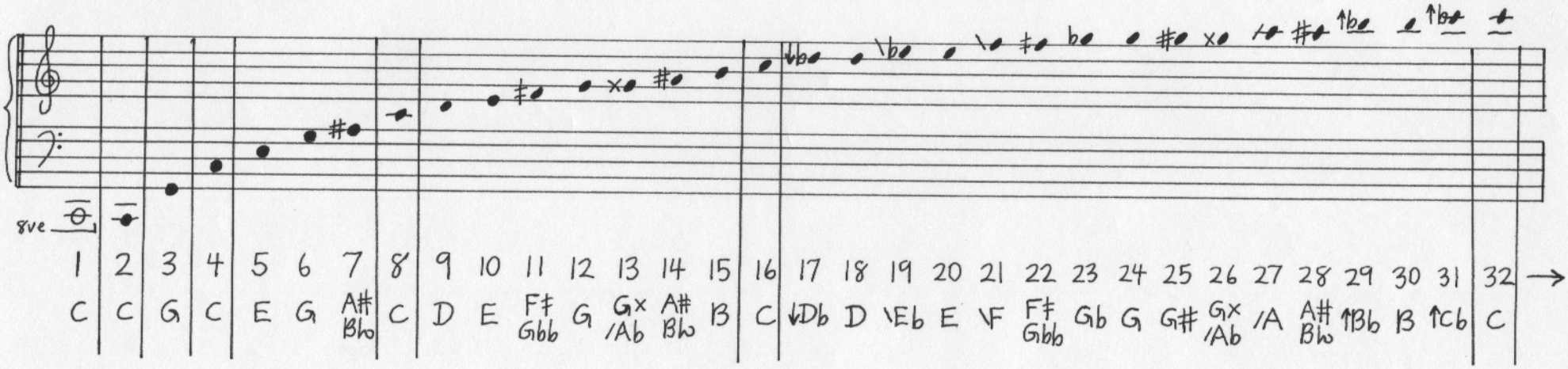
diagrams



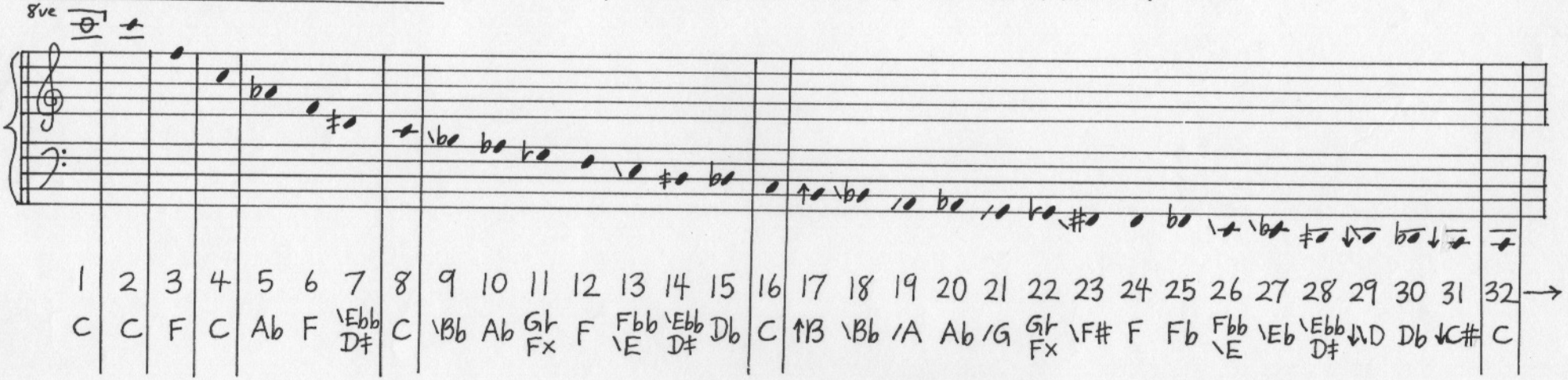
Siemen Terpstra

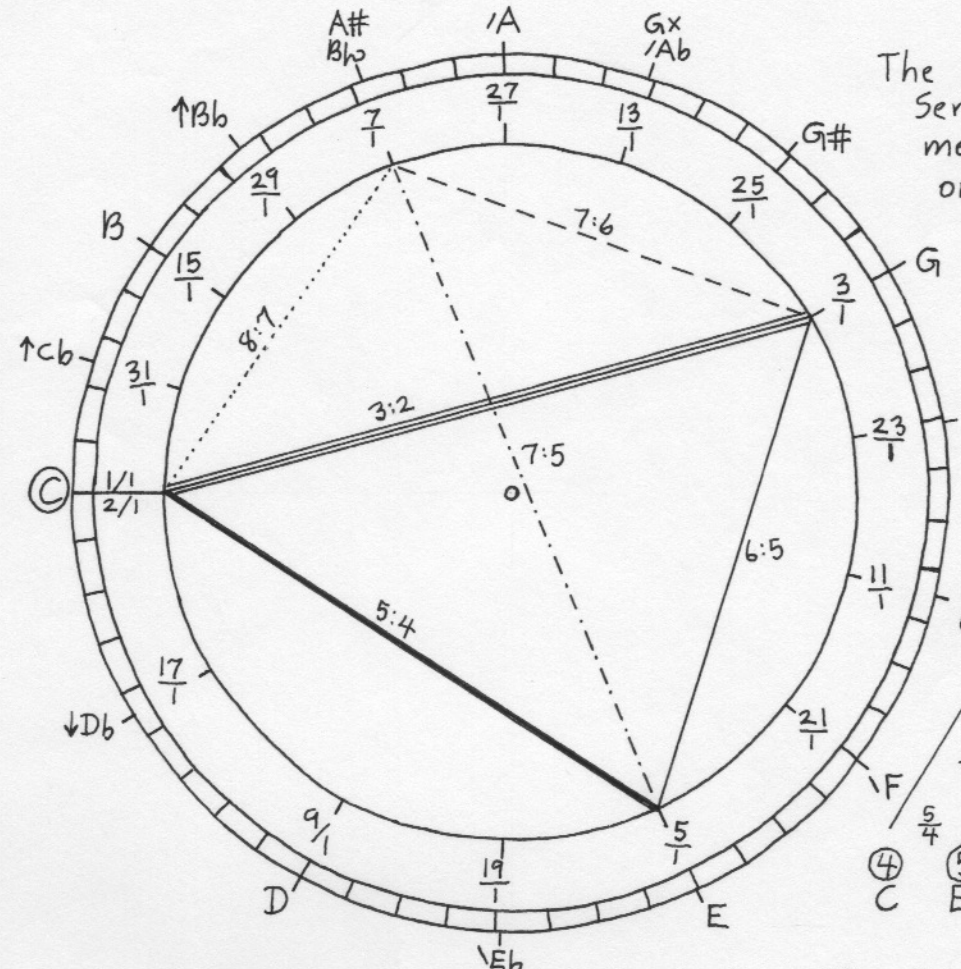
www.siementerpstra.com

The Harmonic Series: The First 32 Harmonics on Fundamental "C"



The Sub-harmonic Series: The First 32 Sub-harmonics on Fundamental "C"

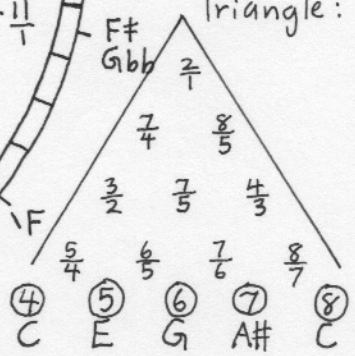


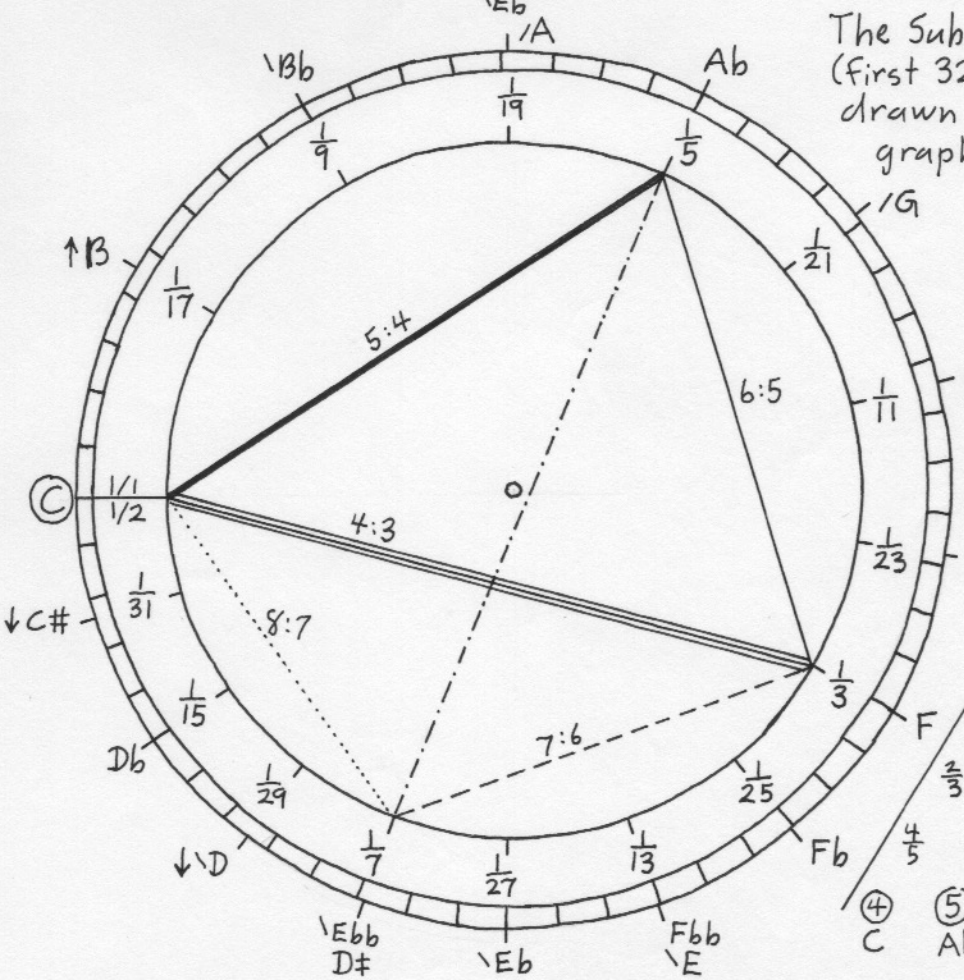


The Harmonic Series (first 32 members) drawn on the circular graph.

The Harmonic Series Tetrad is indicated.

Interval Triangle:



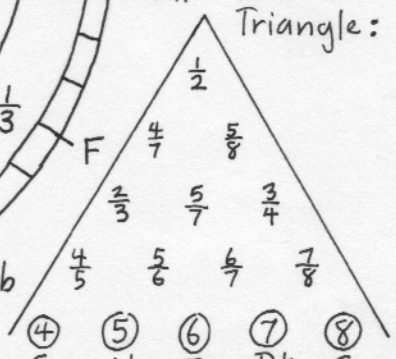


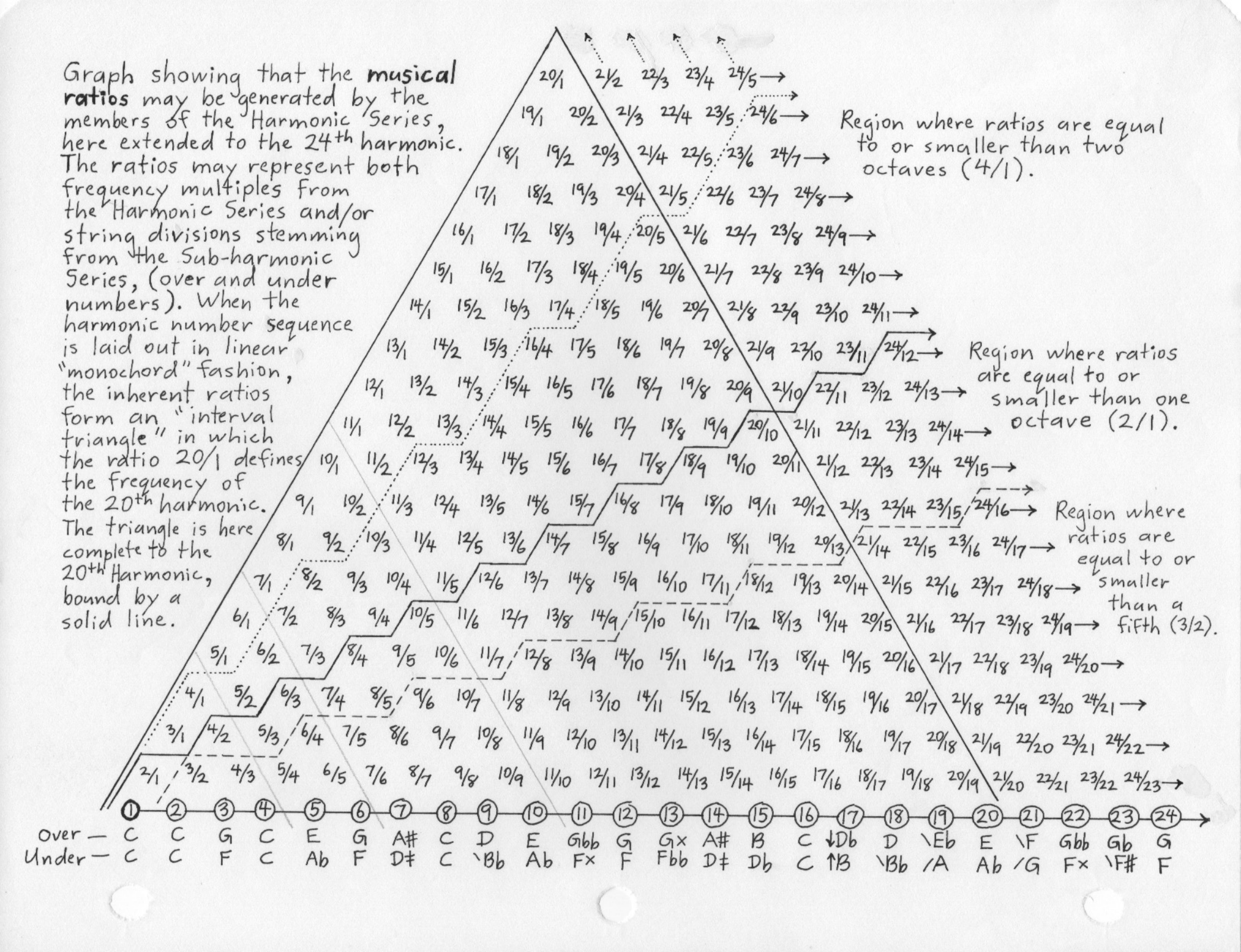
The Sub-harmonic Series (first 32 members) drawn on the circular graph.

The Sub-harmonic Series Tetrad is indicated.

The ratios define monochord string divisions.

Interval Triangle:




Graph showing that the musical ratios may be generated by the members of the Harmonic Series, here extended to the 24th harmonic. The ratios may represent both frequency multiples from the Harmonic Series and/or string divisions stemming from the Sub-harmonic Series, (over and under numbers). When the harmonic number sequence is laid out in linear "monochord" fashion, the inherent ratios form an "interval triangle" in which the ratio 20/1 defines the frequency of the 20th harmonic. The triangle is here complete to the 20th Harmonic, bound by a solid line.
Region where ratios are equal to or smaller than two octaves (4/1).
Region where ratios are equal to or smaller than one octave (2/1).
Region where ratios are equal to or smaller than a fifth (3/2).
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Over — C C G C E G A# C D E Gbb G Gx A# B C ↓Db D \Eb E \F Gbb Gb G
Under — C C F C Ab F D♯ C \Bb Ab Fx F Fbb D♯ Db C ↑B \Bb /A Ab /G Fx \F# F

Octave-equivalent interval size (in Cents + Degrees) For the Harmonics (to 128)

1, 2, 4, 8, 16, 32, 64, 128 = 0^c = 0° (C)

3, 6, 12, 24, 48, 96 = 701.955^c = 210.59° (G)

5, 10, 20, 40, 80 = 386.314^c = 115.89° (E)

7, 14, 28, 56, 112 = 968.826^c = 290.65° (A#)

9, 18, 36, 72, = 203.91^c = 61.17° (D)

11, 22, 44, 88 = 551.318^c = 165.40° (Gbb)

13, 26, 52, 104 = 840.528^c = 252.16° (Gx)

15, 30, 60, 120 = 1088.269^c = 326.48° (B)

17, 34, 68 = 104.955^c = 31.49°

19, 38, 76 = 297.513^c = 89.25°

21, 42, 84 = 470.781^c = 141.23°

23, 46, 92 = 628.274^c = 188.48°

25, 50, 100 = 772.627^c = 231.79° (G#)

27, 54, 108 = 905.865^c = 271.76° (/A)

29, 58, 116 = 1029.577^c = 308.87°

31, 62, 124 = 1145.036^c = 343.51°

33, 66 = 53.273^c = 15.98°

35, 70 = 155.140^c = 46.54°

37, 74 = 251.344^c = 75.40°

39, 78 = 342.483^c = 102.74°

41, 82 = 429.062^c = 128.72°

43, 86 = 511.518^c = 153.46°

45, 90 = 590.224^c = 177.07° (F#)

47, 94 = 665.507^c = 199.65°

49, 98 = 737.652^c = 221.30°

51, 102 = 806.910^c = 242.07°

53, 106 = 873.505^c = 262.05°

55, 110 = 937.632^c = 281.29°

57, 114 = 999.468^c = 299.84°

59, 118 = 1059.172^c = 317.75

61, 122 = 1116.885^c = 335.07°

63, 126 = 1172.736^c = 351.82°

65 = 26.841^c = 8.05°

67 = 79.307^c = 23.79°

69 = 130.229^c = 39.07°

71 = 179.697^c = 53.91°

73 = 227.789^c = 68.34°

75 = 274.582^c = 82.37° (D#)

77 = 320.144^c = 96.04°

79 = 364.537^c = 109.36°

81 = 407.820^c = 122.35° (/E)

83 = 450.047^c = 135.01°

85 = 491.269^c = 147.38°

87 = 531.532^c = 159.46°

89 = 570.880^c = 171.26°

91 = 609.354^c = 182.81°

93 = 646.991^c = 194.10°

95 = 683.827^c = 205.15°

97 = 719.895^c = 215.97°

99 = 755.228° = 226.57°

101 = 789.854^c = 236.96°

103 = 823.801^c = 247.14°

105 = 857.095^c = 257.13°

107 = 889.760^c = 266.93°

109 = 921.821^c = 276.55°

111 = 953.299^c = 285.99°

113 = 984.215^c = 295.26°

115 = 1014.588^c = 304.38°

117 = 1044.438^c = 313.33°

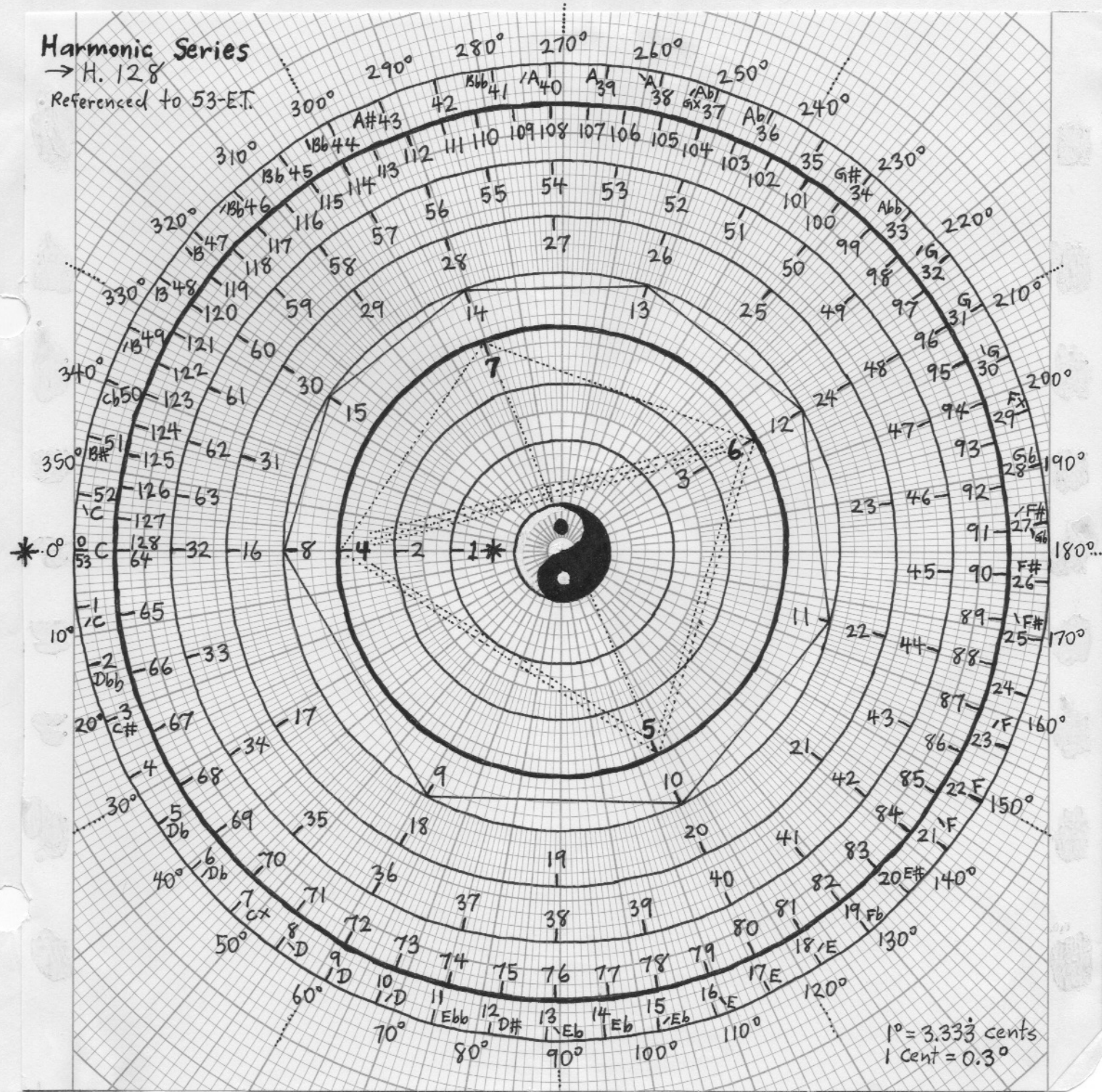
119 = 1073.781^c = 322.13°

121 = 1102.636^c = 330.79°

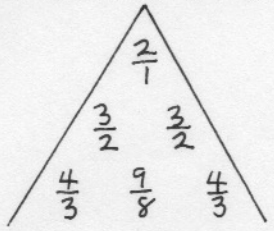
123 = 1131.017^c = 339.31°

125 = 1158.941^c = 347.68° (B#)

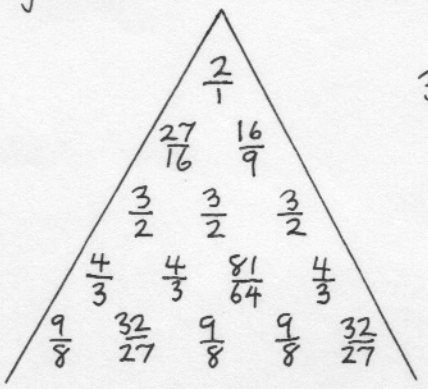
127 = 1186.422^c = 355.93°



Examples of number sequences (ruled by a double), showing that any sequence has an inherent **Interval Triangle**. Any number sequence may be interpreted as a Harmonic Series componant ("over" number) and also as a Sub-harmonic Series componant ("under" number), resulting in both a rising and a falling scale. The Interval Triangle ratios are presented as "over" numbers (in which the numerator is larger than the denominator), but the opposite sense (associated with monochord string division) is understood.

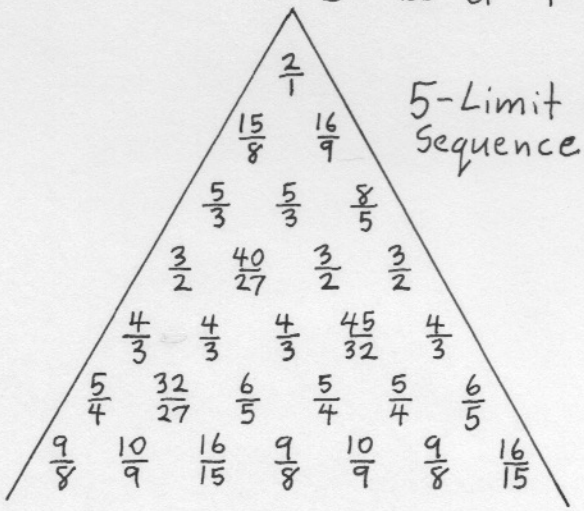


6 : 8 : 9 : 12
O – C F G C
U – C G F C



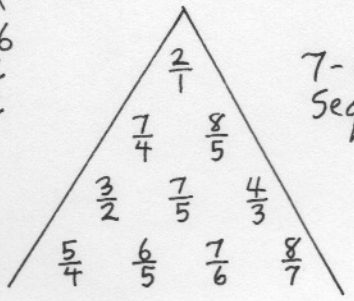
3-Limit sequences

48 : 54 : 64 : 72 : 81 : 96
O – C D F G ^A C
U – C vBb G F vEb C



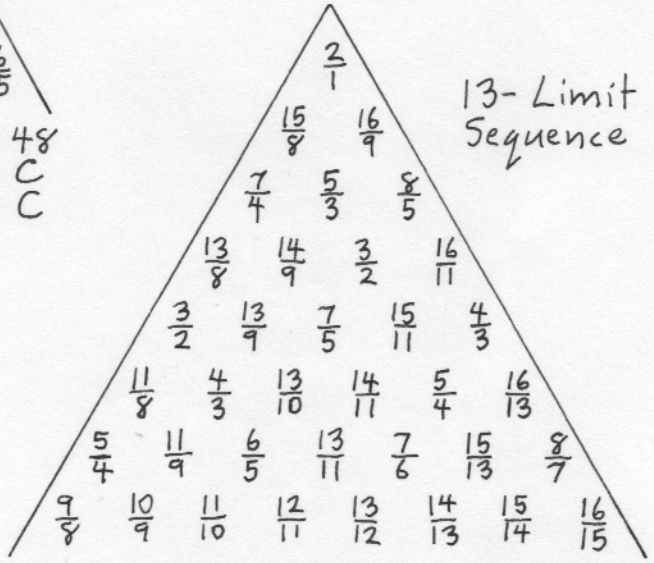
5-Limit Sequence

24 : 27 : 30 : 32 : 36 : 40 : 45 : 48
O – C D E F G A B C
U – C vBb Ab G F Eb Db C



7-Limit Sequence

4 : 5 : 6 : 7 : 8
O – C E G A# C
U – C Ab F D♯ C

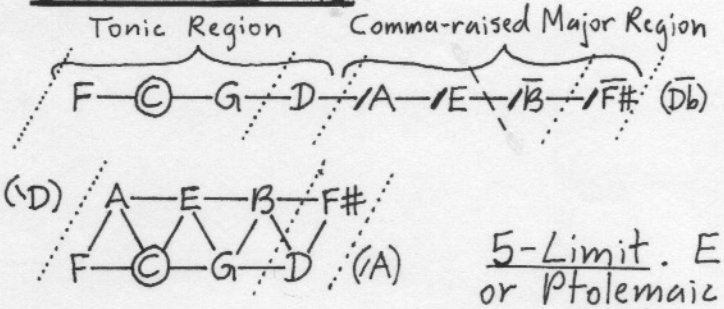


13-Limit Sequence

8 : 9 : 10 : 11 : 12 : 13 : 14 : 15 : 16
O – C D E Gbb G Gx A# B C
U – C vBb Ab Fx F Fbb D♯ Db C

Diagram showing that the progressive integration of higher primes is illustrated by the concept of the three Genera:

Diatonic Genus:

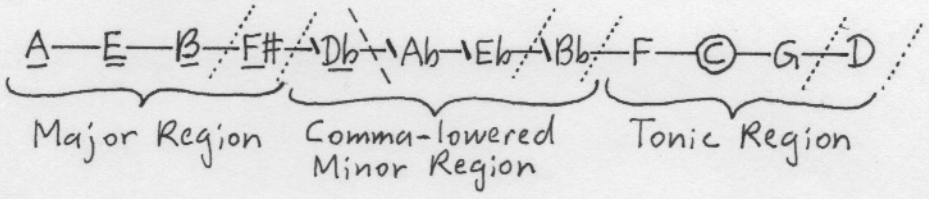


The two paradigms of the diatonic:

3-Limit. Extension beyond eight notes results in quasi-just thirds (eg. between Db and F); Hence the restriction to eight.

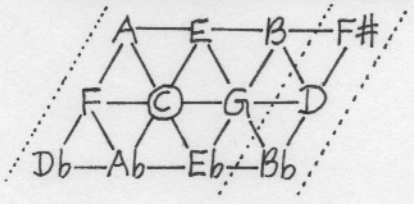
5-Limit. Extension of the "double strand" or Ptolemaic sequence beyond eight notes results in comma-relations, as shown; hence the restriction to eight.

The 5-Limit paradigm within the 3-Limit. Any chromatic extension of the 3-Limit "single strand" embodies the diatonic "double strand" with only schismatic differences. For example, the ratio for E (8192/6561 = 384.360cents) is almost identical to E (5/4 = 386.314cents).



This situation illustrates the "four beat rhythm" within the structure of the single strand. Any wide extension of the "single strand" results in a "double" or even "triple strand."

Chromatic Genus:



Typical "triple strand" Chromatic pattern in 5-Limit ratios.

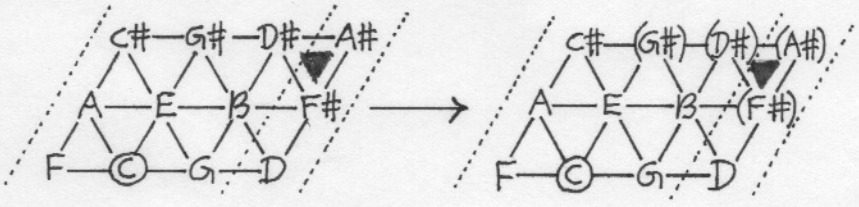
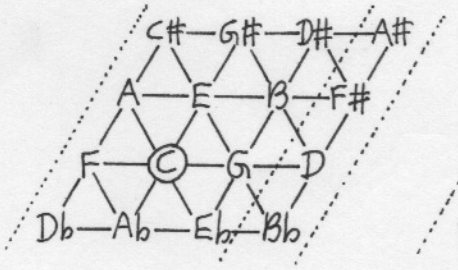
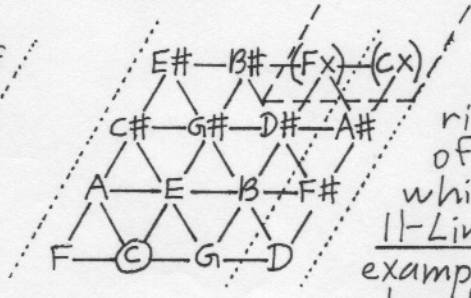


Illustration how the 7-Limit ratios are implied in the "triple strand." For example, the ratio D# (75/64 = 274.583cents) becomes the septimal ratio D# (7/6 = 266.871cents).

Enharmonic Genus:

The "quadruple strand" with its diesis relations is shown at left in its characteristic 5-Limit position. The position at right illustrates the ratios of the Harmonic Antipodes which are amenable to an 11-Limit substitution. For example, Cx (162.850c.) can be replaced by 11/10 (165.004c.).





Generation of "Quarter-tones" or Dieses

Various sizes of enharmonics can be generated by the use of arithmetic-harmonic means. Interval sizes are given in Cents for comparison, starting with the smallest examples and moving to the largest:

27:28 → 54:55:56 — 55/54 = 31.767c., 56/55 = 31.194c. — The small, septimal chromatic semitone generates "sixth-tones".

24:25 → 48:49:50 — 49/48 = 35.697c., 50/49 = 34.976c. — "Quarter-tones" prominant in Plato (Republic).

43/42 = 40.737c. - closest epimoric ratio to Just 5-Limit Diesis.

128/125 = 41.059c. - Just Intonation, 5-Limit Diesis ratio.

20:21 → 40:41:42 — 41/40 = 42.749c., 42/41 = 41.7185c. — 7-Limit medial (4 comma) semitone gives close approximations to Diesis

243:256 → 486:499:512 — 499/486 = 45.700c., 512/499 = 44.525c. — Pythagorean 3-Limit medial semitone gives ratios between the Just Diesis and true 'quarter-tone.'

128:135 → 256:263:270 — 263/256 = 46.703c., 270/263 = 45.476c. — 5-Limit medial semitone close to 3-Limit "cousin"

36/35 = 48.770c. - closest 7-Limit 'quarter-tone' (Archytas).

35/34 = 50.184c. - closest epimoric approximation to true 'quarter-tone' (50c.).

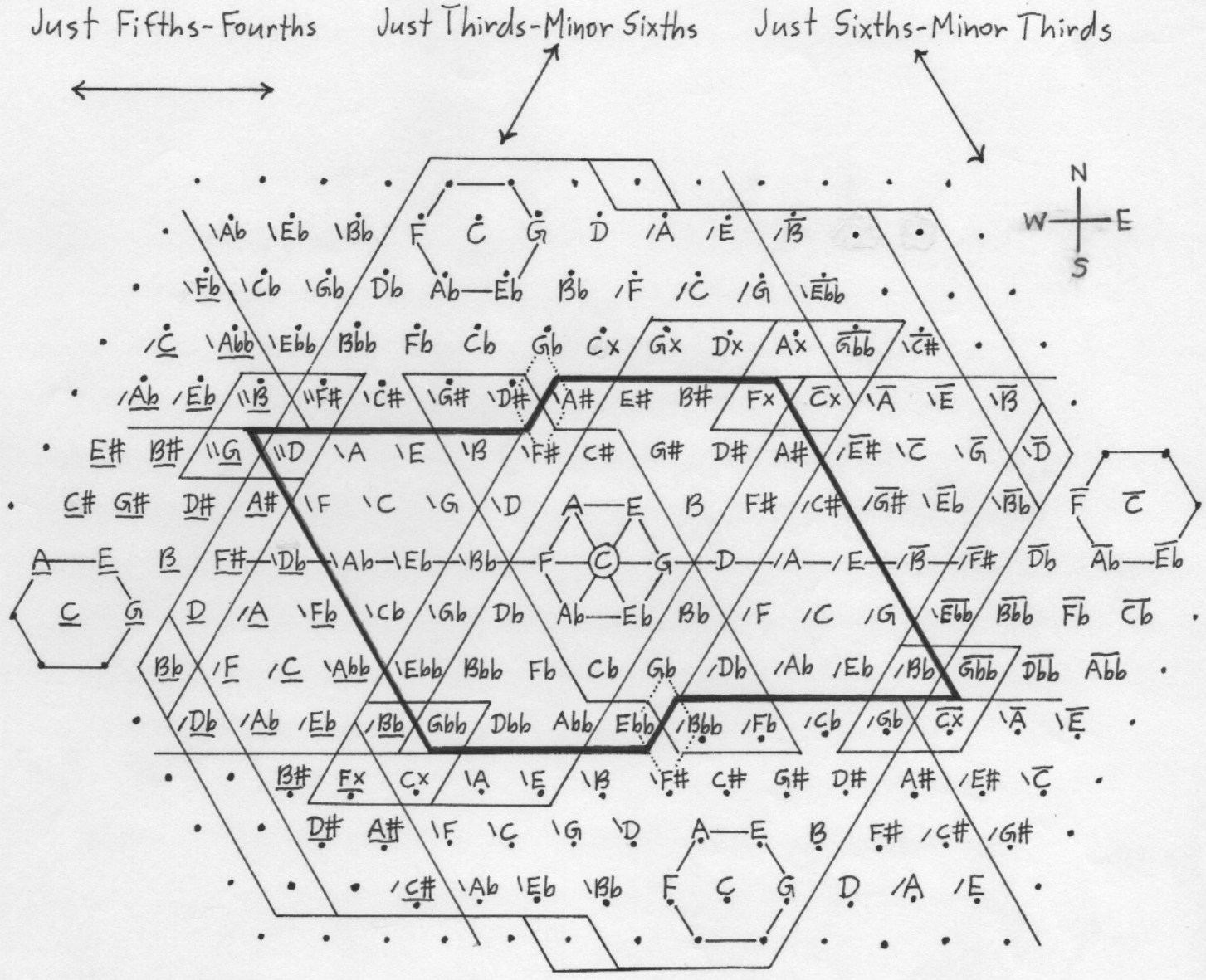
15:16 → 30:31:32 — 31/30 = 56.767c., 32/31 = 54.964c. — Large, diatonic (5 comma) Just semitone, yields 'quarter-tones' associated with Didymus Enharmonic.

14:15 → 28:29:30 — 29/28 = 60.751c., 30/29 = 58.6915c. — Large, 7-Limit diatonic semitone yields over-sized 'quarter-tones.'

25:27 → 25:26:27 — 26/25 = 67.900c., 27/26 = 65.337c. — "Acute" (6 comma) Just over-sized semitone yields large 'quarter-tones.'

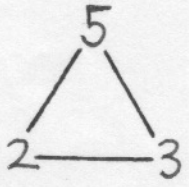
Matrix

The $2^g \cdot 3^b \cdot 5^c$ matrix is derived from the three axes of Just Intonation, indicating the tuning of Just Fifths, Just Major Thirds, and Just Minor Thirds:



The graph shows some of the important boundaries of the varions regions of 5-Limit Just Intonation.

Genesis of the Matrix as Number

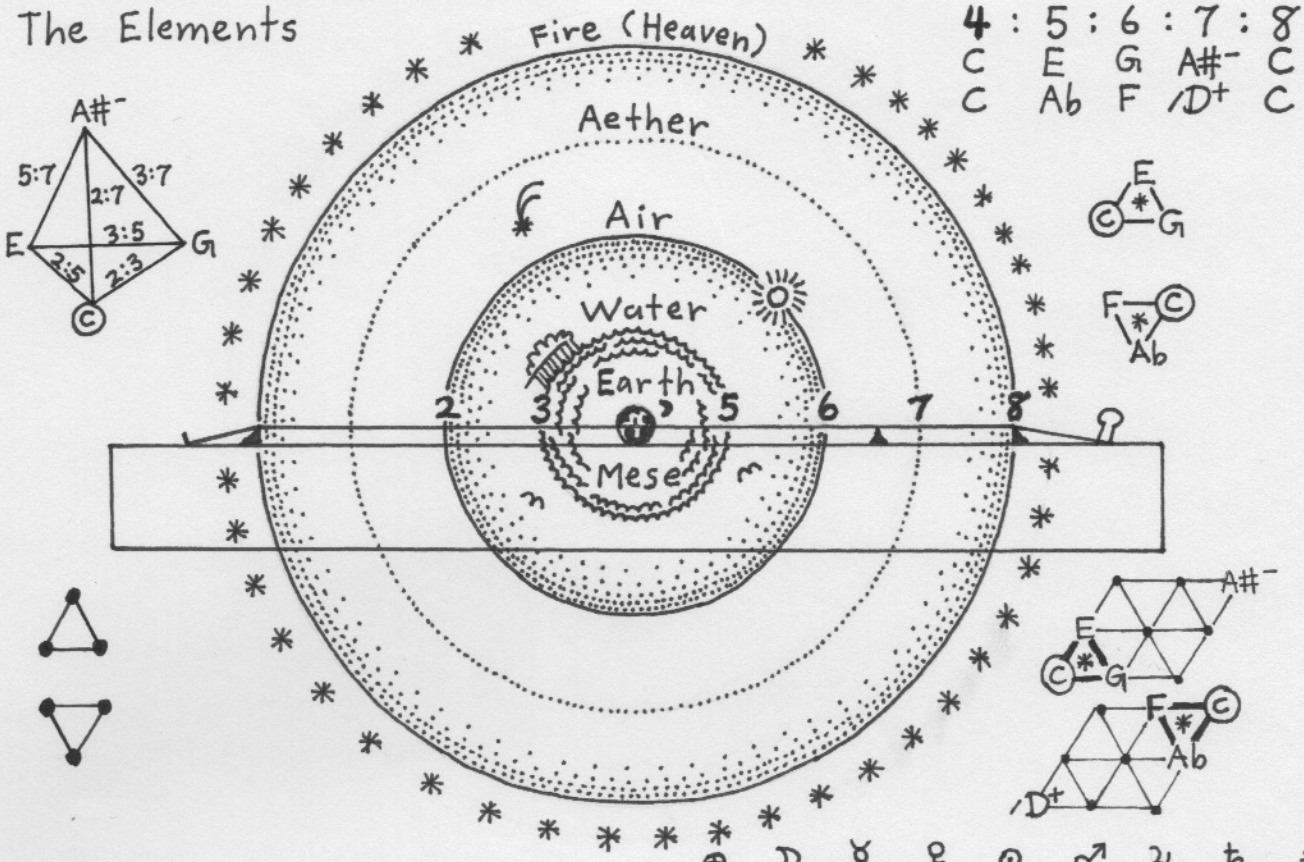


The graph shows
System boundaries for
12-et, 19-et, 31-et, 53-et, 152-et

Two Prominant Monochords

The Cosmological Monochord: 1:2 → 2:4 → 4:8

The Elements



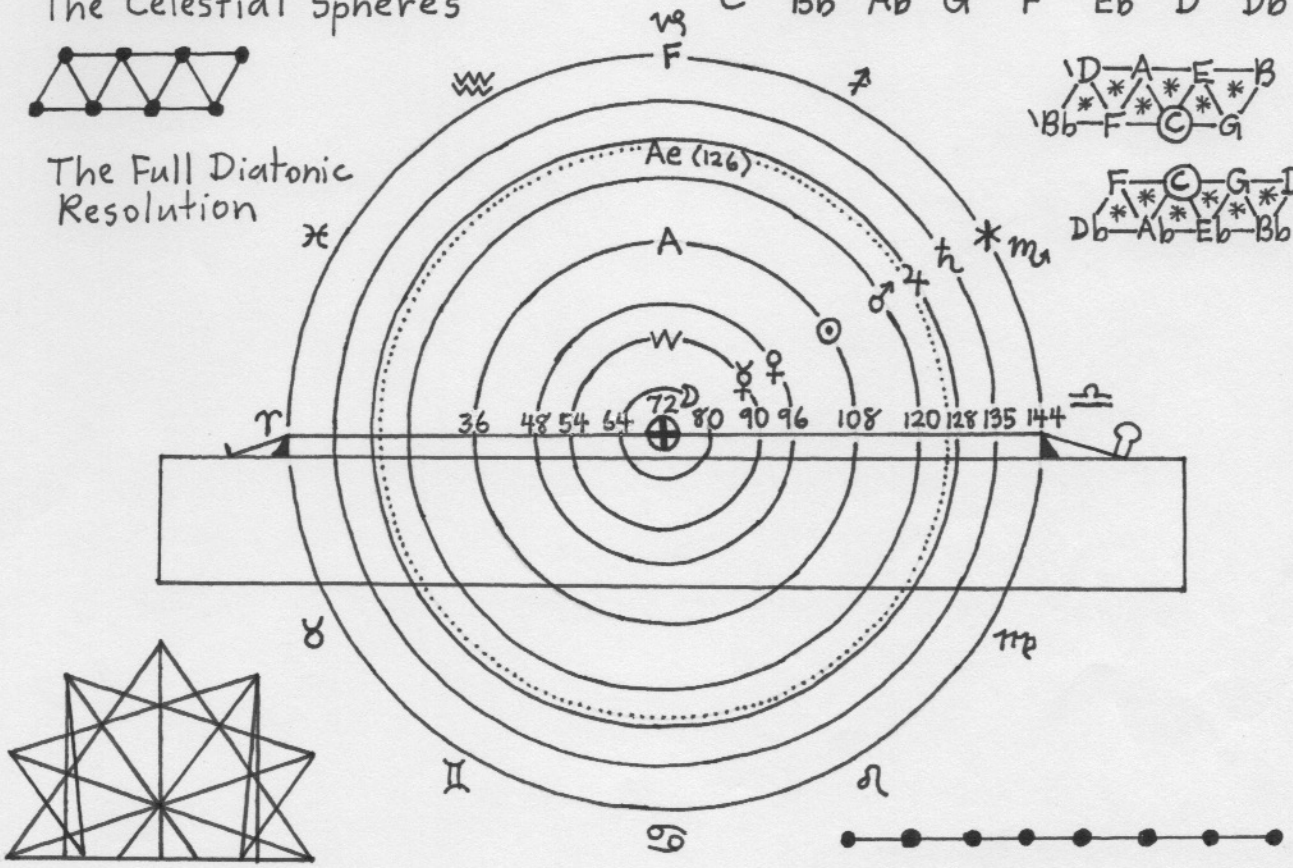
4	5	6	7	8
C	E	G	A#⁻	C
C	Ab	F	/D⁺	C

The Planetary Monochord

⊕	☽	☿	♀	☉	♂	♃	♄	✱
72	80	90	96	108	120	128	135	144
C	\D	E	F	G	A	\Bb	B	C
C	Bb	Ab	G	F	Eb	D	Db	C

The Celestial Spheres

The Full Diatonic Resolution

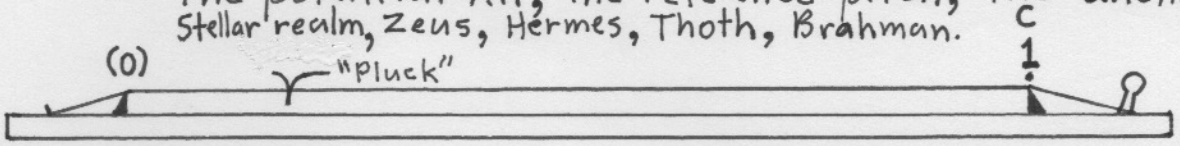


The first Six operations (Emanations) of the sacred Monochord Science

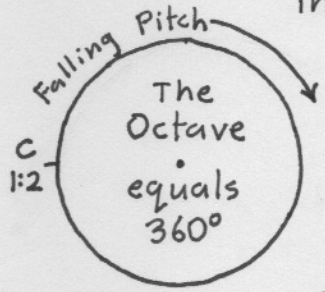
Given: the open string (C), it has two aspects –

A - Unsounded. The Silence (0), the unmanifested state, the primeval waters, the Abyss, the Chaos, the Nun.

B - Sounded. The open string reveals The One, THE MONAD, the element Fire or Heaven, the androgyne, the high god, the first manifest vibratory event, the Source, Unity, the potential All, the reference pitch, the Unchanging, Stellar realm, Zeus, Hermes, Thoth, Brahman.



[1] The 1st operation. Divide the string in half, placing the movable bridge in the middle (The Mese). In the accompanying pebble arithmetic, place the 'mother bead' to establish the Matrix. Emanation: 2 THE DYAD, the element Earth, the female goddess, the Mese, the Point, the Octave (microcosm within macrocosm), Cyclical Identity, the two realms (Harmonics and Sub-harmonics), the Other within the One, the Intelligible Realm, the Intellectual-Principle, the Matrix of relations, the 1st Logos (Ratio 1:2), the Spiral of the Changing, Yin-Yang, Ṛta, Maāt.



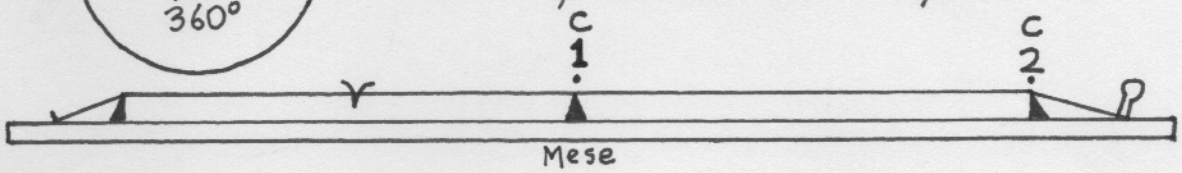
1 : 2

Harmonics (rising) sequence – C C

Monochord (falling) sequence – C C ©

Self-dualing.

The only 2-Limit Harmony

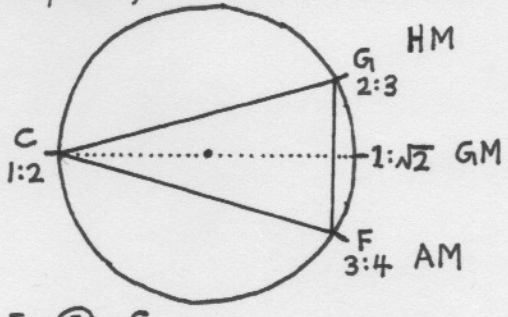


[2] The 2nd operation. Divide the octave in half, place a pebble to form a horizontal line (3-Limit tuning forms a line of fifths). Emanation: 3 THE TRIAD, the element Air, the male god, the hero, the Other manifest, diversity, the Tetrachord frames, the Line, the marraige (of 2 and 3), the Soul-principle, the intervals of Power. Apollo, Shu, Shiva, arrow.

HM = the Harmonic Mean (from Harmonic Series)

AM = the Arithmetic Mean (from Monochord)

GM = the Geometric Mean (the irrational-temper)



3-Limit	2	3	4	Dual	3	4	6
Harmonics –	C	G	C	H –	C	F	C
Monochord –	C	F	C	M –	C	G	C

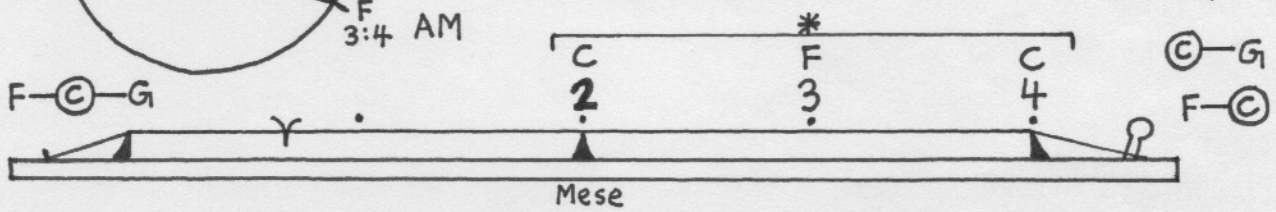


Table of **Prime Factors** for all numbers (up to 1200):

(0)	
1	*
2	*
3	*
4	2^2
5	*
6	$2 \cdot 3$
7	*
8	2^3
9	3^2
10	$2 \cdot 5$
11	*
12	$2^2 \cdot 3$
13	*
14	$2 \cdot 7$
15	$3 \cdot 5$
16	2^4
17	*
18	$2 \cdot 3^2$
19	*
20	$2^2 \cdot 5$
21	$3 \cdot 7$
22	$2 \cdot 11$
23	*
24	$2^3 \cdot 3$
25	5^2
26	$2 \cdot 13$
27	3^3
28	$2^2 \cdot 7$
29	*
30	$2 \cdot 3 \cdot 5$
31	*
32	2^5
33	$3 \cdot 11$
34	$2 \cdot 17$
35	$5 \cdot 7$
36	$2^2 \cdot 3^2$
37	*
38	$2 \cdot 19$
39	$3 \cdot 13$
40	$2^3 \cdot 5$
41	*
42	$2 \cdot 3 \cdot 7$
43	*
44	$2^2 \cdot 11$
45	$3^2 \cdot 5$
46	$2 \cdot 23$
47	*
48	$2^4 \cdot 3$
49	7^2
50	$2 \cdot 5^2$
51	$3 \cdot 17$
52	$2^2 \cdot 13$
53	*
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71	*
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727	*
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787	*
788	$2^2 \cdot 197$
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797	*
798	$2 \cdot 3 \cdot 7 \cdot 19$
799	$17 \cdot 47$

800	$2^5 \cdot 5^2$
801	$3^2 \cdot 89$
802	$2 \cdot 401$
803	$11 \cdot 73$
804	$2^2 \cdot 3 \cdot 67$
805	$5 \cdot 7 \cdot 23$
806	$2 \cdot 13 \cdot 31$
807	$3 \cdot 269$
808	$2^3 \cdot 101$
809	*
810	$2 \cdot 3^4 \cdot 5$
811	*
812	$2^2 \cdot 7 \cdot 29$
813	$3 \cdot 271$
814	$2 \cdot 11 \cdot 37$
815	$5 \cdot 163$
816	$2^4 \cdot 3 \cdot 17$
817	$19 \cdot 43$
818	$2 \cdot 409$
819	$3^2 \cdot 7 \cdot 13$
820	$2^2 \cdot 5 \cdot 41$
821	*
822	$2 \cdot 3 \cdot 137$
823	*
824	$2^3 \cdot 103$
825	$3 \cdot 5^2 \cdot 11$
826	$2 \cdot 7 \cdot 59$
827	*
828	$2^2 \cdot 3^2 \cdot 23$
829	*
830	$2 \cdot 5 \cdot 83$
831	$3 \cdot 277$
832	$2^6 \cdot 13$
833	$7^2 \cdot 17$
834	$2 \cdot 3 \cdot 139$
835	$5 \cdot 167$
836	$2^2 \cdot 11 \cdot 19$
837	$3^3 \cdot 31$
838	$2 \cdot 419$
839	*
840	$2^3 \cdot 3 \cdot 5 \cdot 7$
841	29^2
842	$2 \cdot 421$
843	$3 \cdot 281$
844	$2^2 \cdot 211$
845	$5 \cdot 13^2$
846	$2 \cdot 3^2 \cdot 47$
847	$7 \cdot 11^2$
848	$2^4 \cdot 53$
849	$3 \cdot 283$
850	$2 \cdot 5^2 \cdot 17$
851	$23 \cdot 37$
852	$2^2 \cdot 3 \cdot 71$
853	*
854	$2 \cdot 7 \cdot 61$
855	$3^2 \cdot 5 \cdot 19$
856	$2^3 \cdot 107$
857	*
858	$2 \cdot 3 \cdot 11 \cdot 13$
859	*
860	$2^2 \cdot 5 \cdot 43$
861	$3 \cdot 7 \cdot 41$
862	$2 \cdot 431$
863	*
864	$2^5 \cdot 3^3$
865	$5 \cdot 173$
866	$2 \cdot 433$
867	$3 \cdot 17^2$
868	$2^2 \cdot 7 \cdot 31$
869	$11 \cdot 79$
870	$2 \cdot 3 \cdot 5 \cdot 29$
871	$13 \cdot 67$
872	$2^3 \cdot 109$
873	$3^2 \cdot 97$
874	$2 \cdot 19 \cdot 23$
875	$5^3 \cdot 7$
876	$2^2 \cdot 3 \cdot 73$
877	*
878	$2 \cdot 439$
879	$3 \cdot 293$
880	$2^4 \cdot 5 \cdot 11$
881	*
882	$2 \cdot 3^2 \cdot 7^2$
883	*
884	$2^2 \cdot 13 \cdot 17$
885	$3 \cdot 5 \cdot 59$
886	$2 \cdot 443$
887	*
888	$2^3 \cdot 3 \cdot 37$
889	$7 \cdot 127$
890	$2 \cdot 5 \cdot 89$
891	$3^4 \cdot 11$
892	$2^2 \cdot 223$
893	$19 \cdot 47$
894	$2 \cdot 3 \cdot 149$
895	$5 \cdot 179$
896	$2^7 \cdot 7$
897	$3 \cdot 13 \cdot 23$
898	$2 \cdot 449$
899	$29 \cdot 31$
900	$2^2 \cdot 3^2 \cdot 5^2$
901	$17 \cdot 53$
902	$2 \cdot 11 \cdot 41$
903	$3 \cdot 7 \cdot 43$
904	$2^3 \cdot 113$
905	$5 \cdot 181$
906	$2 \cdot 3 \cdot 151$
907	*
908	$2^2 \cdot 227$
909	$3^2 \cdot 101$
910	$2 \cdot 5 \cdot 7 \cdot 13$
911	*
912	$2^4 \cdot 3 \cdot 19$
913	$11 \cdot 83$
914	$2 \cdot 457$
915	$3 \cdot 5 \cdot 61$
916	$2^2 \cdot 229$
917	$7 \cdot 131$
918	$2 \cdot 3^3 \cdot 17$
919	*
920	$2^3 \cdot 5 \cdot 23$
921	$3 \cdot 307$
922	$2 \cdot 461$
923	$13 \cdot 71$
924	$2^2 \cdot 3 \cdot 7 \cdot 11$
925	$5^2 \cdot 37$
926	$2 \cdot 463$
927	$3^2 \cdot 103$
928	$2^5 \cdot 29$
929	*
930	$2 \cdot 3 \cdot 5 \cdot 31$
931	$7^2 \cdot 19$
932	$2^2 \cdot 233$
933	$3 \cdot 311$
934	$2 \cdot 467$
935	$5 \cdot 11 \cdot 17$
936	$2^3 \cdot 3^2 \cdot 13$
937	*
938	$2 \cdot 7 \cdot 67$
939	$3 \cdot 313$
940	$2^2 \cdot 5 \cdot 47$
941	*
942	$2 \cdot 3 \cdot 157$
943	$23 \cdot 41$
944	$2^4 \cdot 59$
945	$3^3 \cdot 5 \cdot 7$
946	$2 \cdot 11 \cdot 43$
947	*
948	$2^2 \cdot 3 \cdot 79$
949	$13 \cdot 73$
950	$2 \cdot 5^2 \cdot 19$
951	$3 \cdot 317$
952	$2^3 \cdot 7 \cdot 17$
953	*
954	$2 \cdot 3^2 \cdot 53$
955	$5 \cdot 191$
956	$2^2 \cdot 239$
957	$3 \cdot 11 \cdot 29$
958	$2 \cdot 479$
959	$7 \cdot 137$
960	$2^6 \cdot 3 \cdot 5$
961	31^2
962	$2 \cdot 13 \cdot 37$
963	$3^2 \cdot 107$
964	$2^2 \cdot 241$
965	$5 \cdot 193$
966	$2 \cdot 3 \cdot 7 \cdot 23$
967	*
968	$2^3 \cdot 11^2$
969	$3 \cdot 17 \cdot 19$
970	$2 \cdot 5 \cdot 97$
971	*
972	$2^2 \cdot 3^5$
973	$7 \cdot 139$
974	$2 \cdot 487$
975	$3 \cdot 5^2 \cdot 13$
976	$2^4 \cdot 61$
977	*
978	$2 \cdot 3 \cdot 163$
979	$11 \cdot 89$
980	$2^2 \cdot 5 \cdot 7^2$
981	$3^2 \cdot 109$
982	$2 \cdot 491$
983	*
984	$2^3 \cdot 3 \cdot 41$
985	$5 \cdot 197$
986	$2 \cdot 17 \cdot 29$
987	$3 \cdot 7 \cdot 47$
988	$2^2 \cdot 13 \cdot 19$
989	$23 \cdot 41$
990	$2 \cdot 3^2 \cdot 5 \cdot 11$
991	*
992	$2^5 \cdot 31$
993	$3 \cdot 331$
994	$2 \cdot 7 \cdot 71$
995	$5 \cdot 199$
996	$2^2 \cdot 3 \cdot 83$
997	*
998	$2 \cdot 499$
999	$3^2 \cdot 37$

1000	$2^3 \cdot 5^3$
1001	$7 \cdot 11 \cdot 13$
1002	$2 \cdot 3 \cdot 167$
1003	$17 \cdot 59$
1004	$2^2 \cdot 251$
1005	$3 \cdot 5 \cdot 67$
1006	$2 \cdot 503$
1007	$19 \cdot 53$
1008	$2^4 \cdot 3^2 \cdot 7$
1009	*
1010	$2 \cdot 5 \cdot 101$
1011	$3 \cdot 337$
1012	$2^2 \cdot 11 \cdot 23$
1013	*
1014	$2 \cdot 3 \cdot 13^2$
1015	$5 \cdot 7 \cdot 29$
1016	$2^3 \cdot 127$
1017	$3^2 \cdot 113$
1018	$2 \cdot 509$
1019	*
1020	$2^2 \cdot 3 \cdot 5 \cdot 17$
1021	*
1022	$2 \cdot 7 \cdot 73$
1023	$3 \cdot 11 \cdot 31$
1024	2^{10}
1025	$5^2 \cdot 41$
1026	$2 \cdot 3^3 \cdot 19$
1027	$13 \cdot 79$
1028	$2^2 \cdot 257$
1029	$3 \cdot 7^3$
1030	$2 \cdot 5 \cdot 103$
1031	*
1032	$2^3 \cdot 3 \cdot 43$
1033	*
1034	$2 \cdot 11 \cdot 47$
1035	$3^2 \cdot 5 \cdot 23$
1036	$2^2 \cdot 7 \cdot 37$
1037	$17 \cdot 61$
1038	$2 \cdot 3 \cdot 173$
1039	*
1040	$2^4 \cdot 5 \cdot 13$
1041	$3 \cdot 347$
1042	$2 \cdot 521$
1043	$7 \cdot 149$
1044	$2^2 \cdot 3^2 \cdot 29$
1045	$5 \cdot 11 \cdot 19$
1046	$2 \cdot 523$
1047	$3 \cdot 349$
1048	$2^3 \cdot 131$
1049	*
1050	$2 \cdot 3 \cdot 5^2 \cdot 7$
1051	*
1052	$2^2 \cdot 263$
1053	$3^4 \cdot 13$
1054	$2 \cdot 17 \cdot 31$
1055	$5 \cdot 211$
1056	$2^5 \cdot 3 \cdot 11$
1057	$7 \cdot 151$
1058	$2 \cdot 23^2$
1059	$3 \cdot 353$
1060	$2^2 \cdot 5 \cdot 53$
1061	*
1062	$2 \cdot 3^2 \cdot 59$
1063	*
1064	$2^3 \cdot 7 \cdot 19$
1065	$3 \cdot 5 \cdot 71$
1066	$2 \cdot 13 \cdot 41$
1067	$11 \cdot 97$
1068	$2^2 \cdot 3 \cdot 89$
1069	*
1070	$2 \cdot 5 \cdot 107$
1071	$3^2 \cdot 7 \cdot 17$
1072	$2^4 \cdot 67$
1073	$29 \cdot 37$
1074	$2 \cdot 3 \cdot 179$
1075	$5^2 \cdot 43$
1076	$2^2 \cdot 269$
1077	$3 \cdot 359$
1078	$2 \cdot 7^2 \cdot 11$
1079	$13 \cdot 83$
1080	$2^3 \cdot 3^3 \cdot 5$
1081	$23 \cdot 47$
1082	$2 \cdot 541$
1083	$3 \cdot 19^2$
1084	$2^2 \cdot 271$
1085	$5 \cdot 7 \cdot 31$
1086	$2 \cdot 3 \cdot 181$
1087	*
1088	$2^6 \cdot 17$
1089	$3^2 \cdot 11^2$
1090	$2 \cdot 5 \cdot 109$
1091	*
1092	$2^2 \cdot 3 \cdot 7 \cdot 13$
1093	*
1094	$2 \cdot 547$
1095	$3 \cdot 5 \cdot 73$
1096	$2^3 \cdot 137$
1097	*
1098	$2 \cdot 3^2 \cdot 61$
1099	$7 \cdot 157$
1100	$2^2 \cdot 5^2 \cdot 11$
1101	$3 \cdot 367$
1102	$2 \cdot 19 \cdot 29$
1103	*
1104	$2^4 \cdot 3 \cdot 23$
1105	$5 \cdot 13 \cdot 17$
1106	$2 \cdot 7 \cdot 79$
1107	$3^3 \cdot 41$
1108	$2^2 \cdot 277$
1109	*
1110	$2 \cdot 3 \cdot 5 \cdot 37$
1111	$11 \cdot 101$
1112	$2^3 \cdot 139$
1113	$3 \cdot 7 \cdot 53$
1114	$2 \cdot 557$
1115	$5 \cdot 223$
1116	$2^2 \cdot 3^2 \cdot 31$
1117	*
1118	$2 \cdot 13 \cdot 43$
1119	$3 \cdot 373$
1120	$2^5 \cdot 5 \cdot 7$
1121	$19 \cdot 59$
1122	$2 \cdot 3 \cdot 11 \cdot 17$
1123	*
1124	$2^2 \cdot 281$
1125	$3^2 \cdot 5^3$
1126	$2 \cdot 563$
1127	$7^2 \cdot 23$
1128	$2^3 \cdot 3 \cdot 47$
1129	*
1130	$2 \cdot 5 \cdot 113$
1131	$3 \cdot 13 \cdot 29$
1132	$2^2 \cdot 283$
1133	$11 \cdot 103$
1134	$2 \cdot 3^4 \cdot 7$
1135	$5 \cdot 227$
1136	$2^4 \cdot 71$
1137	$3 \cdot 379$
1138	$2 \cdot 569$
1139	$17 \cdot 67$
1140	$2^2 \cdot 3 \cdot 5 \cdot 19$
1141	$7 \cdot 163$
1142	$2 \cdot 571$
1143	$3^2 \cdot 127$
1144	$2^3 \cdot 11 \cdot 13$
1145	$5 \cdot 229$
1146	$2 \cdot 3 \cdot 191$
1147	$31 \cdot 37$
1148	$2^2 \cdot 7 \cdot 41$
1149	$3 \cdot 383$
1150	$2 \cdot 5^2 \cdot 23$
1151	*
1152	$2^7 \cdot 3^2$
1153	*
1154	$2 \cdot 577$
1155	$3 \cdot 5 \cdot 7 \cdot 11$
1156	$2^2 \cdot 17^2$
1157	$13 \cdot 89$
1158	$2 \cdot 3 \cdot 193$
1159	$19 \cdot 61$
1160	$2^3 \cdot 5 \cdot 29$
1161	$3^3 \cdot 43$
1162	$2 \cdot 7 \cdot 83$
1163	*
1164	$2^2 \cdot 3 \cdot 97$
1165	$5 \cdot 233$
1166	$2 \cdot 11 \cdot 53$
1167	$3 \cdot 389$
1168	$2^4 \cdot 73$
1169	$7 \cdot 167$
1170	$2 \cdot 3^2 \cdot 5 \cdot 13$
1171	*
1172	$2^2 \cdot 293$
1173	$3 \cdot 17 \cdot 23$
1174	$2 \cdot 587$
1175	$5^2 \cdot 47$
1176	$2^3 \cdot 3 \cdot 7^2$
1177	$11 \cdot 107$
1178	$2 \cdot 19 \cdot 31$
1179	$3^2 \cdot 131$
1180	$2^2 \cdot 5 \cdot 59$
1181	*
1182	$2 \cdot 3 \cdot 197$
1183	$7 \cdot 13^2$
1184	$2^5 \cdot 37$
1185	$3 \cdot 5 \cdot 79$
1186	$2 \cdot 593$
1187	*
1188	$2^2 \cdot 3^3 \cdot 11$
1189	$29 \cdot 41$
1190	$2 \cdot 5 \cdot 7 \cdot 17$
1191	$3 \cdot 397$
1192	$2^3 \cdot 149$
1193	*
1194	$2 \cdot 3 \cdot 199$
1195	$5 \cdot 239$
1196	$2^2 \cdot 13 \cdot 23$
1197	$3^2 \cdot 7 \cdot 19$
1198	$2 \cdot 599$
1199	$11 \cdot 109$
1200	$2^4 \cdot 3 \cdot 5^2$

Table of Prime Numbers less than 2000 :

(1)	223	503	827	1163	1511	1877
2	227	509	829	1171	1523	1879
3	229	521	839	1181	1531	1889
5	233	523	853	1187	1543	1901
7	239	541	857	1193	1549	1907
11	241	547	859	1201	1553	1913
13	251	557	863	1213	1559	1931
17	257	563	877	1217	1567	1933
19	263	569	881	1223	1571	1949
23	269	571	883	1229	1579	1951
29	271	577	887	1231	1583	1973
31	277	587	907	1237	1597	1979
37	281	593	911	1249	1601	1987
41	283	599	919	1259	1607	1993
43	293	601	929	1277	1609	1997
47	307	607	937	1279	1613	1999
53	311	613	941	1283	1619	
59	313	617	947	1289	1621	
61	317	619	953	1291	1627	
67	331	631	967	1297	1637	
71	337	641	971	1301	1657	
73	347	643	977	1303	1663	
79	349	647	983	1307	1667	
83	353	653	991	1319	1669	
89	359	659	997	1321	1693	
97	367	661	1009	1327	1697	
101	373	673	1013	1361	1699	
103	379	677	1019	1367	1709	
107	383	683	1021	1373	1721	
109	389	691	1031	1381	1723	
113	397	701	1033	1399	1733	
127	401	709	1039	1409	1741	
131	409	719	1049	1423	1747	
137	419	727	1051	1427	1753	
139	421	733	1061	1429	1759	
149	431	739	1063	1433	1777	
151	433	743	1069	1439	1783	
157	439	751	1087	1447	1787	
163	443	757	1091	1451	1789	
167	449	761	1093	1453	1801	
173	457	769	1097	1459	1811	
179	461	773	1103	1471	1823	
181	463	787	1109	1481	1831	
191	467	797	1117	1483	1847	
193	479	809	1123	1487	1861	
197	487	811	1129	1489	1867	
199	491	821	1151	1493	1871	
211	499	823	1153	1499	1873	

Table of mixed Prime and Composite Numbers

Every consecutive pair of primes 'pulls in' a mean composite, which 'mediates' between the primes:

3	*
4	2^2
5	*
6	$2 \cdot 3$
7	*
9	3^2
11	*
12	$2^2 \cdot 3$
13	*
15	$3 \cdot 5$
17	*
18	$2 \cdot 3^2$
19	*
21	$3 \cdot 7$
23	*
26	$2 \cdot 13$
29	*
30	$2 \cdot 3 \cdot 5$
31	*
34	$2 \cdot 17$
37	*
39	$3 \cdot 13$
41	*
42	$2 \cdot 3 \cdot 7$
43	*
45	$3^2 \cdot 5$
47	*
50	$2 \cdot 5^2$
53	*
56	$2^3 \cdot 7$
59	*
60	$2^2 \cdot 3 \cdot 5$
61	*
64	2^6
67	*
69	$3 \cdot 23$
71	*
72	$2^3 \cdot 3^2$
73	*
76	$2^2 \cdot 19$
79	*
81	3^4
83	*
86	$2 \cdot 43$
89	*
93	$3 \cdot 31$
97	*
99	$3^2 \cdot 11$
101	*
102	$2 \cdot 51$
103	*
105	$3 \cdot 5 \cdot 7$
107	*
108	$2^2 \cdot 3^3$
109	*
111	$3 \cdot 37$
113	*
120	$2^3 \cdot 3 \cdot 5$
127	*
129	$3 \cdot 43$
131	*
134	$2 \cdot 67$
137	*
138	$2 \cdot 3 \cdot 23$
139	*
144	$2^4 \cdot 3^2$
149	*
150	$2 \cdot 3 \cdot 5^2$
151	*
154	$2 \cdot 7 \cdot 11$
157	*
160	$2^5 \cdot 5$
163	*
165	$3 \cdot 5 \cdot 11$
167	*
170	$2 \cdot 5 \cdot 17$
173	*
176	$2^4 \cdot 11$
179	*
180	$2^2 \cdot 3^2 \cdot 5$
181	*
186	$2 \cdot 3 \cdot 31$
191	*
192	$2^6 \cdot 3$
193	*
195	$3 \cdot 5 \cdot 13$
197	*
198	$2 \cdot 3^2 \cdot 11$
199	*
205	$5 \cdot 41$
211	*
217	$7 \cdot 31$
223	*
225	$3^2 \cdot 5^2$
227	*
228	$2^2 \cdot 3 \cdot 19$
229	*
231	$3 \cdot 7 \cdot 11$
233	*
236	$2^2 \cdot 59$
239	*
240	$2^4 \cdot 3 \cdot 5$
241	*
246	$2 \cdot 3 \cdot 41$
251	*
254	$2 \cdot 127$
257	*
260	$2^2 \cdot 5 \cdot 13$
263	*
266	$2 \cdot 7 \cdot 19$
269	*
270	$2 \cdot 3^3 \cdot 5$
271	*
274	$2 \cdot 137$
277	*
279	$3^2 \cdot 31$
281	*
282	$2 \cdot 3 \cdot 47$
283	*
288	$2^5 \cdot 3^2$
293	*
300	$2^2 \cdot 3 \cdot 5^2$
307	*
309	$3 \cdot 103$
311	*
312	$2^3 \cdot 3 \cdot 13$
313	*
315	$3^2 \cdot 5 \cdot 7$
317	*
324	$2^2 \cdot 3^4$
331	*
334	$2 \cdot 167$
337	*
342	$2 \cdot 3^2 \cdot 19$
347	*
348	$2^2 \cdot 3 \cdot 29$
349	*
351	$3^3 \cdot 13$
353	*
356	$2^2 \cdot 89$
359	*
363	$3 \cdot 11^2$
367	*
370	$2 \cdot 5 \cdot 37$
373	*
376	$2^3 \cdot 47$
379	*
381	$3 \cdot 127$
383	*
386	$2 \cdot 193$
389	*
393	$3 \cdot 131$
397	*
399	$3 \cdot 7 \cdot 19$
401	*
405	$3^4 \cdot 5$
409	*
414	$2 \cdot 3^2 \cdot 23$
419	*
420	$2^2 \cdot 3 \cdot 5 \cdot 7$
421	*
426	$2 \cdot 3 \cdot 71$
431	*
432	$2^4 \cdot 3^3$
433	*
436	$2^2 \cdot 109$
439	*
441	$3^2 \cdot 7^2$
443	*
446	$2 \cdot 223$
449	*
453	$3 \cdot 151$
457	*
459	$3^2 \cdot 51$

461	*
462	$2 \cdot 3 \cdot 7 \cdot 11$
463	*
465	$3 \cdot 5 \cdot 31$
467	*
473	$11 \cdot 43$
479	*
483	$3 \cdot 7 \cdot 23$
487	*
489	$3 \cdot 163$
491	*
495	$3^2 \cdot 5 \cdot 11$
499	*
501	$3 \cdot 167$
503	*
506	$2 \cdot 11 \cdot 23$
509	*
515	$5 \cdot 103$
521	*
522	$2 \cdot 3^2 \cdot 29$
523	*
532	$2^2 \cdot 7 \cdot 19$
541	*
544	$2^5 \cdot 17$
547	*
552	$2^3 \cdot 3 \cdot 23$
557	*
560	$2^4 \cdot 5 \cdot 7$
563	*
566	$2 \cdot 283$
569	*
570	$2 \cdot 3 \cdot 5 \cdot 19$
571	*
574	$2 \cdot 7 \cdot 41$
577	*
582	$2 \cdot 3 \cdot 97$
587	*
590	$2 \cdot 5 \cdot 59$
593	*
596	$2^2 \cdot 149$
599	*
600	$2^3 \cdot 3 \cdot 5^2$
601	*
604	$2^2 \cdot 151$
607	*
610	$2 \cdot 5 \cdot 61$
613	*
615	$3 \cdot 5 \cdot 41$
617	*
618	$2 \cdot 3 \cdot 103$
619	*
625	5^4
631	*
636	$2^2 \cdot 3 \cdot 53$
641	*
642	$2 \cdot 3 \cdot 107$
643	*
645	$3 \cdot 5 \cdot 43$
647	*
650	$2 \cdot 5^2 \cdot 13$
653	*
656	$2^4 \cdot 41$
659	*
660	$2^2 \cdot 3 \cdot 5 \cdot 11$
661	*
667	$23 \cdot 29$
673	*
675	$3^3 \cdot 5^2$
677	*
680	$2^3 \cdot 5 \cdot 17$
683	*
687	$3 \cdot 229$
691	*
696	$2^3 \cdot 3 \cdot 29$
701	*
705	$3 \cdot 5 \cdot 47$
709	*
714	$2 \cdot 3 \cdot 7 \cdot 17$
719	*
723	$3 \cdot 241$
727	*
730	$2 \cdot 5 \cdot 73$
733	*
736	$2^5 \cdot 23$
739	*
741	$3 \cdot 13 \cdot 19$
743	*
747	$3^2 \cdot 81$
751	*
754	$2 \cdot 13 \cdot 29$
757	*
759	$3 \cdot 11 \cdot 23$
761	*
765	$3 \cdot 5 \cdot 51$
769	*
771	$3 \cdot 257$
773	*
780	$2^2 \cdot 3 \cdot 5 \cdot 13$
787	*
792	$2^3 \cdot 3^2 \cdot 11$
797	*
803	$11 \cdot 73$
809	*
810	$2 \cdot 3^4 \cdot 5$
811	*
816	$2^4 \cdot 51$
821	*
822	$2 \cdot 3 \cdot 137$
823	*
825	$3 \cdot 5^2 \cdot 11$
827	*
828	$2^2 \cdot 3^2 \cdot 23$
829	*
834	$2 \cdot 3 \cdot 139$
839	*
846	$2 \cdot 3^2 \cdot 47$
853	*
855	$3^2 \cdot 5 \cdot 19$
857	*
858	$2 \cdot 3 \cdot 11 \cdot 13$
859	*
861	$3 \cdot 7 \cdot 41$
863	*
870	$2 \cdot 3 \cdot 5 \cdot 29$
877	*
879	$3 \cdot 293$
881	*
882	$2 \cdot 3^2 \cdot 7^2$
883	*
885	$3 \cdot 5 \cdot 59$
887	*
897	$3 \cdot 13 \cdot 23$
907	*
909	$3^2 \cdot 101$
911	*
915	$3 \cdot 5 \cdot 61$
919	*
924	$2^2 \cdot 3 \cdot 7 \cdot 11$
929	*
933	$3 \cdot 311$
937	*
939	$3 \cdot 313$
941	*
944	$2^4 \cdot 59$
947	*
950	$2 \cdot 5^2 \cdot 19$
953	*
960	$2^6 \cdot 3 \cdot 5$
967	*
969	$3 \cdot 17 \cdot 19$
971	*
974	$2 \cdot 487$
977	*
980	$2^2 \cdot 5 \cdot 7^2$
983	*
987	$3 \cdot 7 \cdot 47$
991	*
994	$2 \cdot 7 \cdot 71$
997	*
1003	$17 \cdot 59$
1009	*
1011	$3 \cdot 337$
1013	*
1016	$2^3 \cdot 127$
1019	*
1020	$2^2 \cdot 5 \cdot 51$
1021	*
1026	$2 \cdot 3^3 \cdot 19$
1031	*
1032	$2^3 \cdot 3 \cdot 43$
1033	*
1036	$2^2 \cdot 7 \cdot 37$
1039	*
1044	$2^2 \cdot 3^2 \cdot 29$
1049	*
1050	$2 \cdot 3 \cdot 5^2 \cdot 7$
1051	*
1056	$2^5 \cdot 3 \cdot 11$
1061	*
1062	$2 \cdot 3^2 \cdot 59$
1063	*
1066	$2 \cdot 13 \cdot 41$
1069	*
1078	$2 \cdot 7^2 \cdot 11$
1087	*
1089	$3^2 \cdot 11^2$
1091	*
1092	$2^2 \cdot 3 \cdot 7 \cdot 13$
1093	*
1095	$3 \cdot 5 \cdot 73$
1097	*
1100	$2^2 \cdot 5^2 \cdot 11$

1103	*
1106	$2 \cdot 7 \cdot 79$
1109	*
1113	$3 \cdot 7 \cdot 53$
1117	*
1120	$2^5 \cdot 5 \cdot 7$
1123	*
1126	$2 \cdot 563$
1129	*
1140	$2^2 \cdot 3 \cdot 5 \cdot 19$
1151	*
1152	$2^7 \cdot 3^2$
1153	*
1158	$2 \cdot 3 \cdot 193$
1163	*
1167	$3 \cdot 389$
1171	*
1176	$2^3 \cdot 3 \cdot 7^2$
1181	*
1184	$2^5 \cdot 37$
1187	*
1190	$2 \cdot 5 \cdot 7 \cdot 17$
1193	*

Inventory of various number families generated by this procedure:

3-Limit

6, 9, 12, 18, 64, 72, 81, 108, 144, 192, 288, 324, 432, 1152

5-Limit (including 3-Limit subset)

4, 6, 9, 12, 15, 18, 30, 45, 50 60, 64, 72, 81, 108, 120, 144, 150, 160, 180, 192, 225, 240, 270, 288, 300, 324, 405, 432, 600, 625, 675, 810, 1152

7-Limit (minus the 5- and 3-Limit subsets)

21, 42, 49, 56, 105, 315, 420, 441, 560, 882, 980, 1050, 1120, 1176

11-Limit (minus the smaller Limit subsets)

99, 154, 165, 176, 198, 231, 363, 462, 495, 660, 792, 825, 924, 1056, 1078, 1089, 1100

13-Limit (minus the smaller Limit subsets)

26, 39, 52, 195, 260, 312, 351, 650, 780, 858, 1092

Numbers of the Family grouping $2^a \cdot 3^b$ (Pythagorean or **3-Limit** numbers)

1	*	216	$2^3 \cdot 3^3$	2,592	$2^5 \cdot 3^4$	18,432	$2^{11} \cdot 3^2$
2	*	243	3^5	2,916	$2^2 \cdot 3^6$	19,683	3^9
3	*	256	2^8	3,072	$2^{10} \cdot 3$	20,736	$2^8 \cdot 3^4$
4	2^2	288	$2^5 \cdot 3^2$	3,456	$2^7 \cdot 3^3$	23,328	$2^5 \cdot 3^6$
6	$2 \cdot 3$	324	$2^2 \cdot 3^4$	3,888	$2^4 \cdot 3^5$	24,576	$2^{13} \cdot 3$
8	2^3	384	$2^7 \cdot 3$	4,096	2^{12}	26,244	$2^2 \cdot 3^8$
9	3^2	432	$2^4 \cdot 3^3$	4,374	$2 \cdot 3^7$	27,648	$2^{10} \cdot 3^3$
12	$2^2 \cdot 3$	486	$2 \cdot 3^5$	4,608	$2^9 \cdot 3^2$	31,104	$2^7 \cdot 3^5$
16	2^4	512	2^9	5,184	$2^6 \cdot 3^4$	32,768	2^{15}
18	$2 \cdot 3^2$	576	$2^6 \cdot 3^2$	5,832	$2^3 \cdot 3^6$	34,992	$2^4 \cdot 3^7$
24	$2^3 \cdot 3$	648	$2^3 \cdot 3^4$	6,144	$2^{11} \cdot 3$	36,864	$2^{12} \cdot 3^2$
27	3^3	729	3^6	6561	3^8	39,366	$2 \cdot 3^9$
32	2^5	768	$2^8 \cdot 3$	6,912	$2^8 \cdot 3^3$	41,472	$2^9 \cdot 3^4$
36	$2^2 \cdot 3^2$	864	$2^5 \cdot 3^3$	7,776	$2^5 \cdot 3^5$	46,656	$2^6 \cdot 3^6$
48	$2^4 \cdot 3$	972	$2^2 \cdot 3^5$	8,192	2^{13}	49,152	$2^{14} \cdot 3$
54	$2 \cdot 3^3$	1,024	2^{10}	8748	$2^2 \cdot 3^7$	52,488	$2^3 \cdot 3^8$
64	2^6	1,152	$2^7 \cdot 3^2$	9,216	$2^{10} \cdot 3^2$	55,296	$2^{11} \cdot 3^3$
72	$2^3 \cdot 3^2$	1,296	$2^4 \cdot 3^4$	10,368	$2^7 \cdot 3^4$	59,049	3^{10}
81	3^4	1,458	$2 \cdot 3^6$	11,664	$2^4 \cdot 3^6$	62,208	$2^8 \cdot 3^5$
96	$2^5 \cdot 3$	1,536	$2^9 \cdot 3$	12,288	$2^{12} \cdot 3$	⋮	
108	$2^2 \cdot 3^3$	1,728	$2^6 \cdot 3^3$	13,122	$2 \cdot 3^8$	↓	
128	2^7	1,944	$2^3 \cdot 3^5$	13,824	$2^9 \cdot 3^3$		
144	$2^4 \cdot 3^2$	2,048	2^{11}	15,552	$2^6 \cdot 3^5$	524,288	2^{19}
162	$2 \cdot 3^4$	2,187	3^7	16,384	2^{14}	531,441	3^{12}
192	$2^6 \cdot 3$	2,304	$2^8 \cdot 3^2$	17,496	$2^3 \cdot 3^7$	↓	

Some very prominant 3-Limit monochord number series:

Double 6:12 — 6 : 8 : 9 : 12

Harmonics — C F G C

Monochord — C G F C

The "musical proportion." The Fixed (as opposed to moveable) tones of the ancient tetrachord within the octave. Matrix:

F—Ⓒ—G

Double 72:144 — 72 : 81 : 96 : 108 : 128 : 144

Harmonics — C D F G \Bb C

Monochord — C \Bb G F D C

Ancient Chinese, Egyptian, and Pythagorean pentatonic scale. Matrix: \Bb—F—Ⓒ—G—D

Double 432:864 — 432 : 486 : 512 : 576 : 648 : 729 : 768 : 864

Harmonics — C D \Eb F G /A \Bb C

Monochord — C \Bb /A G F \Eb D C

Pythagorean Heptad. The 1st medieval mode. 'Dorian' diatonic scale.

Matrix: \Eb—\Bb—F—Ⓒ—G—D—/A

Double 5184:10,368 — 5184 : 5832 : 6144 : 6561 : 6912 : 7776 : 8192 : 8748 : 9216 : 10,368

Harmonics — C D \Eb /E F G \Ab /A \Bb C

Monochord — C \Bb /A \Ab G F /E \Eb D C

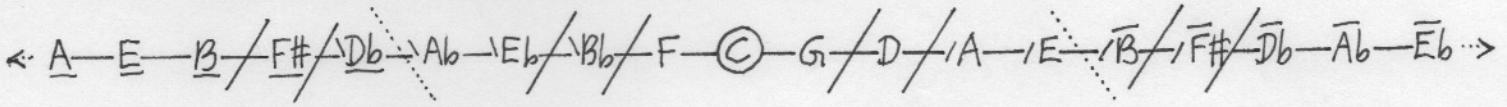
Pythagorean Ennead.

Matrix: \Ab—\Eb—\Bb—F—Ⓒ—G—D—/A—/E

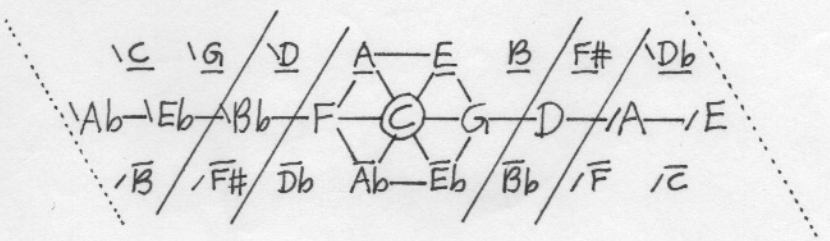
The most important of the Pythagorean Interval Ratios:

Ratio	Prime Factors	Pitch	Cents	Name-remarks
1/1	1/1	C	0.0	Unison — Ancient
2/1	2/1	C	1200.0	Octave — consonances
3/2	3/2	G	701.955	Perfect Fifth
4/3	$2^2/3$	F	498.045	Perfect Fourth (Tetrachord)
9/8	$3^2/2^3$	D	203.910	Major Whole-tone
16/9	$2^4/3^2$	\Bb	996.091	Comma-lowered minor seventh
27/16	$3^3/2^4$	/A	905.865	Comma-raised major sixth
32/27	$2^5/3^3$	\Eb	294.135	Comma-lowered minor third
81/64	$3^4/2^6$	/E	407.820	Comma-raised major third
128/81	$2^7/3^4$	\Ab	792.180	Comma-lowered minor sixth
243/128	$3^5/2^7$	/B	1109.775	Schismatic comma-raised seventh
256/243	$2^8/3^5$	\Db	90.225	Schismatic comma-lowered medial semitone
729/512	$3^6/2^9$	/F#	611.731	Schismatic comma-raised sharp Fourth
1024/729	$2^{10}/3^6$	F#	588.270	Schismatic sharp Fourth
2187/2048	$3^7/2^{11}$	Db	113.685	Schismatic diatonic semitone
4096/2187	$2^{12}/3^7$	B	1086.315	Schismatic Just major seventh
6561/4096	$3^8/2^{12}$	Ab	815.640	Schismatic Just minor sixth
8192/6561	$2^{13}/3^8$	E	384.360	Schismatic Just major third
19,683/16,384	$3^9/2^{14}$	Eb	317.595	Schismatic Just minor third
32,768/19,683	$2^{15}/3^9$	A	882.405	Schismatic Just major sixth
59,049/32,768	$3^{10}/2^{15}$	Bb	1019.550	Schismatic Just minor seventh
65,536/59,049	$2^{16}/3^{10}$	\D	180.450	Schismatic Just minor whole-tone
177,147/131,072	$3^{11}/2^{17}$	/F	521.505	Schismatic comma-raised Fourth
262,144/177,147	$2^{18}/3^{11}$	\G	678.495	Schismatic comma-lowered fifth
531,441/524,288	$3^{12}/2^{19}$	/C	23.460	Schismatic comma (Ditonic)

Pattern on the matrix:



Due to the near identity of schismatic relations, the matrix may be redrawn as:



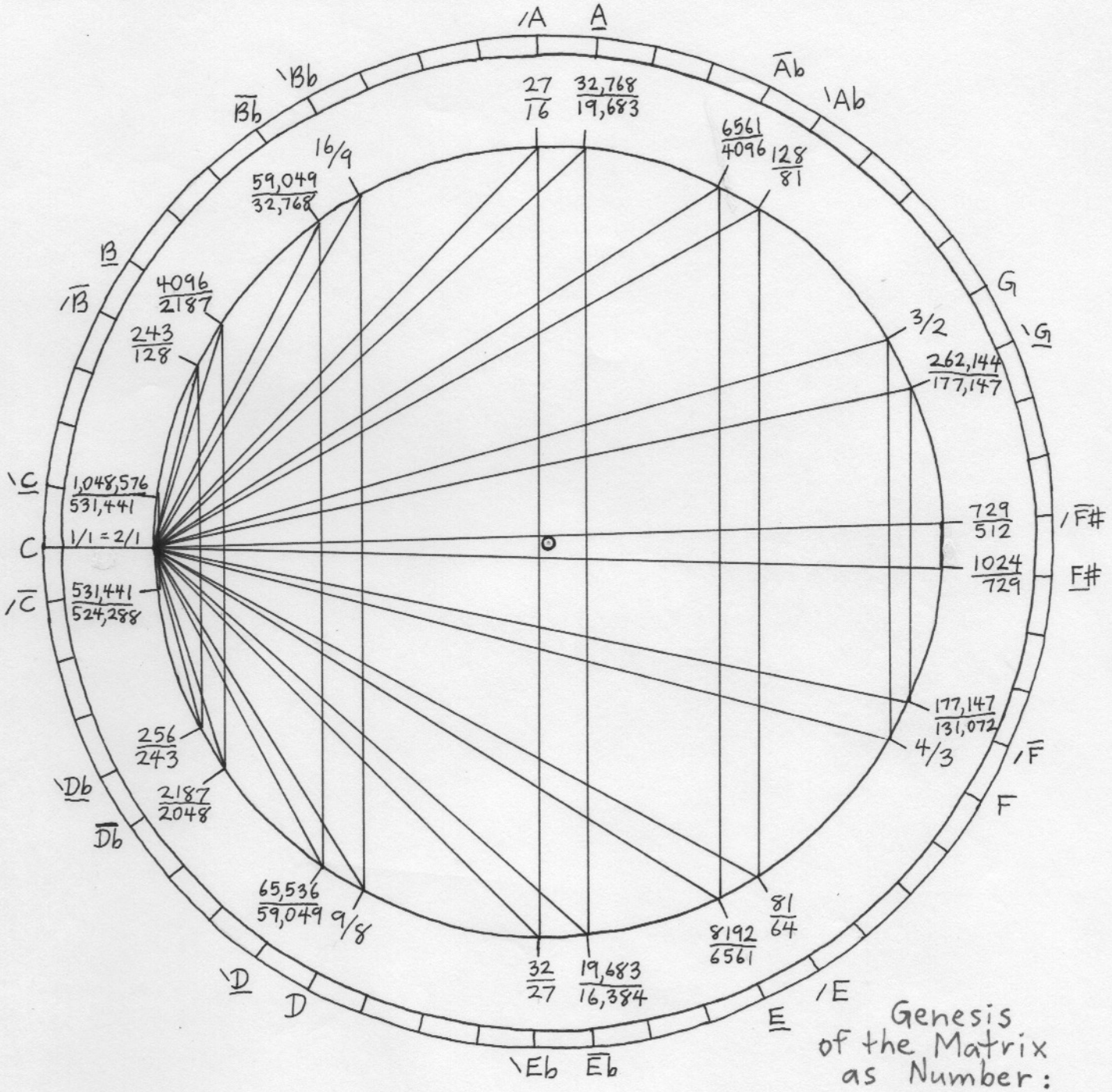
Matrix as an expansion of prime factors:

$$\frac{2^{13}}{3^8}\ \frac{2^{12}}{3^7}\ \frac{2^{10}}{3^6}\ \frac{2^8}{3^5}\ \frac{2^7}{3^4}\ \frac{2^5}{3^3}\ \frac{2^4}{3^2}\ \frac{2^2}{3}\ \boxed{\frac{1}{1}}\ \frac{3}{2}\ \frac{3^2}{2^3}\ \frac{3^3}{2^4}\ \frac{3^4}{2^6}\ \frac{3^5}{2^7}\ \frac{3^6}{2^9}\ \frac{3^7}{2^{11}}\ \frac{3^8}{2^{12}}$$

Prominant Pythagorean Interval Ratios drawn on the circular graph:

showing the close similarity between 3-Limit Just Intonation and the 53-E.T. closed cycle.

Intervals are paired with their inversions by connecting lines.



Genesis of the Matrix as Number:

2——3

Numbers of the family grouping $2^a \cdot 3^b \cdot 5^c$ (Babylonian or **5-Limit**)

1	*
2	*
3	*
4	2^2
5	*
6	$2 \cdot 3$
8	2^3
9	3^2
10	$2 \cdot 5$
12	$2^2 \cdot 3$
15	$3 \cdot 5$
16	2^4
18	$2 \cdot 3^2$
20	$2^2 \cdot 5$
24	$2^3 \cdot 3$
25	5^2
27	3^3
30	$2 \cdot 3 \cdot 5$
32	2^5
36	$2^2 \cdot 3^2$
40	$2^3 \cdot 5$
45	$3^2 \cdot 5$
48	$2^4 \cdot 3$
50	$2 \cdot 5^2$
54	$2 \cdot 3^3$
60	$2^2 \cdot 3 \cdot 5$
64	2^6
72	$2^3 \cdot 3^2$
75	$3 \cdot 5^2$
80	$2^4 \cdot 5$
81	3^4
90	$2 \cdot 3^2 \cdot 5$
96	$2^5 \cdot 3$
100	$2^2 \cdot 5^2$
108	$2^2 \cdot 3^3$
120	$2^3 \cdot 3 \cdot 5$
125	5^3
128	2^7
135	$3^3 \cdot 5$
144	$2^4 \cdot 3^2$
150	$2 \cdot 3 \cdot 5^2$
160	$2^5 \cdot 5$
162	$2 \cdot 3^4$
180	$2^2 \cdot 3^2 \cdot 5$
192	$2^6 \cdot 3$
200	$2^3 \cdot 5^2$
216	$2^3 \cdot 3^3$
225	$3^2 \cdot 5^2$
240	$2^4 \cdot 3 \cdot 5$
243	3^5
250	$2 \cdot 5^3$
256	2^8
270	$2 \cdot 3^3 \cdot 5$
288	$2^5 \cdot 3^2$
300	$2^2 \cdot 3 \cdot 5^2$
320	$2^6 \cdot 5$
324	$2^2 \cdot 3^4$
360	$2^3 \cdot 3^2 \cdot 5$
375	$3 \cdot 5^3$
384	$2^7 \cdot 3$
400	$2^4 \cdot 5^2$
405	$3^4 \cdot 5$
432	$2^4 \cdot 3^3$
450	$2 \cdot 3^2 \cdot 5^2$
480	$2^5 \cdot 3 \cdot 5$
486	$2 \cdot 3^5$
500	$2^2 \cdot 5^3$
512	2^9
540	$2^2 \cdot 3^3 \cdot 5$
576	$2^6 \cdot 3^2$
600	$2^3 \cdot 3 \cdot 5^2$
625	5^4
640	$2^7 \cdot 5$
648	$2^3 \cdot 3^4$
675	$3^3 \cdot 5^2$
720	$2^4 \cdot 3^2 \cdot 5$
729	3^6
750	$2 \cdot 3 \cdot 5^3$
768	$2^8 \cdot 3$
800	$2^5 \cdot 5^2$
810	$2 \cdot 3^4 \cdot 5$
864	$2^5 \cdot 3^3$
900	$2^2 \cdot 3^2 \cdot 5^2$
960	$2^6 \cdot 3 \cdot 5$
972	$2^2 \cdot 3^5$
1000	$2^3 \cdot 5^3$
1024	2^{10}
1080	$2^3 \cdot 3^3 \cdot 5$
1125	$3^2 \cdot 5^3$
1152	$2^7 \cdot 3^2$
1200	$2^4 \cdot 3 \cdot 5^2$
1215	$3^5 \cdot 5$
1250	$2 \cdot 5^4$
1280	$2^8 \cdot 5$
1296	$2^4 \cdot 3^4$
1350	$2 \cdot 3^3 \cdot 5^2$
1440	$2^5 \cdot 3^2 \cdot 5$
1458	$2 \cdot 3^6$
1500	$2^2 \cdot 3 \cdot 5^3$
1536	$2^9 \cdot 3$
1600	$2^6 \cdot 5^2$
1620	$2^2 \cdot 3^4 \cdot 5$
1728	$2^6 \cdot 3^3$
1800	$2^3 \cdot 3^2 \cdot 5^2$
1875	$3 \cdot 5^4$
1920	$2^7 \cdot 3 \cdot 5$
1944	$2^3 \cdot 3^5$
2000	$2^4 \cdot 5^3$
2025	$3^4 \cdot 5^2$
2048	2^{11}
2160	$2^4 \cdot 3^3 \cdot 5$
2187	3^7
2250	$2 \cdot 3^2 \cdot 5^3$
2304	$2^8 \cdot 3^2$
2400	$2^5 \cdot 3 \cdot 5^2$
2430	$2 \cdot 3^5 \cdot 5$
2500	$2^2 \cdot 5^4$
2560	$2^9 \cdot 5$
2592	$2^5 \cdot 3^4$
2700	$2^2 \cdot 3^3 \cdot 5^2$
2880	$2^6 \cdot 3^2 \cdot 5$
2916	$2^2 \cdot 3^6$
3000	$2^3 \cdot 3 \cdot 5^3$
3072	$2^{10} \cdot 3$
3125	5^5
3200	$2^7 \cdot 5^2$
3240	$2^3 \cdot 3^4 \cdot 5$
3375	$3^3 \cdot 5^3$
3456	$2^7 \cdot 3^3$
3600	$2^4 \cdot 3^2 \cdot 5^2$
3645	$3^6 \cdot 5$
3750	$2 \cdot 3 \cdot 5^4$
3840	$2^8 \cdot 3 \cdot 5$
3888	$2^4 \cdot 3^5$
4000	$2^5 \cdot 5^3$
4050	$2 \cdot 3^4 \cdot 5^2$
4096	2^{12}
4320	$2^5 \cdot 3^3 \cdot 5$
4374	$2 \cdot 3^7$
4500	$2^2 \cdot 3^2 \cdot 5^3$
4608	$2^9 \cdot 3^2$
4800	$2^6 \cdot 3 \cdot 5^2$
4860	$2^2 \cdot 3^5 \cdot 5$
5000	$2^3 \cdot 5^4$
5120	$2^{10} \cdot 5$
5184	$2^6 \cdot 3^4$
5400	$2^3 \cdot 3^3 \cdot 5^2$
5625	$3^2 \cdot 5^4$
5760	$2^7 \cdot 3^2 \cdot 5$
5832	$2^3 \cdot 3^6$
6000	$2^4 \cdot 3 \cdot 5^3$
6075	$3^5 \cdot 5^2$
6144	$2^{11} \cdot 3$
6250	$2 \cdot 5^5$
6400	$2^8 \cdot 5^2$
6480	$2^4 \cdot 3^4 \cdot 5$
6561	3^8
6750	$2 \cdot 3^3 \cdot 5^3$
6912	$2^8 \cdot 3^3$
7200	$2^5 \cdot 3^2 \cdot 5^2$
7290	$2 \cdot 3^6 \cdot 5$
7500	$2^2 \cdot 3 \cdot 5^4$
7680	$2^9 \cdot 3 \cdot 5$
7776	$2^5 \cdot 3^5$
8000	$2^6 \cdot 5^3$
8100	$2^2 \cdot 3^4 \cdot 5^2$
8192	2^{13}
8640	$2^6 \cdot 3^3 \cdot 5$
8748	$2^2 \cdot 3^7$
9000	$2^3 \cdot 3^2 \cdot 5^3$
9216	$2^{10} \cdot 3^2$
9375	$3 \cdot 5^5$
9600	$2^7 \cdot 3 \cdot 5^2$
9720	$2^3 \cdot 3^5 \cdot 5$
10,000	$2^4 \cdot 5^4$
10,125	$3^4 \cdot 5^3$
10,240	$2^{11} \cdot 5$
10,368	$2^7 \cdot 3^4$
10,800	$2^4 \cdot 3^3 \cdot 5^2$
10,935	$3^7 \cdot 5$
11,250	$2 \cdot 3^2 \cdot 5^4$
11,520	$2^8 \cdot 3^2 \cdot 5$
11,664	$2^4 \cdot 3^6$
12,000	$2^5 \cdot 3 \cdot 5^3$
12,150	$2 \cdot 3^5 \cdot 5^2$
12,288	$2^{12} \cdot 3$
12,500	$2^2 \cdot 5^5$
12,800	$2^9 \cdot 5^2$
12,960	$2^5 \cdot 3^4 \cdot 5$
13,122	$2 \cdot 3^8$
13,500	$2^2 \cdot 3^3 \cdot 5^3$
13,824	$2^9 \cdot 3^3$

14,400	$2^6 \cdot 3^2 \cdot 5^2$
14,580	$2^2 \cdot 3^6 \cdot 5$
15,000	$2^3 \cdot 3 \cdot 5^4$
15,360	$2^{10} \cdot 3 \cdot 5$
15,552	$2^6 \cdot 3^5$
15,625	5^6
16,000	$2^7 \cdot 5^3$
16,200	$2^3 \cdot 3^4 \cdot 5^2$
16,384	2^{14}
16,875	$3^3 \cdot 5^4$
17,280	$2^7 \cdot 3^3 \cdot 5$
17,496	$2^3 \cdot 3^7$
18,000	$2^4 \cdot 3^2 \cdot 5^3$
18,225	$3^6 \cdot 5^2$
18,432	$2^{11} \cdot 3^2$
18,750	$2 \cdot 3 \cdot 5^5$
19,200	$2^8 \cdot 3 \cdot 5^2$
19,440	$2^4 \cdot 3^5 \cdot 5$
19,683	3^9
20,000	$2^5 \cdot 5^4$
20,250	$3^5 \cdot 5^3$
20,480	$2^{12} \cdot 5$
20,736	$2^8 \cdot 3^4$
21,600	$2^5 \cdot 3^3 \cdot 5^2$
21,870	$2 \cdot 3^7 \cdot 5$
22,500	$2^2 \cdot 3^2 \cdot 5^4$
23,040	$2^9 \cdot 3^2 \cdot 5$
23,328	$2^5 \cdot 3^6$
24,000	$2^6 \cdot 3 \cdot 5^3$
24,300	$2^2 \cdot 3^5 \cdot 5^2$
24,576	$2^{13} \cdot 3$
25,000	$2^3 \cdot 5^5$
25,600	$2^{10} \cdot 5^2$
25,920	$2^6 \cdot 3^4 \cdot 5$
26,244	$2^2 \cdot 3^8$
27,000	$2^3 \cdot 3^3 \cdot 5^3$
27,648	$2^{10} \cdot 3^3$
28,125	$3^2 \cdot 5^5$
28,800	$2^7 \cdot 3^2 \cdot 5^2$
29,160	$2^3 \cdot 3^6 \cdot 5$
30,000	$2^4 \cdot 3 \cdot 5^4$
30,375	$3^5 \cdot 5^3$
30,720	$2^{11} \cdot 3 \cdot 5$
31,104	$2^7 \cdot 3^5$
31,250	$2 \cdot 5^6$
32,000	$2^8 \cdot 5^3$
32,400	$2^4 \cdot 3^4 \cdot 5^2$
32,768	2^{15}
32,805	$3^8 \cdot 5$
33,750	$2 \cdot 3^3 \cdot 5^4$
34,560	$2^8 \cdot 3^3 \cdot 5$
34,992	$2^4 \cdot 3^7$
36,000	$2^5 \cdot 3^2 \cdot 5^3$
36,450	$2 \cdot 3^6 \cdot 5^2$
36,864	$2^{12} \cdot 3^2$
37,500	$2^2 \cdot 3 \cdot 5^5$
38,400	$2^9 \cdot 3 \cdot 5^2$
38,880	$2^5 \cdot 3^5 \cdot 5$
39,366	$2 \cdot 3^9$
40,000	$2^6 \cdot 5^4$
40,500	$2 \cdot 3^5 \cdot 5^3$
40,960	$2^{13} \cdot 5$
41,472	$2^9 \cdot 3^4$
43,200	$2^6 \cdot 3^3 \cdot 5^2$
43,740	$2^2 \cdot 3^7 \cdot 5$
45,000	$2^3 \cdot 3^2 \cdot 5^4$
46,080	$2^{10} \cdot 3^2 \cdot 5$
46,656	$2^6 \cdot 3^6$
46,875	$3 \cdot 5^6$
48,000	$2^7 \cdot 3 \cdot 5^3$
48,600	$2^3 \cdot 3^5 \cdot 5^2$
49,152	$2^{14} \cdot 3$
50,000	$2^4 \cdot 5^5$
50,625	$3^4 \cdot 5^4$
51,200	$2^{11} \cdot 5^2$
51,840	$2^7 \cdot 3^4 \cdot 5$
52,488	$2^3 \cdot 3^8$
54,000	$2^4 \cdot 3^3 \cdot 5^3$
54,675	$3^7 \cdot 5^2$
55,296	$2^{11} \cdot 3^3$
56,250	$2 \cdot 3^2 \cdot 5^5$
57,600	$2^8 \cdot 3^2 \cdot 5^2$
58,320	$2^4 \cdot 3^6 \cdot 5$
59,049	3^{10}
60,000	$2^5 \cdot 3 \cdot 5^4$
60,750	$2 \cdot 3^5 \cdot 5^3$
61,440	$2^{12} \cdot 3 \cdot 5$
62,208	$2^8 \cdot 3^5$
62,500	$2^2 \cdot 5^6$
64,000	$2^9 \cdot 5^3$
64,800	$2^5 \cdot 3^4 \cdot 5^2$
65,536	2^{16}
65,610	$2 \cdot 3^8 \cdot 5$
67,500	$2^2 \cdot 3^3 \cdot 5^4$
69,120	$2^9 \cdot 3^3 \cdot 5$
69,984	$2^5 \cdot 3^7$
72,000	$2^6 \cdot 3^2 \cdot 5^3$
72,900	$2^2 \cdot 3^6 \cdot 5^2$
73,728	$2^{13} \cdot 3^2$
75,000	$2^3 \cdot 3 \cdot 5^5$
76,800	$2^{10} \cdot 3 \cdot 5^2$
77,760	$2^6 \cdot 3^5 \cdot 5$
78,125	5^7
78,732	$2^2 \cdot 3^9$
80,000	$2^7 \cdot 5^4$
81,000	$2^2 \cdot 3^5 \cdot 5^3$
81,920	$2^{14} \cdot 5$
82,944	$2^{10} \cdot 3^4$
84,375	$3^3 \cdot 5^5$
86,400	$2^7 \cdot 3^3 \cdot 5^2$
87,480	$2^3 \cdot 3^7 \cdot 5$
90,000	$2^4 \cdot 3^2 \cdot 5^4$
91,125	$3^6 \cdot 5^3$
92,160	$2^{11} \cdot 3^2 \cdot 5$
93,312	$2^7 \cdot 3^6$
93,750	$2 \cdot 3 \cdot 5^6$
96,000	$2^8 \cdot 3 \cdot 5^3$
97,200	$2^4 \cdot 3^5 \cdot 5^2$
98,304	$2^{15} \cdot 3$
98,415	$3^9 \cdot 5$
100,000	$2^5 \cdot 5^5$
101,250	$2 \cdot 3^4 \cdot 5^4$
102,400	$2^{12} \cdot 5^2$
103,680	$2^8 \cdot 3^4 \cdot 5$
104,976	$2^4 \cdot 3^8$
108,000	$2^5 \cdot 3^3 \cdot 5^3$
109,350	$2 \cdot 3^7 \cdot 5^2$
110,592	$2^{12} \cdot 3^3$
112,500	$2^2 \cdot 3^2 \cdot 5^5$
115,200	$2^9 \cdot 3^2 \cdot 5^2$
116,640	$2^5 \cdot 3^6 \cdot 5$
118,098	$2 \cdot 3^{10}$
120,000	$2^6 \cdot 3 \cdot 5^4$
121,500	$2^2 \cdot 3^5 \cdot 5^3$
122,880	$2^{13} \cdot 3 \cdot 5$
124,416	$2^9 \cdot 3^5$
125,000	$2^3 \cdot 5^6$
128,000	$2^{10} \cdot 5^3$
129,600	$2^6 \cdot 3^4 \cdot 5^2$
131,072	2^{17}
131,220	$2^2 \cdot 3^8 \cdot 5$
135,000	$2^3 \cdot 3^3 \cdot 5^4$
138,240	$2^{10} \cdot 3^3 \cdot 5$
139,968	$2^6 \cdot 3^7$
140,625	$3^2 \cdot 5^6$
144,000	$2^7 \cdot 3^2 \cdot 5^3$
145,800	$2^3 \cdot 3^6 \cdot 5^2$
147,456	$2^{14} \cdot 3^2$
150,000	$2^4 \cdot 3 \cdot 5^5$
151,875	$3^5 \cdot 5^4$

153,600	$2^{11} \cdot 3 \cdot 5^2$
155,520	$2^7 \cdot 3^5 \cdot 5$
156,250	$2 \cdot 5^7$
157,464	$2^3 \cdot 3^9$
160,000	$2^8 \cdot 5^4$
162,000	$2^3 \cdot 3^5 \cdot 5^3$
163,840	$2^{15} \cdot 5$
164,025	$3^8 \cdot 5^2$
165,888	$2^{11} \cdot 3^4$
168,750	$2 \cdot 3^3 \cdot 5^5$
172,800	$2^8 \cdot 3^3 \cdot 5^2$
174,960	$2^4 \cdot 3^7 \cdot 5$
177,147	3^{11}
180,000	$2^5 \cdot 3^2 \cdot 5^4$
182,250	$2 \cdot 3^6 \cdot 5^3$
184,320	$2^{12} \cdot 3^2 \cdot 5$
186,624	$2^8 \cdot 3^6$
187,500	$2^2 \cdot 3 \cdot 5^6$
192,000	$2^9 \cdot 3 \cdot 5^3$
194,400	$2^5 \cdot 3^5 \cdot 5^2$
196,608	$2^{16} \cdot 3$
196,830	$2 \cdot 3^9 \cdot 5$
200,000	$2^6 \cdot 5^5$
202,500	$2^2 \cdot 3^4 \cdot 5^4$
204,800	$2^{13} \cdot 5^2$
207,360	$2^9 \cdot 3^4 \cdot 5$
209,952	$2^5 \cdot 3^8$
216,000	$2^6 \cdot 3^3 \cdot 5^3$
218,700	$2 \cdot 3^7 \cdot 5^2$
221,184	$2^{13} \cdot 3^3$
225,000	$2^3 \cdot 3^2 \cdot 5^5$
230,400	$2^{10} \cdot 3^2 \cdot 5^2$
233,280	$2^6 \cdot 3^6 \cdot 5$
234,375	$3 \cdot 5^7$
236,196	$2^2 \cdot 3^{10}$
240,000	$2^7 \cdot 3 \cdot 5^4$
243,000	$2^3 \cdot 3^5 \cdot 5^3$
245,760	$2^{14} \cdot 3 \cdot 5$
248,832	$2^{10} \cdot 3^5$
250,000	$2^4 \cdot 5^6$
253,125	$3^4 \cdot 5^5$
256,000	$2^{11} \cdot 5^3$
259,200	$2^7 \cdot 3^4 \cdot 5^2$
262,144	2^{18}
262,440	$2^3 \cdot 3^8 \cdot 5$
270,000	$2^4 \cdot 3^3 \cdot 5^4$
273,375	$3^7 \cdot 5^3$
276,480	$2^{11} \cdot 3^3 \cdot 5$
279,936	$2^7 \cdot 3^7$
281,250	$2 \cdot 3^2 \cdot 5^6$
288,000	$2^8 \cdot 3^2 \cdot 5^3$
291,600	$2^4 \cdot 3^6 \cdot 5^2$
294,912	$2^{15} \cdot 3^2$
295,245	$3^{10} \cdot 5$
300,000	$2^5 \cdot 3 \cdot 5^5$
303,750	$2 \cdot 3^5 \cdot 5^4$
307,200	$2^{12} \cdot 3 \cdot 5^2$
311,040	$2^8 \cdot 3^5 \cdot 5$
312,500	$2^2 \cdot 5^7$
314,928	$2^4 \cdot 3^9$
320,000	$2^9 \cdot 5^4$
324,000	$2^4 \cdot 3^5 \cdot 5^3$
327,680	$2^{16} \cdot 5$
328,050	$2 \cdot 3^8 \cdot 5^2$
331,776	$2^{12} \cdot 3^4$
337,500	$2^2 \cdot 3^3 \cdot 5^5$
345,600	$2^9 \cdot 3^3 \cdot 5^2$
349,920	$2^5 \cdot 3^7 \cdot 5$
354,294	$2 \cdot 3^{11}$
360,000	$2^6 \cdot 3^2 \cdot 5^4$
364,500	$2^2 \cdot 3^6 \cdot 5^3$
368,640	$2^{13} \cdot 3^2 \cdot 5$
373,248	$2^9 \cdot 3^6$
375,000	$2^3 \cdot 3 \cdot 5^6$
384,000	$2^{10} \cdot 3 \cdot 5^3$
388,800	$2^6 \cdot 3^5 \cdot 5^2$
390,625	5^8
393,216	$2^{17} \cdot 3$
393,660	$2^2 \cdot 3^9 \cdot 5$
400,000	$2^7 \cdot 5^5$
405,000	$2^3 \cdot 3^4 \cdot 5^4$
409,600	$2^{14} \cdot 5^2$
414,720	$2^{10} \cdot 3^4 \cdot 5$
419,904	$2^6 \cdot 3^8$
421,875	$3^3 \cdot 5^6$
432,000	$2^7 \cdot 3^3 \cdot 5^3$
437,400	$2^2 \cdot 3^7 \cdot 5^2$
442,368	$2^{14} \cdot 3^3$
450,000	$2^4 \cdot 3^2 \cdot 5^5$
455,625	$3^6 \cdot 5^4$
460,800	$2^{11} \cdot 3^2 \cdot 5^2$
466,560	$2^7 \cdot 3^6 \cdot 5$
468,750	$2 \cdot 3 \cdot 5^7$
472,392	$2^3 \cdot 3^{10}$
480,000	$2^8 \cdot 3 \cdot 5^4$
486,000	$2^4 \cdot 3^5 \cdot 5^3$
491,520	$2^{15} \cdot 3 \cdot 5$
492,075	$3^9 \cdot 5^2$
497,664	$2^{11} \cdot 3^5$
500,000	$2^5 \cdot 5^6$
506,250	$2 \cdot 3^4 \cdot 5^5$
512,000	$2^{12} \cdot 5^3$
518,400	$2^8 \cdot 3^4 \cdot 5^2$
524,288	2^{19}
524,880	$2^4 \cdot 3^8 \cdot 5$
531,441	3^{12}
540,000	$2^5 \cdot 3^3 \cdot 5^4$
546,750	$2 \cdot 3^7 \cdot 5^3$
552,960	$2^{12} \cdot 3^3 \cdot 5$
559,872	$2^8 \cdot 3^7$
562,500	$2^2 \cdot 3^2 \cdot 5^6$
576,000	$2^9 \cdot 3^2 \cdot 5^3$
583,200	$2^5 \cdot 3^6 \cdot 5^2$
589,824	$2^{16} \cdot 3^2$
590,490	$3^{11} \cdot 5$
600,000	$2^6 \cdot 3 \cdot 5^5$
607,500	$2^2 \cdot 3^5 \cdot 5^4$
614,400	$2^{13} \cdot 3 \cdot 5^2$
622,080	$2^9 \cdot 3^5 \cdot 5$
625,000	$2^3 \cdot 5^7$
629,856	$2^5 \cdot 3^9$
640,000	$2^{10} \cdot 5^4$
648,000	$2^5 \cdot 3^5 \cdot 5^3$
655,360	$2^{17} \cdot 5$
656,100	$2^2 \cdot 3^8 \cdot 5^2$
663,552	$2^{13} \cdot 3^4$
675,000	$2^3 \cdot 3^3 \cdot 5^5$
691,200	$2^{10} \cdot 3^3 \cdot 5^2$
699,840	$2^6 \cdot 3^7 \cdot 5$
703,125	$3^2 \cdot 5^7$
708,588	$2^2 \cdot 3^{11}$
720,000	$2^7 \cdot 3^2 \cdot 5^4$
729,000	$2^3 \cdot 3^6 \cdot 5^3$
737,280	$2^{14} \cdot 3^2 \cdot 5$
746,496	$2^{10} \cdot 3^6$
750,000	$2^4 \cdot 3 \cdot 5^6$
768,000	$2^{11} \cdot 3 \cdot 5^3$
777,600	$2^7 \cdot 3^5 \cdot 5^2$
781,250	$2 \cdot 5^8$
786,432	$2^{18} \cdot 3$
787,320	$2^3 \cdot 3^9 \cdot 5$
800,000	$2^8 \cdot 5^5$
810,000	$2^4 \cdot 3^4 \cdot 5^4$
819,200	$2^{15} \cdot 5^2$
820,125	$3^8 \cdot 5^3$
829,440	$2^{11} \cdot 3^4 \cdot 5$
839,808	$2^7 \cdot 3^8$
843,750	$2 \cdot 3^3 \cdot 5^6$
864,000	$2^8 \cdot 3^3 \cdot 5^3$

Important **5-Limit ratios** between 1/1 and 2/1: (includes 3-Limit)

Ratio	Prime Factors	Pitch	Cents	Name-remarks
1/1	1/1	C	0.0	Unison } Ancient
2/1	2/1	C	1200.0	Octave } Perfect
3/2	3/2	G	701.955	Perfect Fifth } Consonances
4/3	$2^2/3$	F	498.045	Perfect Fourth }
5/3	5/3	A	884.359	Just Major Sixth } Medial and
5/4	$5/2^2$	E	386.314	Just Major Third } Imperfect
6/5	$2\cdot3/5$	Eb	315.641	Just Minor Third } Consonances
8/5	$2^3/5$	Ab	813.687	Just Minor Sixth }
9/5	$3^2/5$	Bb	1017.597	Just Minor Seventh
9/8	$3^2/2^3$	D	203.910	Just Major Wholetone
10/9	$2\cdot5/3^2$	\D	182.404	Just Minor Wholetone
15/8	$3\cdot5/2^3$	B	1088.269	Just Major Seventh
16/9	$2^4/3^2$	\Bb	996.090	Comma-lowered (Pythagorean) minor seventh
16/15	$2^4/3\cdot5$	Db	111.731	Just Diatonic Semitone
25/16	$5^2/2^4$	G#	772.627	Just Augmented Fifth
25/18	$5^2/2\cdot3^2$	\F#	568.718	Comma-lowered Augmented Fourth
25/24	$5^2/2^3\cdot3$	C#	70.673	Just Chromatic Semitone
27/16	$3^3/2^4$	/A	905.865	Comma-raised (Pythagorean) Major Sixth
27/20	$3^3/2^2\cdot5$	/F	519.552	Comma-raised Fourth
27/25	$3^3/5^2$	/Db	133.237	Comma-raised Diatonic Semitone
32/25	$2^5/5^2$	Fb	427.342	Just Diminished Fourth
32/27	$2^5/3^3$	\Eb	294.135	Comma-lowered (Pythagorean) Minor Third
36/25	$2^2\cdot3^2/5^2$	Gb	631.283	Just Diminished Fifth
40/27	$2^3\cdot5/3^3$	\G	680.449	Comma-lowered Fifth
45/32	$3^2\cdot5/2^5$	F#	590.224	Just Augmented Fourth
48/25	$2^4\cdot3/5^2$	Cb	1129.327	Just Diminished Octave
50/27	$2\cdot5^2/3^3$	\B	1066.762	Comma-lowered Major Seventh
64/45	$2^6/3^2\cdot5$	\Gb	609.777	Comma-lowered Diminished Fifth
75/64	$3\cdot5^2/2^6$	D#	274.583	Just Augmented Second
81/50	$3^4/2\cdot5^2$	/Ab	835.192	Comma-raised Minor Sixth
81/64	$3^4/2^6$	/E	407.820	Comma-raised (Pythagorean) Major Third
81/80	$3^4/2^4\cdot5$	/C	21.506	Syntonic Comma
100/81	$2^2\cdot5^2/3^4$	\E	364.806	Comma-lowered Major Third
125/64	$5^3/2^6$	B#	1158.941	Diesis-lowered Octave (Augmented Seventh)
125/72	$5^3/2^3\cdot3^2$	\A#	955.031	Comma-lowered Augmented Sixth
125/81	$5^3/3^4$	\G#	751.118	Clisma-raised, Comma-lowered Augmented Fifth
125/96	$5^3/2^5\cdot3$	E#	456.986	Augmented Major Third
125/108	$5^3/2^2\cdot3^3$	\D#	253.076	Clisma-raised, Comma-lowered Augmented Second
128/75	$2^7/3\cdot5^2$	Bbb	925.416	Double-diminished Seventh
128/81	$2^7/3^4$	\Ab	792.180	Comma-lowered (Pythagorean) Minor Sixth
128/125	$2^7/5^3$	Dbb	41.059	Diesis (Enharmonic)
135/128	$3^3\cdot5/2^7$	/C#	92.179	Comma-raised Chromatic (Medial) Semitone
144/125	$2^4\cdot3^2/5^3$	Ebb	244.968	Double-diminished Third
160/81	$2^5\cdot5/3^4$	\C	1178.493	Comma-lowered Octave
162/125	$2\cdot3^4/5^3$	/Fb	448.881	Clisma-lowered, Comma-raised Diminished Fourth
192/125	$2^6\cdot3/5^3$	Abb	743.014	Double-diminished Sixth
216/125	$2^3\cdot3^3/5^3$	/Bbb	946.924	Clisma-lowered, Comma-raised, Double-diminished seventh
225/128	$3^2\cdot5^2/2^7$	A#	976.537	Just Augmented Sixth

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243/125	$3^5/5^3$	/C.b	1150.836	Clisma-lowered, Comma-raised Diminished Octave
243/128	$3^5/2^7$	/B	1109.775	Schisma-raised, Comma-raised (Pythagorean) Seventh
243/160	$3^5/2^5.5$	/G	723.461	Comma-raised Fifth
243/200	$3^5/2^3.5^2$	/Eb	337.148	Comma-raised Minor Third
250/243	$2\cdot5^3/3^5$	\C#	49.162	Clisma-raised, Comma-lowered Chromatic Semitone
256/135	$2^8/3^3.5$	\Cb	1107.821	Comma-lowered, Diminished Octave
256/225	$2^8/3^2.5^2$	\Ebb	223.461	Comma-lowered, Double-diminished Third
256/243	$2^8/3^5$	\Db	90.225	Schisma-lowered, Comma-lowered (Pythagorean) Semitone
320/243	$2^6.5/3^5$	\F	476.539	Comma-lowered Fourth
375/256	$3\cdot5^3/2^8$	Fx	660.895	Double-sharpened Fourth
400/243	$2^4.5^2/3^5$	\A	862.852	Comma-lowered Major Sixth
405/256	$3^4.5/2^8$	/G#	794.134	Schisma-raised, Comma-raised Augmented Fifth
512/375	$2^9/3\cdot5^3$	Gbb	539.103	Double-diminished Fifth
512/405	$2^9/3^4.5$	/E·\Fb	405.866	Comma-raised, Schisma-lowered Major Third
625/324	$5^4/2^2.3^4$	\B#	1137.433	Clisma-raised, Comma-lowered, Augmented Seventh
625/384	$5^4/2^7.3$	Gx	843.297	Double-sharpened Fifth
625/432	$5^4/2^4.3^3$	Gb	639.388	Clisma-raised Diminished Fifth
625/486	$5^4/2\cdot3^5$	\E#	435.478	Clisma-raised, Comma-lowered Augmented Third
625/512	$5^4/2^9$	Dx	345.253	Double-sharp Second
625/576	$5^4/2^6.3^2$	\Cx	141.342	Comma-lowered, Double-sharp Unison
648/625	$2^3.3^4/5^4$	/Dbb	62.568	Clisma-lowered, Comma-raised, Double diminished second
675/512	$3^3.5^2/2^9$	/E#	478.493	Schisma-raised, Comma-raised, Augmented Third
729/400	$3^6/2^4.5^2$	/Bb	1039.102	Comma-raised Minor Seventh
729/500	$3^6/2^2.5^3$	/Gb	652.790	Clisma-raised, Comma-raised Diminished Fifth
729/512	$3^6/2^9$	/F#	611.731	Schisma-raised, Comma-raised Augmented Fourth
729/625	$3^6/5^4$	/Ebb	266.477	Clisma-lowered, Comma-raised Double-diminished Third
729/640	$3^6/2^7.5$	\Ebb	225.415	Schisma-raised, Comma-lowered Double-diminished Third
768/625	$2^8.3/5^4$	Fbb	356.701	Double-diminished Fourth
800/729	$2^5.5^2/3^6$	\\D	160.896	Double-comma-lowered Second
864/625	$2^5.3^3/5^4$	/Gbb	560.613	Clisma-lowered, Comma-raised, Double-diminished Fifth
972/625	$2^2.3^5/5^4$	G#	764.522	Clisma-lowered, Augmented Fifth
1000/729	$2^3.5^3/3^6$	\\F#	547.208	Clisma-raised, Double-comma-lowered Augmented Fourth
1024/625	$2^{10}/5^4$	Bbbb	854.747	Clisma-lowered, Triple-diminished Seventh
1024/675	$2^{10}/3^3.5^2$	\Abb	721.507	Schisma-lowered, Comma-lowered, Double-diminished Sixth
1024/729	$2^{10}/3^6$	F#	588.270	Schisma-lowered Augmented Fourth
1125/1024	$3^2.5^3/2^{10}$	Cx	162.850	Schisma-raised, Double-sharp Unison
1152/625	$2^7.3^2/5^4$	C.bb	1058.657	Clisma-lowered, Double-diminished Octave
1215/1024	$3^5.5/2^{10}$	\Eb	296.089	Schisma-raised, Comma-lowered Minor Third
1250/729	$2\cdot5^4/3^6$	Bbb	933.525	Clisma-raised, Double-diminished Seventh
1280/729	$2^8.5/3^6$	A#	974.583	Schisma-lowered Augmented Sixth
2025/1024	$3^4.5^2/2^{10}$	\C	1180.447	Schisma-raised, Comma-lowered Octave
2048/1125	$2^{11}/3^2.5^3$	\Cbb	1037.148	Schisma-raised, Comma-lowered, Double-diminished Octave
2048/1215	$2^{11}/3^5.5$	/A	903.911	Schisma-lowered, Comma-raised, Major Sixth
2048/1875	$2^{11}/3\cdot5^4$	Ebbb	152.791	Clisma-lowered, Triple-diminished Third
2048/2025	$2^{11}/3^4.5^2$	/C	19.553	Schisma-lowered Comma (Diaschisma)
2187/1250	$3^7/2\cdot5^4$	A#	968.430	Clisma-lowered Augmented Sixth
2187/1280	$3^7/2^8.5$	Bbb	927.371	Schisma-raised, Double-diminished Seventh
2187/1600	$3^7/2^6.5^2$	Gbb	541.057	Schisma-raised, Double-diminished Fifth
2187/2000	$3^7/2^4.5^3$	C.x	154.745	Schisma-raised, clisma-lowered, Double-sharp Unison

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2187/2048	$3^7/2^{11}$	Db	113.685	Schisma-raised Diatonic Semitone
2500/2187	$2^2 \cdot 5^4/3^7$	/D	231.570	Schisma-lowered, Clisma-raised, Comma-raised wholestep
2560/2187	$2^9 \cdot 5/3^7$	D#	272.629	Schisma-lowered Augmented Second
3125/1728	$5^5/2^6 \cdot 3^3$	Bb	1025.704	Clisma-raised Minor Seventh
3125/1944	$5^5/2^3 \cdot 3^5$	Ab	821.794	Clisma-raised Minor Sixth
3125/2048	$5^5/2^{11}$	/G	731.569	Clisma-raised, Comma-raised Fifth
3125/2187	$5^5/3^7$	\G	617.884	Clisma-raised, Comma-lowered Fifth
3125/2304	$5^5/2^8 \cdot 3^2$	/F	527.659	Clisma-raised, Comma-raised Fourth
3125/2592	$5^5/2^5 \cdot 3^4$	Eb	323.749	Clisma-raised Minor Third
3125/2916	$5^5/2^2 \cdot 3^6$	Db	119.839	Clisma-raised Diatonic Semitone
3125/3072	$5^5/2^{10} \cdot 3$	/C	29.136	Clisma-raised, Comma-raised Unison
3200/2187	$2^7 \cdot 5^2/3^7$	Fx	658.941	Schisma-lowered, Double-augmented Fourth
3375/2048	$3^3 \cdot 5^3/2^{11}$	\A	864.806	Schisma-raised, Comma-lowered Sixth
3456/3125	$2^7 \cdot 3^3/5^5$	\D	174.296	Clisma-lowered, Comma-lowered Second
3645/2048	$3^6 \cdot 5/2^{11}$	\Bb	998.044	Schisma-raised, Comma-lowered Minor Seventh
3888/3125	$2^4 \cdot 3^5/5^5$	E	378.206	Clisma-lowered Major Third
4000/2187	$2^5 \cdot 5^3/3^7$	\B	1045.253	Schisma-lowered, Clisma-raised, Two Comma-lowered seventh
4096/2187	$2^{12}/3^7$	B	1086.315	Schisma-lowered Major Seventh (Pythagorean)
4096/3125	$2^{12}/5^5$	\F	468.431	Clisma-lowered, Comma-lowered Fourth
4096/3375	$2^{12}/3^3 \cdot 5^3$	/Eb	335.194	Schisma-lowered, Comma-raised Minor Third
4096/3645	$2^{12}/3^6 \cdot 5$	D	201.956	Schisma-lowered Wholestep
4374/3125	$2 \cdot 3^7/5^5$	F#	582.116	Clisma-lowered Augmented Fourth
4608/3125	$2^9 \cdot 3^2/5^5$	\G	672.341	Clisma-lowered, Comma-lowered Fifth
5184/3125	$2^6 \cdot 3^4/5^5$	A	876.251	Clisma-lowered Major Sixth
5625/4096	$3^2 \cdot 5^4/2^{12}$	Gbb	549.165	Schisma+Clisma-raised, Double-diminished Fifth
5832/3125	$2^3 \cdot 3^6/5^5$	B	1080.161	Clisma-lowered Major Seventh
6075/4096	$3^5 \cdot 5^2/2^{12}$	\G	682.402	Schisma-raised, Comma-lowered Fifth
6144/3125	$2^{11} \cdot 3/5^5$	\C	1170.386	Clisma-lowered, Comma-lowered Octave
6561/4000	$3^8/2^5 \cdot 5^3$	\A	856.699	Schisma-raised, Clisma-lowered, Comma-lowered sixth
6561/4096	$3^8/2^{12}$	Ab	815.640	Schisma-raised Minor Sixth (Pythagorean)
6561/5000	$3^8/2^3 \cdot 5^4$	\F	470.385	Schisma-raised, Clisma-lowered, Comma-lowered Fourth
6561/5120	$3^8/2^{10} \cdot 5$	Fb	429.326	Schisma-raised Diminished Fourth
6561/6250	$3^8/2 \cdot 5^5$	/C#	84.071	Clisma-lowered, Comma-raised Chromatic semitone
6561/6400	$3^8/2^8 \cdot 5^2$	Dbb	43.013	Schisma-raised Double-diminished Wholestep
8000/6561	$2^6 \cdot 5^3/3^8$	Dx	343.301	Schisma-lowered, Clisma-raised, Double-sharp wholestep
8192/5625	$2^{13}/3^2 \cdot 5^4$	Fx	650.835	Schisma+Clisma-lowered, Double-sharp Fourth
8192/6075	$2^{13}/3^5 \cdot 5^2$	/F	517.598	Schisma-lowered, Comma-raised Fourth
8192/6561	$2^{13}/3^8$	E	384.360	Schisma-lowered Major Third
9375/8192	$3 \cdot 5^5/2^{13}$	/D	233.524	Schisma+Clisma-raised, Comma-raised Second
10,000/6561	$2^4 \cdot 5^4/3^8$	/G	729.615	Schisma-lowered, Comma+Clisma-raised Fifth
10,125/8192	$3^4 \cdot 5^3/2^{13}$	\E	366.761	Schisma-raised, Comma-lowered Major Third
10,240/6561	$2^{11} \cdot 5/3^8$	G#	770.674	Schisma-lowered Augmented Fifth
10,935/8192	$3^7 \cdot 5/2^{13}$	F	499.999	Schisma-raised Fourth
12,500/6561	$2^2 \cdot 5^5/3^8$	\Cb	1115.929	Clisma-raised, Comma-lowered Diminished Octave
12,800/6561	$2^9 \cdot 5^2/3^8$	B#	1156.987	Schisma-lowered Augmented Seventh
15,625/8192	$5^6/2^{13}$	/B	1117.882	Schisma-Clisma-Comma-raised Major Seventh
15,625/8748	$5^6/2^2 \cdot 3^7$	\Bb	1004.197	Clisma-raised, Comma-lowered Minor Seventh
15,625/9216	$5^6/2^{10} \cdot 3^2$	/A	913.972	Clisma-raised, Comma-raised Major Sixth
15,625/10,368	$5^6/2^7 \cdot 3^4$	G	710.062	Clisma-raised Fifth

continued over page

15,625/11,664	$5^6/2^4.3^6$	F	506.152	Clisma-raised Fourth
15,625/12,288	$5^6/2^{12}.3$	/E	415.927	Clisma-raised, Comma-raised Third
15,625/13,122	$5^6/2.3^8$	\Eb	302.242	Clisma-raised, Comma-lowered Minor Third
15,625/13,824	$5^6/2^9.3^3$	D	212.017	Clisma-raised Wholestep
15,625/15,552	$5^6/2^6.3^5$	C	8.107	Clisma-raised Unison (The Clisma)
16,384/9375	$2^{14}/3.5^5$	A#	966.476	Schisma + Clisma-lowered Augmented Sixth
16,384/10,125	$2^{14}/3^4.5^3$	/Ab	833.239	Schisma-lowered, Comma-raised Minor Sixth
16,384/10,935	$2^{14}/3^7.5$	G	700.001	Schisma-lowered Fifth
16,384/15,625	$2^{14}/5^6$	\Db	82.118	Schisma + Clisma-lowered, Comma-lowered semitone
16,875/16,384	$3^3.5^4/2^{14}$	Dbb	51.120	Schisma + Clisma-raised, Double-diminished Second
17,496/15,625	$2^3.3^7/5^6$	D	195.803	Clisma-lowered Wholestep
18,225/16,384	$3^6.5^2/2^{14}$	\D	184.357	Schisma-raised, Comma-lowered Wholestep
18,432/15,625	$2^{11}.3^2/5^6$	\Eb	286.028	Clisma-lowered, Comma-lowered Minor Third
19,683/10,000	$3^9/2^4.5^4$	\C	1172.340	Schisma-raised, Clisma + Comma-lowered Octave
19,683/10,240	$3^9/2^{11}.5$	Cb	1131.281	Schisma-raised Diminished Octave
19,683/12,500	$3^9/2^2.5^5$	\Ab	786.026	Schisma-raised, Clisma + Comma-lowered Minor sixth
19,683/12,800	$3^9/2^9.5^2$	Abb	744.968	Schisma-raised, Double-diminished Sixth
19,683/15,625	$3^9/5^6$	/E	399.713	Clisma-lowered, Comma-raised Major Third
19,683/16,000	$3^9/2^7.5^3$	\E	358.654	Schisma-raised, Clisma + Comma-lowered Third
19,683/16,384	$3^9/2^{14}$	Eb	317.595	Schisma-raised Minor Third
20,000/19,683	$2^5.5^4/3^9$	/C	27.660	Schisma-lowered, Clisma + Comma-raised Unison
20,480/19,683	$2^{12}.5/3^9$	C#	68.719	Schisma-lowered Chromatic Semitone
20,736/15,625	$2^8.3^4/5^6$	F	489.938	Clisma-lowered Fourth
23,328/15,625	$2^5.3^6/5^6$	G	693.848	Clisma-lowered Fifth
24,576/15,625	$2^{13}.3/5^6$	\Ab	784.073	Clisma-lowered, Comma-lowered Minor Sixth
25,000/19,683	$2^3.5^5/3^9$	/E	413.974	Schisma-raised, Clisma + Comma-raised Third
25,600/19,683	$2^{10}.5^2/3^9$	E#	455.032	Schisma-lowered Augmented Third
26,244/15,625	$2^2.3^8/5^6$	/A	897.758	Clisma-lowered, Comma-raised Major Sixth
27,648/15,625	$2^{10}.3^3/5^6$	\Bb	987.983	Clisma + Comma-lowered Minor Seventh
28,125/16,384	$3^2.5^5/2^{14}$	Bbb	935.479	Schisma + Clisma-raised Double-flat Seventh
30,375/16,384	$3^5.5^3/2^{14}$	\B	1068.716	Schisma-raised, Comma-lowered Major Seventh
31,104/15,625	$2^7.3^5/5^6$	C	1191.893	Clisma-lowered Octave
31,250/19,683	$2.5^6/3^9$	\Ab	800.287	Clisma-raised, Comma-lowered Minor Sixth
32,000/19,683	$2^8.5^3/3^9$	/Ab	841.346	Schisma-lowered, Clisma + Comma-raised Minor Sixth
32,768/16,875	$2^{15}/3^3.5^4$	B#	1148.880	Schisma + Clisma-lowered Augmented Seventh
32,768/18,225	$2^{15}/3^6.5^2$	Bb	1015.643	Schisma-lowered Minor Seventh
32,768/19,683	$2^{15}/3^9$	A	882.405	Schisma-lowered Major Sixth
32,768/28,125	$2^{15}/3^2.5^5$	D#	264.521	Schisma + Clisma-lowered Augmented Second
32,768/30,375	$2^{15}/3^5.5^3$	/Db	131.284	Schisma-lowered, Comma-raised semitone
32,805/32,768	$3^8.5/2^{15}$	C	1.954	The Schisma

*- Errata : One ratio is missing :

1,875/1024	$3.5^4/2^{10}$	Ax	1047.210	Clisma-raised double-sharp sixth

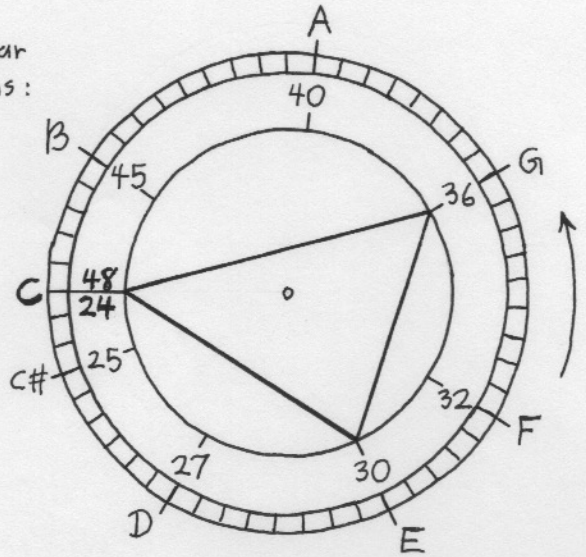
A prominant 5-Limit monochord series: 24-48

Number Series— 24 : 25 : 27 : 30 : 32 : 36 : 40 : 45 : 48

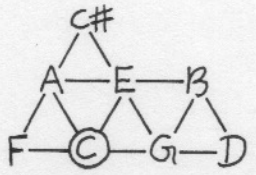
Rising pitch-harmonic (frequency) series— **C** C# D E F G A B **C**

Falling pitch-sub-harmonic (monochord) series— **C** Cb \Bb Ab G F Eb Db **C**

Circular graphs:

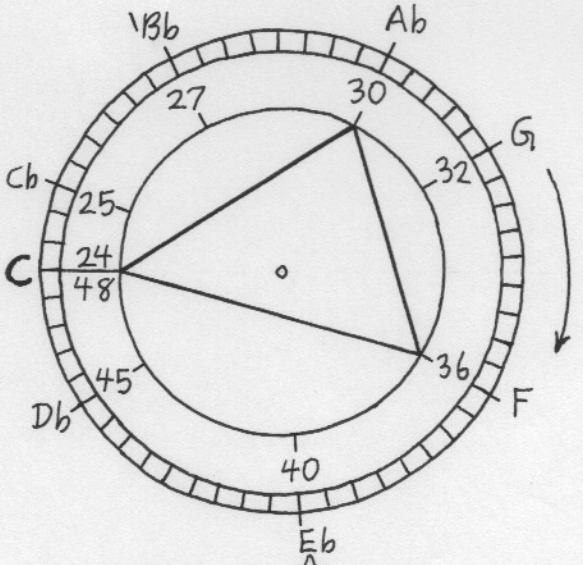


The rising (frequency) series:

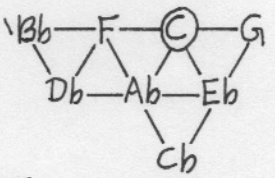


Tuning matrix

If the C# is omitted, the pattern defines a Just Intonation Major Scale. The Just Major Triad is filled in.

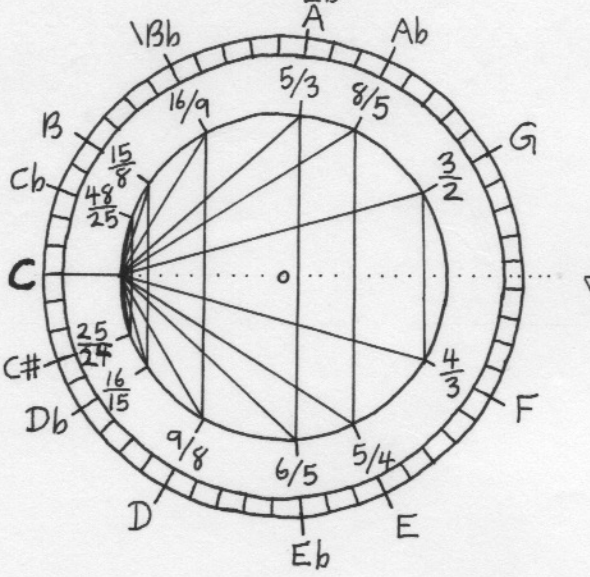


The falling (monochord) series:

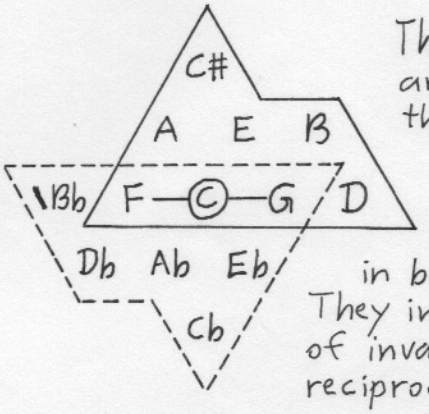


Tuning Matrix

If the Cb is omitted, the pattern defines a Just Intonation Phrygian Minor Scale. The Just Minor Triad is filled in.



The combined field:



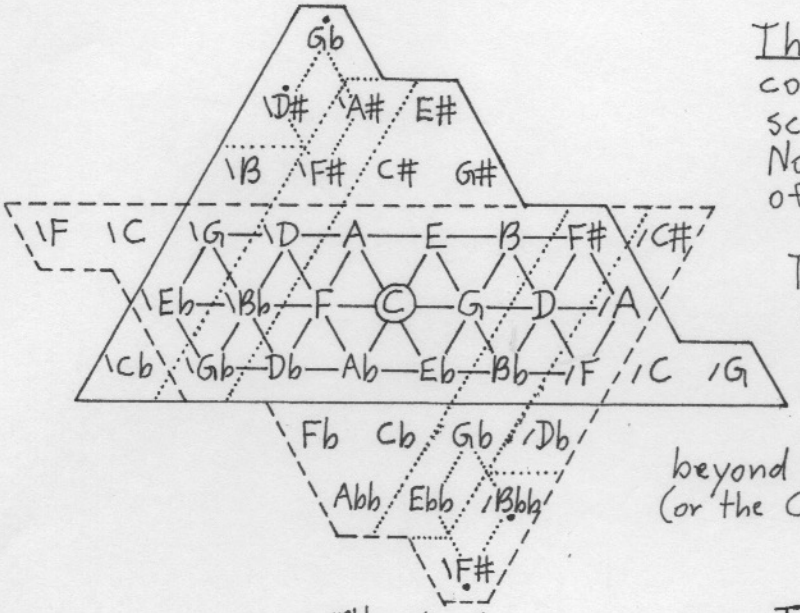
The inversion pairs are connected on the circular graph.

The pitches F and G are present in both reciprocals. They inhabit a zone of invariance under reciprocation.

A significant complex 5-Limit monochord : Double 4320-8640

4320:4374:4500:4608:4800:4860:5000:5120:5180:5400:5625:5760:5832:6000:6075 .....
0 - C /C C# Db \D D \D# \Eb Eb E E# F /F \F# F# .....
U - C \C Db B Bb \Bb /Bbb /A A Ab Abb G \G Gb \Gb .....

..... 6144:6250:6400:6480:6561:6750:6912:7200:7290:7500:7680:7776:8000:8100:8192:8640
..... \Gb Gb \G G /G G# Ab A /A \A# \Bb Bb \B B \Cb C
..... F# \F# /F F \F Fb E Eb \Eb Ebb D \D /Db Db /C# C

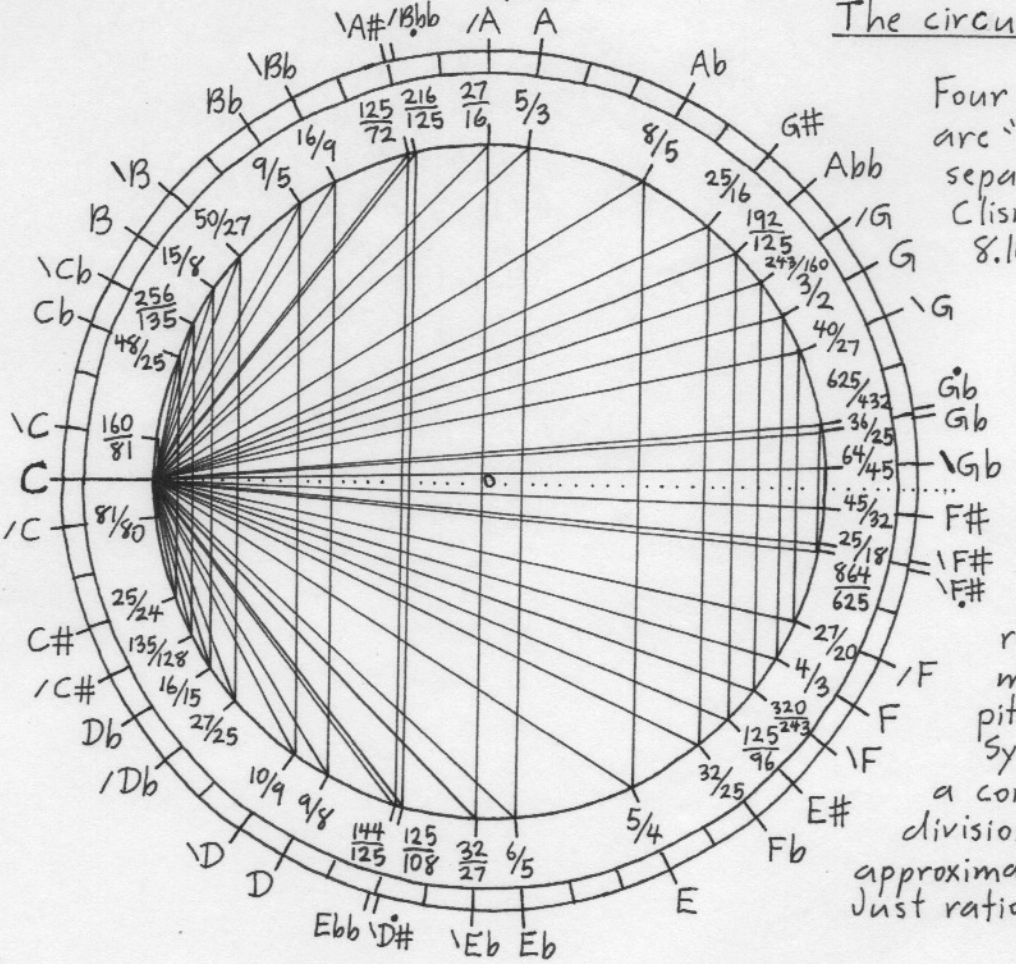


The tuning matrix which combines both the rising scale and the falling scale. Note the significant zone of invariance under reciprocation.

The dotted lines show important boundaries to the Field of Just Intonation.

Note that four pitches lie beyond the North-South boundary (or the Clisma boundary).

The circular graph



Four pairs of pitches are "near misses" separated by the Clisma interval (about 8.107 cents).

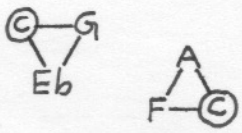
The pitches \A# (955.031 cents) and /Bbb (946.924 cents) surround the irrational ratio $\sqrt{3}/1$ (950.977 cents).

Apart from Clisma-related "near unisons", most of the adjacent pitches relate by a Syntonic Comma; hence a comma-based equal division (53-E.T.) adequately approximates this field of Just ratios.

Some additional Monochord Series mentioned:

Series **10:20**
Dual of 5-Limit 3:6

10	12	15	20
C	Eb	G	C
C	A	F	C



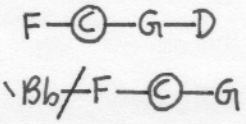
Series **15:30**
Dual of 5-Limit 4:8

15	20	24	30
C	F	Ab	C
C	G	E	C



Series **24:48**
3-Limit Tetrad
Its Dual is 90:180

24	27	32	36	48
C	D	F	G	C
C	\Bb	G	F	C



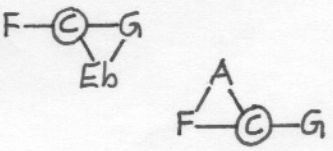
Series **24:48**
Diatonic Scale in its smallest integers
Ionian Mode
Phrygian Mode

24	27	30	32	36	40	45	48
C	D	E	F	G	A	B	C
C	\Bb	Ab	G	F	Eb	Db	C



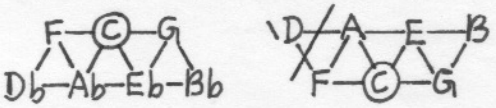
Series **30:60**
Dual of 5-Limit 6:12

30	36	40	45	60
C	Eb	F	G	C
C	A	G	F	C



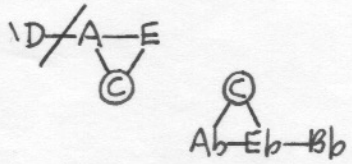
Series **30:60**
Babylonian Series
Prominant Diatonic Modes
Dual is 72:144
Phrygian Mode
Ionian Mode

30	32	36	40	45	48	54	60
C	Db	Eb	F	G	Ab	Bb	C
C	B	A	G	F	E	\D	C



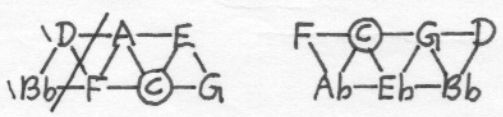
Series **36:72**
Dual of 5-Limit 5:10

36	40	45	60	72
C	\D	E	A	C
C	Bb	Ab	Eb	C



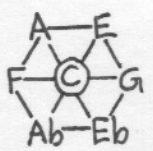
Series **36:72**
Prominant Diatonic Modes
Mixolydian Mode
Aeolian Mode

36	40	45	48	54	60	64	72
C	\D	E	F	G	A	\Bb	C
C	Bb	Ab	G	F	Eb	D	C



Series **60:120**
Symmetrical
Consonance Heptagon
Core Chromatic

60	72	75	80	90	96	100	120
C	Eb	E	F	G	Ab	A	C
C	A	Ab	G	F	E	Eb	C

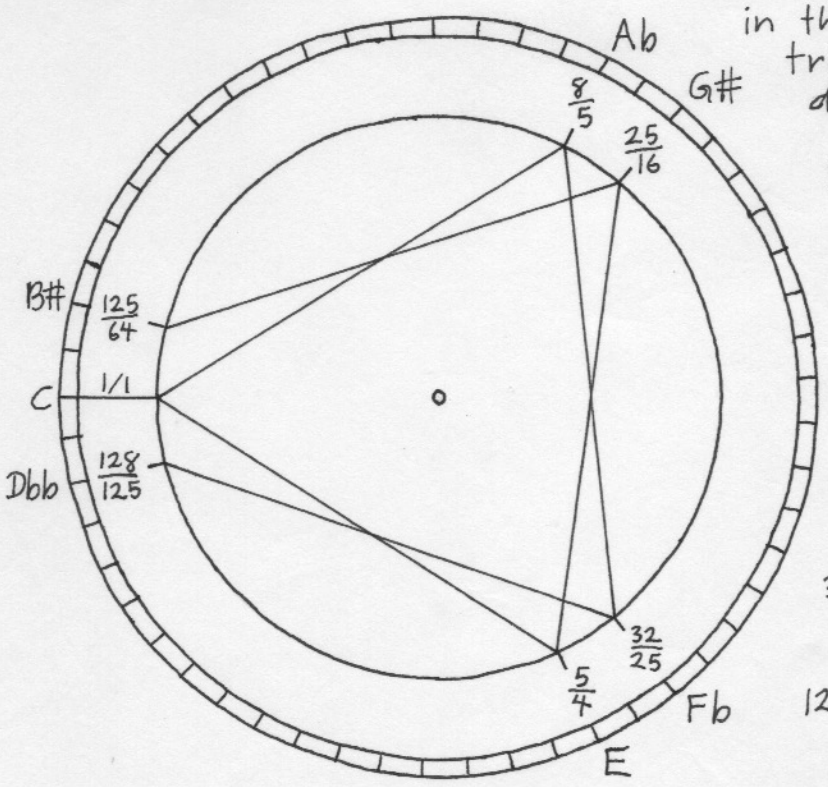


Numbers of the family grouping $2^a \cdot 5^b$ (another subset of Babylonian numbers)

1	*	250	$2 \cdot 5^3$	3200	$2^7 \cdot 5^2$	25,600	$2^{10} \cdot 5^2$
2	*	256	2^8	4000	$2^5 \cdot 5^3$	31,250	$2 \cdot 5^6$
4	2^2	320	$2^6 \cdot 5$	4096	2^{12}	32,000	$2^8 \cdot 5^3$
5	*	400	$2^4 \cdot 5^2$	5000	$2^3 \cdot 5^4$	32,768	2^{15}
8	2^3	500	$2^2 \cdot 5^3$	5120	$2^{10} \cdot 5$	40,000	$2^6 \cdot 5^4$
10	$2 \cdot 5$	512	2^9	6250	$2 \cdot 5^5$	40,960	$2^{13} \cdot 5$
16	2^4	625	5^4	6400	$2^8 \cdot 5^2$	50,000	$2^4 \cdot 5^5$
20	$2^2 \cdot 5$	640	$2^7 \cdot 5$	8000	$2^6 \cdot 5^3$	51,200	$2^{11} \cdot 5^2$
25	5^2	800	$2^5 \cdot 5^2$	8192	2^{13}	62,500	$2^2 \cdot 5^6$
32	2^5	1000	$2^3 \cdot 5^3$	10,000	$2^4 \cdot 5^4$	64,000	$2^9 \cdot 5^3$
40	$2^3 \cdot 5$	1024	2^{10}	10,240	$2^{11} \cdot 5$	65,536	2^{16}
50	$2 \cdot 5^2$	1250	$2 \cdot 5^4$	12,500	$2^2 \cdot 5^5$	78,125	5^7
64	2^6	1280	$2^8 \cdot 5$	12,800	$2^9 \cdot 5^2$	80,000	$2^7 \cdot 5^4$
80	$2^4 \cdot 5$	1600	$2^6 \cdot 5^2$	15,625	5^6	81,920	$2^{14} \cdot 5$
100	$2^2 \cdot 5^2$	2000	$2^4 \cdot 5^3$	16,000	$2^7 \cdot 5^3$	100,000	$2^5 \cdot 5^5$
125	5^3	2048	2^{11}	16,384	2^{14}	102,400	$2^{12} \cdot 5^2$
128	2^7	2500	$2^2 \cdot 5^4$	20,000	$2^5 \cdot 5^4$	125,000	$2^3 \cdot 5^6$
160	$2^5 \cdot 5$	2560	$2^9 \cdot 5$	20,480	$2^{12} \cdot 5$	128,000	$2^{10} \cdot 5^3$
200	$2^3 \cdot 5^2$	3125	5^5	25,000	$2^3 \cdot 5^5$	131,072	2^{17}

A characteristic monochord number series:

	64	80	100	125	128
harmonics –	C	E	G#	B#	C
monochord –	C	Ab	Fb	Dbb	C



The grouping represents expansion along the major third-minor sixth axis of Just Intonation, resulting in the characteristic "augmented triad" structure and the diesis step interval.

Some representative ratios:

5/4 (386.314 cents)
Just Major Third

8/5 (813.687 cents)
Just Minor Sixth

25/16 (772.627 cents)
Just augmented sixth

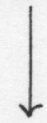
32/25 (427.342 cents)
Just diminished Fourth

128/125 (41.059 cents)
Diesis or enharmonic

125/64 (1158.941 cents)
Diesis-Flat octave

Numbers of the family grouping $2^a \cdot 3^b \cdot 5^c \cdot 7^d$ (Archytas or 7-Limit)

1	*	112	$2^4 \cdot 7$	432	$2^4 \cdot 3^3$	1050	$2 \cdot 3 \cdot 5^2 \cdot 7$	2160	$2^4 \cdot 3^3 \cdot 5$	3840	$2^8 \cdot 3 \cdot 5$
2	*	120	$2^3 \cdot 3 \cdot 5$	441	$3^2 \cdot 7^2$	1080	$2^3 \cdot 3^3 \cdot 5$	2187	3^7	3888	$2^4 \cdot 3^5$
3	*	125	5^3	448	$2^6 \cdot 7$	1120	$2^5 \cdot 5 \cdot 7$	2205	$3^2 \cdot 5 \cdot 7^2$	3920	$2^4 \cdot 5 \cdot 7^2$
4	2^2	126	$2 \cdot 3^2 \cdot 7$	450	$2 \cdot 3^2 \cdot 5^2$	1125	$3^2 \cdot 5^3$	2240	$2^6 \cdot 5 \cdot 7$	3969	$3^4 \cdot 7^2$
5	*	128	2^7	480	$2^5 \cdot 3 \cdot 5$	1134	$2 \cdot 3^4 \cdot 7$	2250	$2 \cdot 3^2 \cdot 5^3$	4000	$2^5 \cdot 5^3$
6	$2 \cdot 3$	135	$3^3 \cdot 5$	486	$2 \cdot 3^5$	1152	$2^7 \cdot 3^2$	2268	$2^2 \cdot 3^4 \cdot 7$	4032	$2^6 \cdot 3^2 \cdot 7$
7	*	140	$2^2 \cdot 5 \cdot 7$	490	$2 \cdot 5 \cdot 7^2$	1176	$2^3 \cdot 3 \cdot 7^2$	2304	$2^8 \cdot 3^2$	4050	$2 \cdot 3^4 \cdot 5^2$
8	2^3	144	$2^4 \cdot 3^2$	500	$2^2 \cdot 5^3$	1200	$2^4 \cdot 3 \cdot 5^2$	2352	$2^4 \cdot 3 \cdot 7^2$	4096	2^{12}
9	3^2	147	$3 \cdot 7^2$	504	$2^3 \cdot 3^2 \cdot 7$	1215	$3^5 \cdot 5$	2400	$2^5 \cdot 3 \cdot 5^2$	4116	$2^2 \cdot 3 \cdot 7^3$
10	$2 \cdot 5$	150	$2 \cdot 3 \cdot 5^2$	512	2^9	1225	$5^2 \cdot 7^2$	2401	7^4	4200	$2^3 \cdot 3 \cdot 5^2 \cdot 7$
12	$2^2 \cdot 3$	160	$2^5 \cdot 5$	525	$3 \cdot 5^2 \cdot 7$	1250	$2 \cdot 5^4$	2430	$2 \cdot 3^5 \cdot 5$	4320	$2^5 \cdot 3^3 \cdot 5$
14	$2 \cdot 7$	162	$2 \cdot 3^4$	540	$2^2 \cdot 3^3 \cdot 5$	1260	$2^2 \cdot 3^2 \cdot 5 \cdot 7$	2450	$2 \cdot 5^2 \cdot 7^2$	4374	$2 \cdot 3^7$
15	$3 \cdot 5$	168	$2^3 \cdot 3 \cdot 7$	560	$2^4 \cdot 5 \cdot 7$	1280	$2^8 \cdot 5$	2500	$2^2 \cdot 5^4$	4375	$5^4 \cdot 7$
16	2^4	175	$5^2 \cdot 7$	567	$3^4 \cdot 7$	1296	$2^4 \cdot 3^4$	2520	$2^3 \cdot 3^2 \cdot 5 \cdot 7$	4410	$2 \cdot 3^2 \cdot 5 \cdot 7^2$
18	$2 \cdot 3^2$	180	$2^2 \cdot 3^2 \cdot 5$	576	$2^6 \cdot 3^2$	1323	$3^3 \cdot 7^2$	2560	$2^9 \cdot 5$	4480	$2^7 \cdot 5 \cdot 7$
20	$2^2 \cdot 5$	189	$3^3 \cdot 7$	588	$2^2 \cdot 3 \cdot 7^2$	1344	$2^6 \cdot 3 \cdot 7$	2592	$2^5 \cdot 3^4$	4500	$2^2 \cdot 3^2 \cdot 5^3$
21	$3 \cdot 7$	192	$2^6 \cdot 3$	600	$2^3 \cdot 3 \cdot 5^2$	1350	$2 \cdot 3^3 \cdot 5^2$	2625	$3 \cdot 5^3 \cdot 7$	4536	$2^3 \cdot 3^4 \cdot 7$
24	$2^3 \cdot 3$	196	$2^2 \cdot 7^2$	625	5^4	1372	$2^2 \cdot 7^3$	2646	$2 \cdot 3^3 \cdot 7^2$	4608	$2^9 \cdot 3^2$
25	5^2	200	$2^3 \cdot 5^2$	630	$2 \cdot 3^2 \cdot 5 \cdot 7$	1400	$2^3 \cdot 5^2 \cdot 7$	2688	$2^7 \cdot 3 \cdot 7$	4704	$2^5 \cdot 3 \cdot 7^2$
27	3^3	210	$2 \cdot 3 \cdot 5 \cdot 7$	640	$2^7 \cdot 5$	1440	$2^5 \cdot 3^2 \cdot 5$	2700	$2^2 \cdot 3^3 \cdot 5^2$	4725	$3^3 \cdot 5^2 \cdot 7$
28	$2^2 \cdot 7$	216	$2^3 \cdot 3^3$	648	$2^3 \cdot 3^4$	1458	$2 \cdot 3^6$	2744	$2^3 \cdot 7^3$	4800	$2^6 \cdot 3 \cdot 5^2$
30	$2 \cdot 3 \cdot 5$	224	$2^5 \cdot 7$	672	$2^5 \cdot 3 \cdot 7$	1470	$2 \cdot 3 \cdot 5 \cdot 7^2$	2800	$2^4 \cdot 5^2 \cdot 7$	4802	$2 \cdot 7^4$
32	2^5	225	$3^2 \cdot 5^2$	675	$3^3 \cdot 5^2$	1500	$2^2 \cdot 3 \cdot 5^3$	2835	$3^4 \cdot 5 \cdot 7$	4860	$2^2 \cdot 3^5 \cdot 5$
35	$5 \cdot 7$	240	$2^4 \cdot 3 \cdot 5$	686	$2 \cdot 7^3$	1512	$2^3 \cdot 3^3 \cdot 7$	2880	$2^6 \cdot 3^2 \cdot 5$	4900	$2^2 \cdot 5^2 \cdot 7^2$
36	$2^2 \cdot 3^2$	243	3^5	700	$2^2 \cdot 5^2 \cdot 7$	1536	$2^9 \cdot 3$	2916	$2^2 \cdot 3^6$	5000	$2^3 \cdot 5^4$
40	$2^3 \cdot 5$	245	$5 \cdot 7^2$	720	$2^4 \cdot 3^2 \cdot 5$	1568	$2^5 \cdot 7^2$	2940	$2^2 \cdot 3 \cdot 5 \cdot 7^2$	5040	$2^4 \cdot 3^2 \cdot 5 \cdot 7$
42	$2 \cdot 3 \cdot 7$	250	$2 \cdot 5^3$	729	3^6	1575	$3^2 \cdot 5^2 \cdot 7$	3000	$2^3 \cdot 3 \cdot 5^3$		
45	$3^2 \cdot 5$	252	$2^2 \cdot 3^2 \cdot 7$	735	$3 \cdot 5 \cdot 7^2$	1600	$2^6 \cdot 5^2$	3024	$2^4 \cdot 3^3 \cdot 7$		
48	$2^4 \cdot 3$	256	2^8	750	$2 \cdot 3 \cdot 5^3$	1620	$2^2 \cdot 3^4 \cdot 5$	3072	$2^{10} \cdot 3$		
49	7^2	270	$2 \cdot 3^3 \cdot 5$	756	$2^2 \cdot 3^3 \cdot 7$	1680	$2^4 \cdot 3 \cdot 5 \cdot 7$	3087	$3^2 \cdot 7^3$		
50	$2 \cdot 5^2$	280	$2^3 \cdot 5 \cdot 7$	768	$2^8 \cdot 3$	1701	$3^5 \cdot 7$	3125	5^5		
54	$2 \cdot 3^3$	288	$2^5 \cdot 3^2$	784	$2^4 \cdot 7^2$	1715	$5 \cdot 7^3$	3136	$2^6 \cdot 7^2$		
56	$2^3 \cdot 7$	294	$2 \cdot 3 \cdot 7^2$	800	$2^5 \cdot 5^2$	1728	$2^6 \cdot 3^3$	3150	$2 \cdot 3^2 \cdot 5^2 \cdot 7$		
60	$2^2 \cdot 3 \cdot 5$	300	$2^2 \cdot 3 \cdot 5^2$	810	$2 \cdot 3^4 \cdot 5$	1750	$2 \cdot 5^3 \cdot 7$	3200	$2^7 \cdot 5^2$		
63	$3^2 \cdot 7$	315	$3^2 \cdot 5 \cdot 7$	840	$2^3 \cdot 3 \cdot 5 \cdot 7$	1764	$2^2 \cdot 3^2 \cdot 7^2$	3240	$2^3 \cdot 3^4 \cdot 5$		
64	2^6	320	$2^6 \cdot 5$	864	$2^5 \cdot 3^3$	1792	$2^8 \cdot 7$	3360	$2^5 \cdot 3 \cdot 5 \cdot 7$		
70	$2 \cdot 5 \cdot 7$	324	$2^2 \cdot 3^4$	875	$5^3 \cdot 7$	1800	$2^3 \cdot 3^2 \cdot 5^2$	3375	$3^3 \cdot 5^3$		
72	$2^3 \cdot 3^2$	336	$2^4 \cdot 3 \cdot 7$	882	$2 \cdot 3^2 \cdot 7^2$	1875	$3 \cdot 5^4$	3402	$2 \cdot 3^5 \cdot 7$		
75	$3 \cdot 5^2$	343	7^3	896	$2^7 \cdot 7$	1890	$2 \cdot 3^3 \cdot 5 \cdot 7$	3430	$2 \cdot 5 \cdot 7^3$		
80	$2^4 \cdot 5$	350	$2 \cdot 5^2 \cdot 7$	900	$2^2 \cdot 3^2 \cdot 5^2$	1920	$2^7 \cdot 3 \cdot 5$	3456	$2^7 \cdot 3^3$		
81	3^4	360	$2^3 \cdot 3^2 \cdot 5$	945	$3^3 \cdot 5 \cdot 7$	1944	$2^3 \cdot 3^5$	3500	$2^2 \cdot 5^3 \cdot 7$		
84	$2^2 \cdot 3 \cdot 7$	375	$3 \cdot 5^3$	960	$2^6 \cdot 3 \cdot 5$	1960	$2^3 \cdot 5 \cdot 7^2$	3528	$2^3 \cdot 3^2 \cdot 7^2$		
90	$2 \cdot 3^2 \cdot 5$	378	$2 \cdot 3^3 \cdot 7$	972	$2^2 \cdot 3^5$	2000	$2^4 \cdot 5^3$	3584	$2^9 \cdot 7$		
96	$2^5 \cdot 3$	384	$2^7 \cdot 3$	980	$2^2 \cdot 5 \cdot 7^2$	2016	$2^5 \cdot 3^2 \cdot 7$	3600	$2^4 \cdot 3^2 \cdot 5^2$		
98	$2 \cdot 7^2$	392	$2^3 \cdot 7^2$	1000	$2^3 \cdot 5^3$	2025	$3^4 \cdot 5^2$	3645	$3^6 \cdot 5$		
100	$2^2 \cdot 5^2$	400	$2^4 \cdot 5^2$	1008	$2^4 \cdot 3^2 \cdot 7$	2048	2^{11}	3675	$3 \cdot 5^2 \cdot 7^2$		
105	$3 \cdot 5 \cdot 7$	405	$3^4 \cdot 5$	1024	2^{10}	2058	$2 \cdot 3 \cdot 7^3$	3750	$2 \cdot 3 \cdot 5^4$		
108	$2^2 \cdot 3^3$	420	$2^2 \cdot 3 \cdot 5 \cdot 7$	1029	$3 \cdot 7^3$	2100	$2^2 \cdot 3 \cdot 5^2 \cdot 7$	3780	$2^2 \cdot 3^3 \cdot 5 \cdot 7$		



7-Limit Ratios (between 1/1 and 2/1) not found in the 5-Limit Fabric:

Ratio	Prime Factors	Cents	Pitch
7/4	$7/2^2$	968.826	
7/5	7/5	582.512	
7/6	$7/2\cdot3$	266.871	
8/7	$2^3/7$	231.174	
9/7	$3^2/7$	435.084	
10/7	$2\cdot5/7$	617.488	
12/7	$2^2\cdot3/7$	933.129	
14/9	$2\cdot7/3^2$	764.916	
15/14	$3\cdot5/2\cdot7$	119.443	
21/16	$3\cdot7/2^4$	470.781	
21/20	$3\cdot7/2^2\cdot5$	84.467	
25/14	$5^2/2\cdot7$	1003.802	
25/21	$5^2/3\cdot7$	301.847	
27/14	$3^3/2\cdot7$	1137.039	
28/15	$2^2\cdot7/3\cdot5$	1080.557	
28/25	$2^2\cdot7/5^2$	196.198	
28/27	$2^2\cdot7/3^3$	62.961	
32/21	$2^5/3\cdot7$	729.219	
35/18	$5\cdot7/2\cdot3^2$	1151.230	
35/24	$5\cdot7/2^3\cdot3$	653.185	
35/27	$5\cdot7/3^3$	449.275	
35/32	$5\cdot7/2^5$	155.140	
36/35	$2^2\cdot3^2/5\cdot7$	48.770	
40/21	$2^3\cdot5/3\cdot7$	1115.533	
42/25	$2\cdot3\cdot7/5^2$	898.153	
45/28	$3^2\cdot5/2^2\cdot7$	821.398	
48/35	$2^4\cdot3/5\cdot7$	546.815	
49/25	$7^2/5^2$	1165.024	
49/27	$7^2/3^3$	1031.787	
49/30	$7^2/2\cdot3\cdot5$	849.383	
49/32	$7^2/2^5$	737.652	
49/36	$7^2/2^2\cdot3^2$	533.742	
49/40	$7^2/2^3\cdot5$	351.338	
49/45	$7^2/3^2\cdot5$	147.428	
49/48	$7^2/2^4\cdot3$	35.697	
50/49	$2\cdot5^2/7^2$	34.976	
54/35	$2\cdot3^3/5\cdot7$	750.725	
54/49	$2\cdot3^3/7^2$	168.213	
56/45	$2^3\cdot7/3^2\cdot5$	378.602	
60/49	$2^2\cdot3\cdot5/7^2$	350.617	
63/32	$3^2\cdot7/2^5$	1172.736	
63/40	$3^2\cdot7/2^3\cdot5$	786.422	
63/50	$3^2\cdot7/2\cdot5^2$	400.108	
64/35	$2^6/5\cdot7$	1044.860	
64/49	$2^6/7^2$	462.348	
64/63	$2^6/3^2\cdot7$	27.264	
72/49	$2^3\cdot3^2/7^2$	666.258	
75/49	$3\cdot5^2/7^2$	736.931	
75/56	$3\cdot5^2/2^3\cdot7$	505.757	
80/49	$2^4\cdot5/7^2$	848.662	
80/63	$2^4\cdot5/3^2\cdot7$	413.578	
81/49	$3^4/7^2$	870.168	
81/56	$3^4/2^3\cdot7$	638.994	
81/70	$3^4/2\cdot5\cdot7$	252.680	
90/49	$2\cdot3^2\cdot5/7^2$	1052.572	
96/49	$2^5\cdot3/7^2$	1164.303	
98/75	$2\cdot7^2/3\cdot5^2$	463.069	
98/81	$2\cdot7^2/3^4$	329.832	
100/63	$2^2\cdot5^2/3^2\cdot7$	799.892	
105/64	$3\cdot5\cdot7/2^6$	857.095	
112/75	$2^4\cdot7/3\cdot5^2$	694.243	
112/81	$2^4\cdot7/3^4$	561.006	
125/63	$5^3/3^2\cdot7$	1186.205	
125/84	$5^3/2^2\cdot3\cdot7$	688.160	
125/98	$5^3/2\cdot7^2$	421.289	
125/112	$5^3/2^4\cdot7$	190.115	
126/125	$2\cdot3^2\cdot7/5^3$	13.795	
128/105	$2^7/3\cdot5\cdot7$	342.905	
135/98	$3^3\cdot5/2\cdot7^2$	554.527	
135/112	$3^3\cdot5/2^4\cdot7$	323.353	
140/81	$2^2\cdot5\cdot7/3^4$	947.320	
147/80	$3\cdot7^2/2^4\cdot5$	1053.293	
147/100	$3\cdot7^2/2^2\cdot5^2$	666.979	
147/125	$3\cdot7^2/5^3$	280.666	
147/128	$3\cdot7^2/2^7$	239.607	
160/147	$2^5\cdot5/3\cdot7^2$	146.707	
168/125	$2^3\cdot3\cdot7/5^3$	511.840	
175/96	$5^2\cdot7/2^5\cdot3$	1039.498	
175/108	$5^2\cdot7/2^2\cdot3^3$	835.588	
175/128	$5^2\cdot7/2^7$	541.453	
175/144	$5^2\cdot7/2^4\cdot3^2$	337.543	
175/162	$5^2\cdot7/2\cdot3^4$	133.633	
189/100	$3^3\cdot7/2^2\cdot5^2$	1102.064	
189/125	$3^3\cdot7/5^3$	715.750	
189/128	$3^3\cdot7/2^7$	674.691	
189/160	$3^3\cdot7/2^5\cdot5$	288.377	
192/175	$2^6\cdot3/5^2\cdot7$	160.502	
196/125	$2^2\cdot7^2/5^3$	778.711	
196/135	$2^2\cdot7^2/3^3\cdot5$	645.473	
200/147	$2^3\cdot5^2/3\cdot7^2$	533.021	
200/189	$2^3\cdot5^2/3^3\cdot7$	97.937	
216/175	$2^3\cdot3^3/5^2\cdot7$	364.412	
224/125	$2^5\cdot7/5^3$	1009.885	

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224/135	$2^5.7/3^3.5$	876.647
225/196	$3^2.5^2/2^2.7^2$	238.886
225/224	$3^2.5^2/2^5.7$	7.712
243/140	$3^5/2^2.5.7$	954.635
243/175	$3^5/5^2.7$	568.322
243/196	$3^5/2^2.7^2$	372.123
243/224	$3^5/2^5.7$	140.949
245/128	$5.7^2/2^7$	1123.966
245/144	$5.7^2/2^4.3^2$	920.056
245/162	$5.7^2/2.3^4$	716.146
245/192	$5.7^2/2^6.3$	422.011
245/216	$5.7^2/2^3.3^3$	218.101
245/243	$5.7^2/3^5$	14.191
250/147	$2.5^3/3.7^2$	919.334
250/189	$2.5^3/3^3.7$	484.250
256/147	$2^8/3.7^2$	960.393
256/175	$2^8/5^2.7$	658.547
256/189	$2^8/3^3.7$	525.309
256/245	$2^8/5.7^2$	76.034
280/243	$2^3.5.7/3^5$	245.365
288/175	$2^5.3^2/5^2.7$	862.457
288/245	$2^5.3^2/5.7^2$	279.944
315/256	$3^2.5.7/2^8$	359.050
320/189	$2^6.5/3^3.7$	911.623
324/175	$2^2.3^4/5^2.7$	1066.367
324/245	$2^2.3^4/5.7^2$	483.854
343/180	$7^3/2^2.3^2.5$	1116.254
343/192	$7^3/2^6.3$	1004.523
343/200	$7^3/2^3.5^2$	933.850
343/216	$7^3/2^3.3^3$	800.613
343/225	$7^3/3^2.5^2$	729.940
343/243	$7^3/3^5$	596.703
343/250	$7^3/2.5^3$	547.537
343/256	$7^3/2^8$	506.478
343/270	$7^3/2.3^3.5$	414.299
343/288	$7^3/2^5.3^2$	302.568
343/300	$7^3/2^2.3.5^2$	231.895
343/320	$7^3/2^6.5$	120.164
343/324	$7^3/2^2.3^4$	98.658
350/243	$2.5^2.7/3^5$	631.678
360/343	$2^3.3^2.5/7^3$	83.746
375/196	$3.5^3/2^2.7^2$	1123.244
375/224	$3.5^3/2^5.7$	892.070
375/343	$3.5^3/7^3$	154.418
384/245	$2^7.3/5.7^2$	777.989
384/343	$2^7.3/7^3$	195.477
392/225	$2^3.7^2/3^2.5^2$	961.114
392/243	$2^3.7^2/3^5$	827.877
392/375	$2^3.7^2/3.5^3$	76.756
400/343	$2^4.5^2/7^3$	266.150
405/224	$3^4.5/2^5.7$	1025.308
405/343	$3^4.5/7^3$	287.656
432/245	$2^4.3^3/5.7^2$	981.899
432/343	$2^4.3^3/7^3$	399.387
441/250	$3^2.7^2/2.5^3$	982.621
441/256	$3^2.7^2/2^8$	941.562
441/320	$3^2.7^2/2^6.5$	555.248
441/400	$3^2.7^2/2^4.5^2$	168.934
448/225	$2^6.7/3^2.5^2$	1192.288
448/243	$2^6.7/3^5$	1059.051
448/375	$2^6.7/3.5^3$	307.930
448/405	$2^6.7/3^4.5$	174.692
450/343	$2.3^2.5^2/7^3$	470.060
480/343	$2^5.3.5/7^3$	581.791
486/245	$2.3^5/5.7^2$	1185.809
486/343	$2.3^5/7^3$	603.297
500/343	$2^2.5^3/7^3$	652.463
512/315	$2^9/3^2.5.7$	840.950
512/343	$2^9/7^3$	693.522
512/441	$2^9/3^2.7^2$	258.438
525/512	$3.5^2.7/2^9$	43.408
540/343	$2^2.3^3.5/7^3$	785.701
567/320	$3^4.7/2^6.5$	990.332
567/400	$3^4.7/2^4.5^2$	604.018
567/500	$3^4.7/2^2.5^3$	217.705
567/343	$2^6.3^2/7^3$	870.168
600/343	$2^3.3.5^2/7^3$	968.105
625/336	$5^4/2^4.3.7$	1074.474
625/343	$5^4/7^3$	1038.777
625/378	$5^4/2.3^3.7$	870.564
625/392	$5^4/2^3.7^2$	807.603
625/441	$5^4/3^2.7^2$	603.693
625/448	$5^4/2^6.7$	576.429
625/504	$5^4/2^3.3^2.7$	372.519
625/567	$5^4/3^4.7$	168.609
625/588	$5^4/2^2.3.7^2$	105.648
640/343	$2^7.5/7^3$	1079.836
640/441	$2^7.5/3^2.7^2$	644.752
640/567	$2^7.5/3^4.7$	209.668
648/343	$2^3.3^4/7^3$	1101.342
672/625	$2^5.3.7/5^4$	125.526
675/343	$3^3.5^2/7^3$	1172.015
675/392	$3^3.5^2/2^3.7^2$	940.841
675/448	$3^3.5^2/2^6.7$	709.667
686/375	$2.7^3/3.5^3$	1045.582
686/405	$2.7^3/3^4.5$	912.344
686/625	$2.7^3/5^4$	161.223
729/392	$3^6/2^3.7^2$	1074.078

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729/448	$3^6/2^6.7$	842.904
729/490	$3^6/2.5.7^2$	687.764
729/560	$3^6/2^4.5.7$	456.590
729/686	$3^6/2.7^3$	105.252
729/700	$3^6/2^2.5^2.7$	70.277
735/512	$3.5.7^2/2^9$	625.921
756/625	$2^2.3^3.7/5^4$	329.436
784/405	$2^4.7^2/3^4.5$	1143.518
784/625	$2^4.7^2/5^4$	392.397
784/675	$2^4.7^2/3^3.5^2$	259.159
784/729	$2^4.7^2/3^6$	125.922
800/441	$2^5.5^2/3^2.7^2$	1031.066
800/567	$2^5.5^2/3^4.7$	595.982
875/486	$5^3.7/2.3^5$	1017.992
875/512	$5^3.7/2^9$	927.767
875/576	$5^3.7/2^6.3^2$	723.857
875/648	$5^3.7/2^3.3^4$	519.947
875/729	$5^3.7/3^6$	316.037
875/768	$5^3.7/2^8.3$	225.812
875/864	$5^3.7/2^5.3^3$	21.902
882/625	$2.3^2.7^2/5^4$	596.307
896/625	$2^7.7/5^4$	623.571
896/675	$2^7.7/3^3.5^2$	490.333
896/729	$2^7.7/3^6$	357.096
945/512	$3^3.5.7/2^9$	1061.005
972/875	$2^2.3^5/5^3.7$	182.008
980/729	$2^2.5.7^2/3^6$	512.236
1000/567	$2^3.5^3/3^4.7$	982.295
1008/625	$2^4.3^2.7/5^4$	827.481
1024/525	$2^{10}/3.5^2.7$	1156.592
1024/567	$2^{10}/3^4.7$	1023.354
1024/735	$2^{10}/3.5.7^2$	574.079
1024/875	$2^{10}/5^3.7$	272.233
1024/945	$2^{10}/3^3.5.7$	138.995
1029/625	$3.7^3/5^4$	863.178
1029/640	$3.7^3/2^7.5$	822.119
1029/800	$3.7^3/2^5.5^2$	435.805
1029/1000	$3.7^3/2^3.5^3$	49.492
1029/1024	$3.7^3/2^{10}$	8.433
1120/729	$2^5.5.7/3^6$	743.410
1125/686	$3^2.5^3/2.7^3$	856.373
1125/784	$3^2.5^3/2^4.7^2$	625.199
1125/896	$3^2.5^3/2^7.7$	394.025
1134/625	$2.3^4.7/5^4$	1031.391
1152/875	$2^7.3^2/5^3.7$	476.143
1176/625	$2^3.3.7^2/5^4$	1094.352
1215/686	$3^5.5/2.7^3$	989.611
1215/784	$3^5.5/2^4.7^2$	758.437
1215/896	$3^5.5/2^7.7$	527.263
1225/648	$5^2.7^2/2^3.3^4$	1102.459
1225/729	$5^2.7^2/3^6$	898.549
1225/768	$5^2.7^2/2^8.3$	808.324
1225/864	$5^2.7^2/2^5.3^3$	604.414
1225/972	$5^2.7^2/2^2.3^5$	400.504
1225/1024	$5^2.7^2/2^{10}$	310.279
1225/1152	$5^2.7^2/2^7.3^2$	106.369
1250/1029	$2.5^4/3.7^3$	336.822
1280/1029	$2^8.5/3.7^3$	377.881
1296/875	$2^4.3^4/5^3.7$	680.053
1296/1225	$2^4.3^4/5^2.7^2$	97.541
1323/800	$3^3.7^2/2^5.5^2$	870.889
1323/1000	$3^3.7^2/2^3.5^3$	484.576
1323/1024	$3^3.7^2/2^{10}$	443.517
1323/1250	$3^3.7^2/2.5^4$	98.262
1323/1280	$3^3.7^2/2^8.5$	57.203
1372/729	$2^2.7^3/3^6$	1094.748
1372/1125	$2^2.7^3/3^2.5^3$	343.627
1372/1215	$2^2.7^3/3^5.5$	210.389
1400/729	$2^3.5^2.7/3^6$	1129.723
1458/875	$2.3^6/5^3.7$	883.963
1458/1225	$2.3^6/5^2.7^2$	301.451
1536/875	$2^9.3/5^3.7$	974.188
1536/1225	$2^9.3/5^2.7^2$	539.104
1568/1125	$2^5.7^2/3^2.5^3$	574.801
1568/1215	$2^5.7^2/3^5.5$	441.563
1575/1024	$3^2.5^2.7/2^{10}$	745.363
1600/1029	$2^6.5^2/3.7^3$	764.195
1600/1323	$2^6.5^2/3^3.7^2$	329.111
1701/1000	$3^5.7/2^3.5^3$	919.660
1701/1024	$3^5.7/2^{10}$	878.601
1701/1250	$3^5.7/2.5^4$	533.346
1701/1280	$3^5.7/2^8.5$	492.287
1701/1600	$3^5.7/2^6.5^2$	105.973
1715/864	$5.7^3/2^5.3^3$	1186.926
1715/972	$5.7^3/2^2.3^5$	983.016
1715/1024	$5.7^3/2^{10}$	892.791
1715/1152	$5.7^3/2^7.3^2$	688.881
1715/1296	$5.7^3/2^4.3^4$	484.971
1715/1458	$5.7^3/2.3^6$	281.061
1715/1536	$5.7^3/2^9.3$	190.836
1728/875	$2^6.3^3/5^3.7$	1178.098
1728/1225	$2^6.3^3/5^2.7^2$	595.586
1792/1125	$2^8.7/3^2.5^3$	805.975
1792/1215	$2^8.7/3^5.5$	672.737
1875/1372	$3.5^4/2^2.7^3$	540.732
1875/1568	$3.5^4/2^5.7^2$	309.558
1875/1792	$3.5^4/2^8.7$	78.384
1944/1225	$2^3.3^5/5^2.7^2$	799.496

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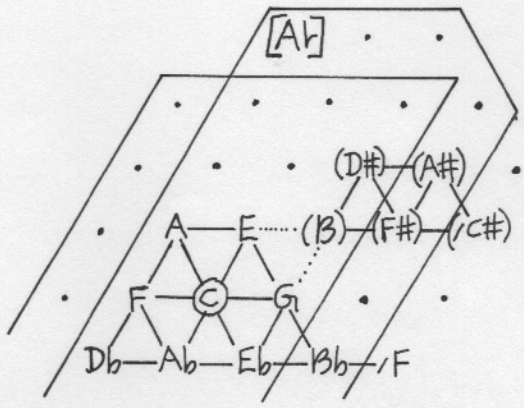
1944/1715	$2^3 \cdot 3^5 / 5 \cdot 7^3$	216.984
2000/1029	$2^4 \cdot 5^3 / 3 \cdot 7^3$	1150.508
2000/1323	$2^4 \cdot 5^3 / 3^3 \cdot 7^2$	715.424
2000/1701	$2^4 \cdot 5^3 / 3^5 \cdot 7$	280.340
2025/1372	$3^4 \cdot 5^2 / 2^2 \cdot 7^3$	673.970
2025/1568	$3^4 \cdot 5^2 / 2^5 \cdot 7^2$	442.796
2025/1792	$3^4 \cdot 5^2 / 2^8 \cdot 7$	211.622
2048/1029	$2^{11} / 3 \cdot 7^3$	1191.567
2048/1225	$2^{11} / 5^2 \cdot 7^2$	889.721
2048/1323	$2^{11} / 3^3 \cdot 7^2$	756.483
2048/1575	$2^{11} / 3^2 \cdot 5^2 \cdot 7$	454.637
2048/1701	$2^{11} / 3^5 \cdot 7$	321.399
2048/1715	$2^{11} / 5 \cdot 7^3$	307.209
2187/1120	$3^7 / 2^5 \cdot 5 \cdot 7$	1158.545
2187/1225	$3^7 / 5^2 \cdot 7^2$	1003.406
2187/1372	$3^7 / 2^2 \cdot 7^3$	807.207
2187/1400	$3^7 / 2^3 \cdot 5^2 \cdot 7$	772.232
2187/1568	$3^7 / 2^5 \cdot 7^2$	576.033
2187/1715	$3^7 / 5 \cdot 7^3$	420.894
2187/1750	$3^7 / 2 \cdot 5^3 \cdot 7$	385.918
2187/1792	$3^7 / 2^8 \cdot 7$	344.859
2187/1960	$3^7 / 2^3 \cdot 5 \cdot 7^2$	189.719
2187/2016	$3^7 / 2^5 \cdot 3^2 \cdot 7$	140.949
2205/2048	$3^2 \cdot 5 \cdot 7^2 / 2^{11}$	127.876
2240/2187	$2^6 \cdot 5 \cdot 7 / 3^7$	41.455
2304/1225	$2^8 \cdot 3^2 / 5^2 \cdot 7^2$	1093.631
2304/1715	$2^8 \cdot 3^2 / 5 \cdot 7^3$	511.119
2401/1215	$7^4 / 3^5 \cdot 5$	1179.215
2401/1250	$7^4 / 2 \cdot 5^4$	1130.049
2401/1280	$7^4 / 2^8 \cdot 5$	1088.990
2401/1296	$7^4 / 2^4 \cdot 3^4$	1067.484
2401/1350	$7^4 / 2 \cdot 3^3 \cdot 5^2$	996.811
2401/1440	$7^4 / 2^5 \cdot 3^2 \cdot 5$	885.080
2401/1458	$7^4 / 2 \cdot 3^6$	863.574
2401/1500	$7^4 / 2^2 \cdot 3 \cdot 5^3$	814.407
2401/1536	$7^4 / 2^9 \cdot 3$	773.349
2401/1600	$7^4 / 2^6 \cdot 5^2$	702.676
2401/1620	$7^4 / 2^2 \cdot 3^4 \cdot 5$	681.170
2401/1728	$7^4 / 2^6 \cdot 3^3$	569.439
2401/1800	$7^4 / 2^3 \cdot 3^2 \cdot 5^2$	498.766
2401/1875	$7^4 / 3 \cdot 5^4$	428.094
2401/1920	$7^4 / 2^7 \cdot 3 \cdot 5$	387.035
2401/1944	$7^4 / 2^3 \cdot 3^5$	365.529
2401/2000	$7^4 / 2^4 \cdot 5^3$	316.362
2401/2025	$7^4 / 3^4 \cdot 5^2$	294.856
2401/2048	$7^4 / 2^{11}$	275.304
2401/2160	$7^4 / 2^4 \cdot 3^3 \cdot 5$	183.125
2401/2187	$7^4 / 3^7$	161.619
2401/2250	$7^4 / 2 \cdot 3^2 \cdot 5^3$	112.452
2401/2304	$7^4 / 2^8 \cdot 3^2$	71.394
2401/2400	$7^4 / 2^5 \cdot 3 \cdot 5^2$	0.721
2430/2401	$2 \cdot 3^5 \cdot 5 / 7^4$	20.785
2450/2187	$2 \cdot 5^2 \cdot 7^2 / 3^7$	196.594
2500/1323	$2^2 \cdot 5^4 / 3^3 \cdot 7^2$	1101.738
2500/1701	$2^2 \cdot 5^4 / 3^5 \cdot 7$	666.654
2500/2401	$2^2 \cdot 5^4 / 7^4$	69.951
2560/1323	$2^9 \cdot 5 / 3^3 \cdot 7^2$	1142.797
2560/1701	$2^9 \cdot 5 / 3^5 \cdot 7$	707.713
2560/2401	$2^9 \cdot 5 / 7^4$	111.010
2592/1715	$2^5 \cdot 3^4 / 5 \cdot 7^3$	715.029
2592/2401	$2^5 \cdot 3^4 / 7^4$	132.516
2625/2048	$3 \cdot 5^3 \cdot 7 / 2^{11}$	429.722
2700/2401	$2^2 \cdot 3^3 \cdot 5^2 / 7^4$	203.189
2744/1875	$2^3 \cdot 7^3 / 3 \cdot 5^4$	659.268
2744/2025	$2^3 \cdot 7^3 / 3^4 \cdot 5^2$	526.030
2744/2187	$2^3 \cdot 7^3 / 3^7$	392.793
2800/2187	$2^4 \cdot 5^2 \cdot 7 / 3^7$	427.768
2835/2048	$3^4 \cdot 5 \cdot 7 / 2^{11}$	562.960
2880/2401	$2^6 \cdot 3^2 \cdot 5 / 7^4$	314.920
2916/1715	$2^2 \cdot 3^6 / 5 \cdot 7^3$	918.939
2916/2401	$2^2 \cdot 3^6 / 7^4$	336.426
3000/2401	$2^3 \cdot 3 \cdot 5^3 / 7^4$	385.593
3072/1715	$2^{10} \cdot 3 / 5 \cdot 7^3$	1009.164
3072/2401	$2^{10} \cdot 3 / 7^4$	426.651
3087/1600	$3^2 \cdot 7^3 / 2^6 \cdot 5^2$	1137.760
3087/2000	$3^2 \cdot 7^3 / 2^4 \cdot 5^3$	751.447
3087/2048	$3^2 \cdot 7^3 / 2^{11}$	710.388
3087/2500	$3^2 \cdot 7^3 / 2^2 \cdot 5^4$	365.133
3087/2560	$3^2 \cdot 7^3 / 2^9 \cdot 5$	324.074
3125/1568	$5^5 / 2^5 \cdot 7^2$	1193.917
3125/1701	$5^5 / 3^5 \cdot 7$	1052.968
3125/1792	$5^5 / 2^8 \cdot 7$	962.743
3125/2058	$5^5 / 2 \cdot 3 \cdot 7^3$	723.136
3125/2268	$5^5 / 2^2 \cdot 3^4 \cdot 7$	554.923
3125/2352	$5^5 / 2^4 \cdot 3 \cdot 7^2$	491.962
3125/2401	$5^5 / 7^4$	456.265
3125/2646	$5^5 / 2 \cdot 3^3 \cdot 7^2$	288.052
3125/2688	$5^5 / 2^7 \cdot 3 \cdot 7$	260.788
3125/3024	$5^5 / 2^4 \cdot 3^3 \cdot 7$	56.878
3125/3087	$5^5 / 3^2 \cdot 7^3$	21.181
3136/1875	$2^6 \cdot 7^2 / 3 \cdot 5^4$	890.442
3136/2025	$2^6 \cdot 7^2 / 3^4 \cdot 5^2$	757.204
3136/2187	$2^6 \cdot 7^2 / 3^7$	623.967
3136/3125	$2^6 \cdot 7^2 / 5^5$	6.083
3200/1701	$2^7 \cdot 5^2 / 3^5 \cdot 7$	1094.027
3200/2401	$2^7 \cdot 5^2 / 7^4$	497.324
3200/3087	$2^7 \cdot 5^2 / 3^2 \cdot 7^3$	62.240
3375/1792	$3^3 \cdot 5^3 / 2^8 \cdot 7$	1095.980

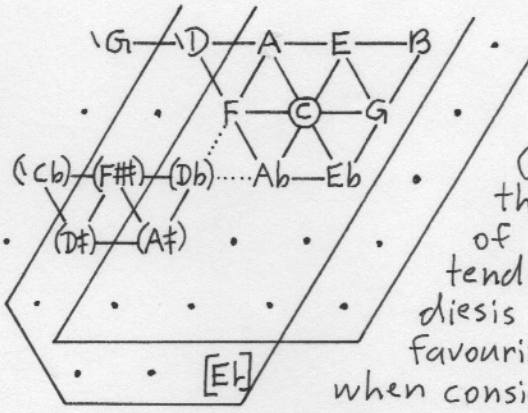
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3375/2401	$3^3 \cdot 5^3 / 7^4$	589.503
3375/2744	$3^3 \cdot 5^3 / 2^3 \cdot 7^3$	358.328
3375/3136	$3^3 \cdot 5^3 / 2^6 \cdot 7^2$	127.154
3402/3125	$2 \cdot 3^5 \cdot 7 / 5^5$	147.032
3430/2187	$2 \cdot 5 \cdot 7^3 / 3^7$	779.106
3456/2401	$2^7 \cdot 3^3 / 7^4$	630.561
3500/2187	$2^2 \cdot 5^3 \cdot 7 / 3^7$	814.082
3528/3125	$2^3 \cdot 3^2 \cdot 7^2 / 5^5$	209.993
3584/1875	$2^9 \cdot 7 / 3 \cdot 5^4$	1121.616
3584/2025	$2^9 \cdot 7 / 3^4 \cdot 5^2$	988.378
3584/2187	$2^9 \cdot 7 / 3^7$	855.141
3584/3125	$2^9 \cdot 7 / 5^5$	237.257
3584/3375	$2^9 \cdot 7 / 3^3 \cdot 5^3$	104.020
3600/2401	$2^4 \cdot 3^2 \cdot 5^2 / 7^4$	701.234
3645/2401	$3^6 \cdot 5 / 7^4$	722.740
3645/2744	$3^6 \cdot 5 / 2^3 \cdot 7^3$	491.566
3645/3136	$3^6 \cdot 5 / 2^6 \cdot 7^2$	260.392
3645/3584	$3^6 \cdot 5 / 2^9 \cdot 7$	29.218
3675/2048	$3 \cdot 5^2 \cdot 7^2 / 2^{11}$	1012.234
3750/2401	$2 \cdot 3 \cdot 5^4 / 7^4$	771.906
3840/2401	$2^8 \cdot 3 \cdot 5 / 7^4$	812.965
3888/2401	$2^4 \cdot 3^5 / 7^4$	834.471
3920/2187	$2^4 \cdot 5 \cdot 7^2 / 3^7$	1010.281
3969/2500	$3^4 \cdot 7^2 / 2^2 \cdot 5^4$	800.217
3969/2560	$3^4 \cdot 7^2 / 2^9 \cdot 5$	759.158
3969/3125	$3^4 \cdot 7^2 / 5^5$	413.903
3969/3200	$3^4 \cdot 7^2 / 2^7 \cdot 5^2$	372.844
4000/2401	$2^5 \cdot 5^3 / 7^4$	883.637
4000/3087	$2^5 \cdot 5^3 / 3^2 \cdot 7^3$	448.553
4000/3969	$2^5 \cdot 5^3 / 3^4 \cdot 7^2$	13.469
4032/3125	$2^6 \cdot 3^2 \cdot 7 / 5^5$	441.167
4050/2401	$2 \cdot 3^4 \cdot 5^2 / 7^4$	905.144
4096/2205	$2^{12} / 3^2 \cdot 5 \cdot 7^2$	1072.124
4096/2401	$2^{12} / 7^4$	924.696
4096/2625	$2^{12} / 3 \cdot 5^3 \cdot 7$	770.278
4096/2835	$2^{12} / 3^4 \cdot 5 \cdot 7$	637.040
4096/3087	$2^{12} / 3^2 \cdot 7^3$	489.612
4096/3675	$2^{12} / 3 \cdot 5^2 \cdot 7^2$	187.766
4096/3969	$2^{12} / 3^4 \cdot 7^2$	54.528
4116/3125	$2^2 \cdot 3 \cdot 7^3 / 5^4$	476.864
4320/2401	$2^5 \cdot 3^3 \cdot 5 / 7^4$	1016.875
4374/2401	$2 \cdot 3^7 / 7^4$	1038.381
4375/2304	$5^4 \cdot 7 / 2^8 \cdot 3^2$	1110.171
4375/2592	$5^4 \cdot 7 / 2^5 \cdot 3^4$	906.261
4375/2916	$5^4 \cdot 7 / 2^2 \cdot 3^6$	702.351
4375/3072	$5^4 \cdot 7 / 2^{10} \cdot 3$	612.126
4375/3456	$5^4 \cdot 7 / 2^7 \cdot 3^3$	408.216
4375/3888	$5^4 \cdot 7 / 2^4 \cdot 3^5$	204.306
4375/4096	$5^4 \cdot 7 / 2^{12}$	114.081
4375/4374	$5^4 \cdot 7 / 2 \cdot 3^7$	0.396
4500/2401	$2^2 \cdot 3^2 \cdot 5^3 / 7^4$	1087.548
4536/3125	$2^3 \cdot 3^4 \cdot 7 / 5^5$	645.077
4608/2401	$2^9 \cdot 3^2 / 7^4$	1128.606
4608/4375	$2^9 \cdot 3^2 / 5^4 \cdot 7$	89.829
4704/3125	$2^5 \cdot 3 \cdot 7^2 / 5^5$	708.038
4725/4096	$3^3 \cdot 5^2 \cdot 7 / 2^{12}$	247.318
4800/2401	$2^6 \cdot 3 \cdot 5^2 / 7^4$	1199.279
4802/3125	$2 \cdot 7^4 / 5^5$	743.735
4802/3375	$2 \cdot 7^4 / 3^3 \cdot 5^3$	610.497
4802/3645	$2 \cdot 7^4 / 3^6 \cdot 5$	477.260
5000/3087	$2^3 \cdot 5^4 / 3^2 \cdot 7^3$	834.867
5000/3969	$2^3 \cdot 5^4 / 3^4 \cdot 7^2$	399.783

A characteristic 7-Limit Monochord : Double 60-120

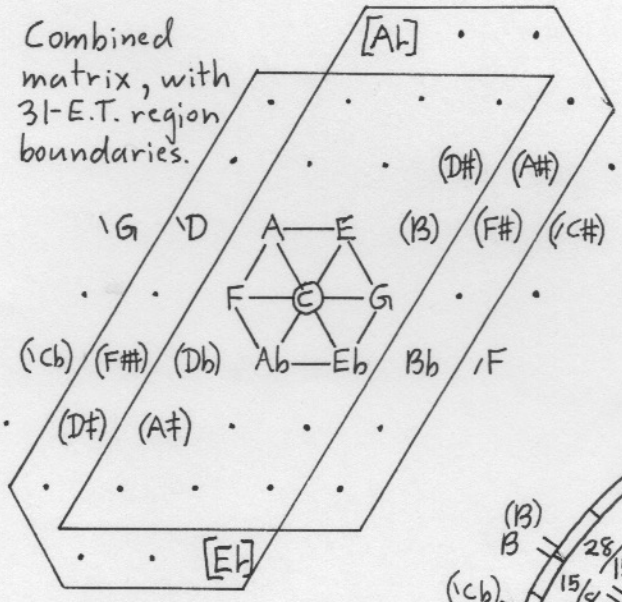
60	: (63) :	64	: (70) :	72	: 75 :	80	: 81 :	(84) :	90	: 96 :	[98] :	100	: (105) :	108	: (112) :	120
Up ~ C		↑C#	Db	D#	Eb	E	F	↑F	F#	G	Ab	A↓	A	A#	Bb	B	C
Down ~ C		↓Cb	B	A♯	A	Ab	G	↓G	F##	F	E	E↓	Eb	D♯	↓D	Db	C





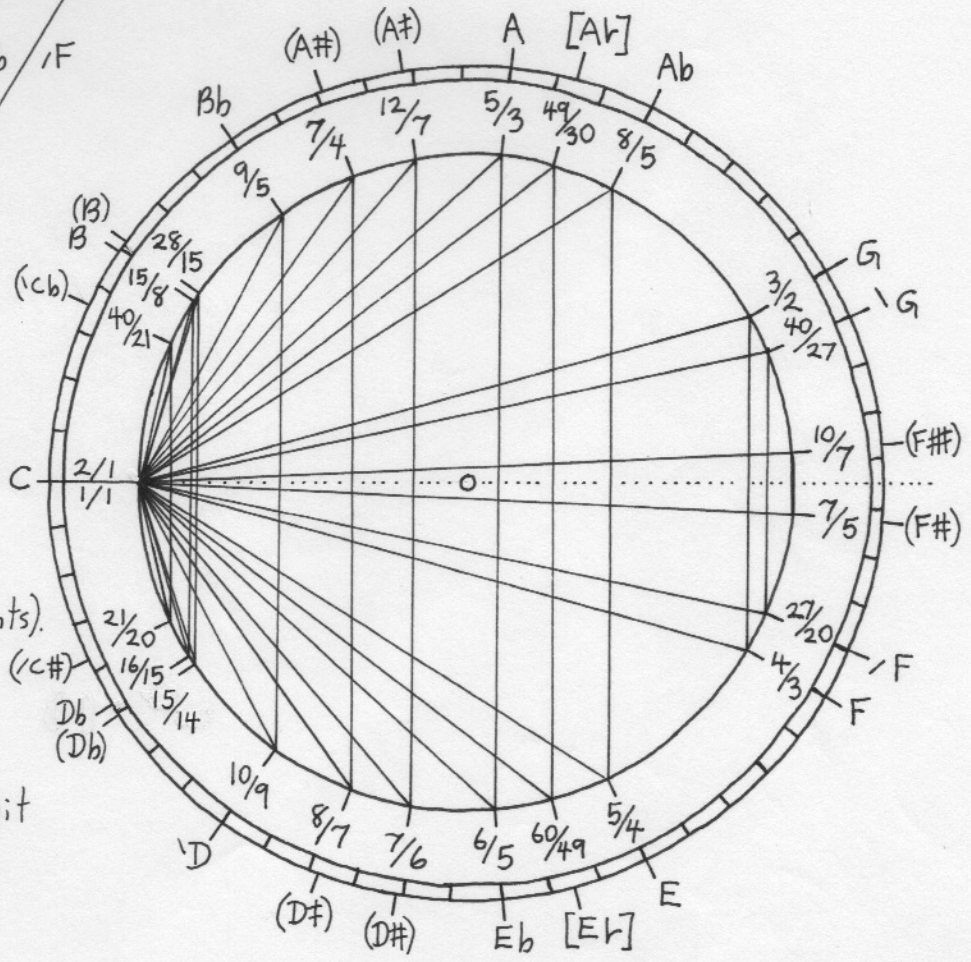
Tuning matrices, For the rising scale (left) and the falling scale (right). They show the "vertical" nature of 7-Limit tunings, tending to exhibit diesis relations and favouring a 31 division when considering temperament.

Combined matrix, with 31-E.T. region boundaries.



Septimal elements in the scale are bracketed. The element which embodies the second power septimal element is enclosed in a square bracket.

The circular graph is shown below :

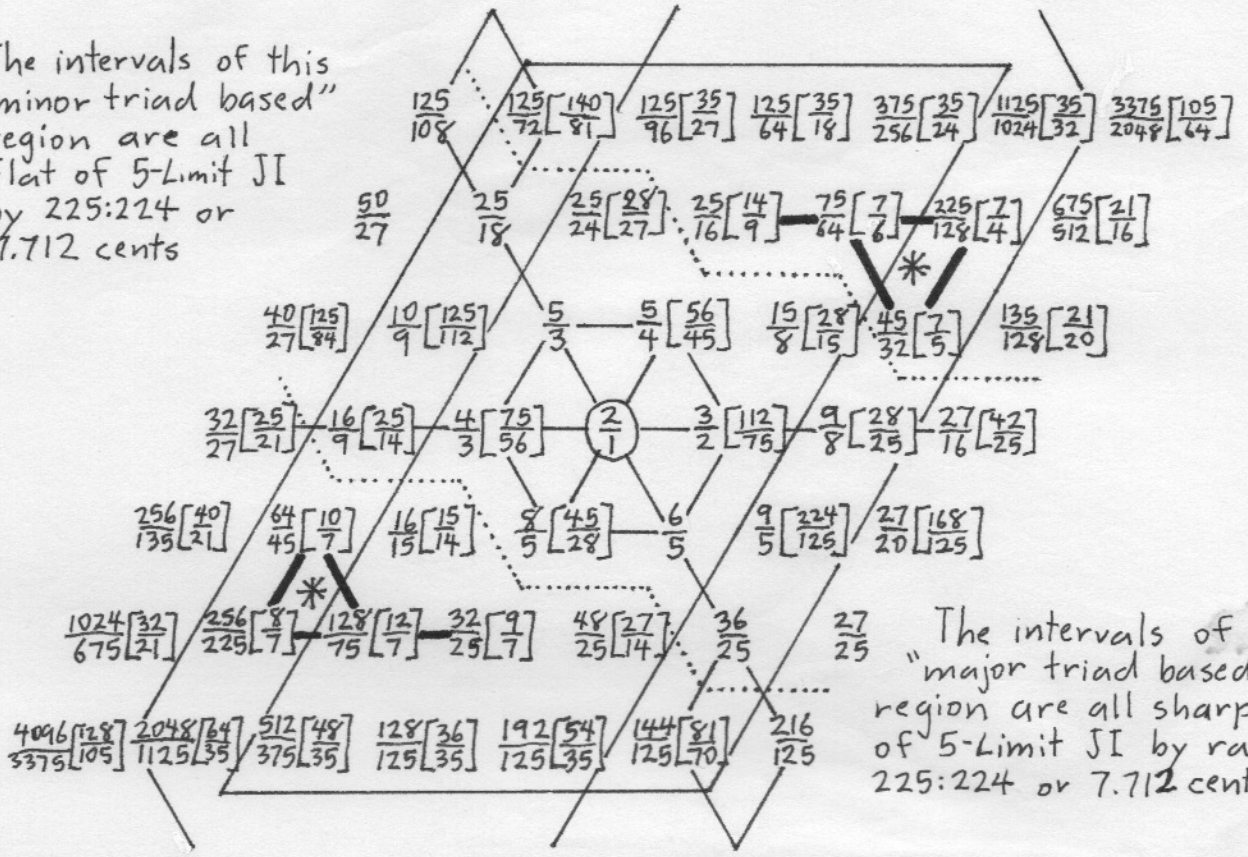


Note on the circular graph the pitches both labelled Db : 16/15 (111.731 cents) and 15/14 (119.443 cents). The microtone between them is the septimal ratio 225:224 (7.712 cents). This ratio "controls" the difference in pitch between the 5-Limit triangular graph and the 7-Limit modification.

Figure

The feasibility of **mapping 7-Limit ratios** on the two-dimensional grid. Only "upward" ratios are shown. Septimal substitutions are put in square brackets.

The intervals of this "minor triad based" region are all flat of 5-Limit JI by 225:224 or 7.712 cents

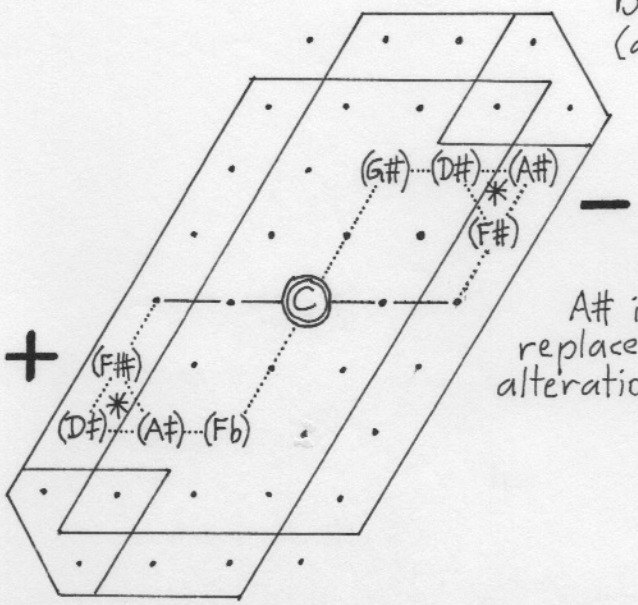


The intervals of this "major triad based" region are all sharp of 5-Limit JI by ratio 225:224 or 7.712 cents.

The central two zones of septimal "consonances" (smallest ratios) are highlighted. The substitutions become the centres for a septimal matrix expansion. This expansion has a natural check when the septimal ratios become larger than the 5-Limit ratios. For example, the septimal ratio 15:14 and the JI ratio 16:15 are almost balanced, but the septimal ratio 45:28 is decidedly inferior to 8:5. The dotted line shows the boundary between the two paradigms.

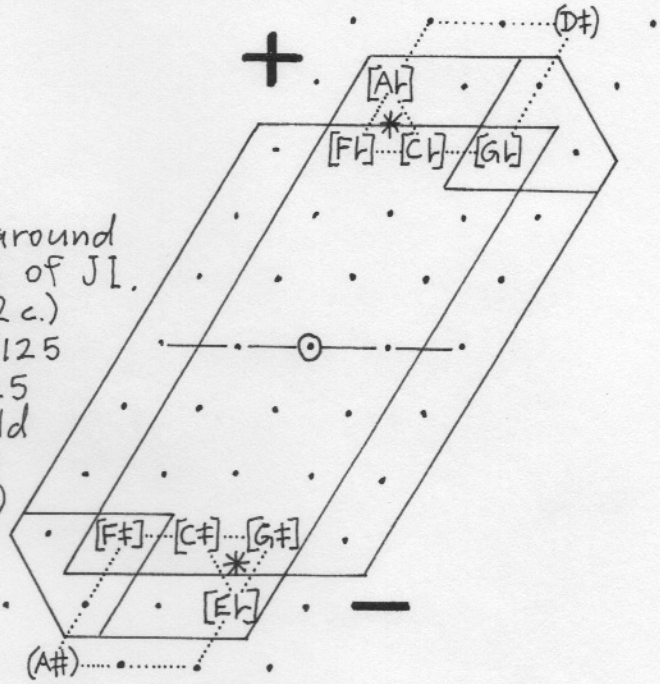
In tempered tuning systems a defined step number gives the representation of 7:4 (or any of the other septimal ratios). In most tempered systems which represent 7:4 well, the position is as above, ie in 31-E.T., 36-E.T., 41-E.T., 53-E.T., 72-E.T., 74-E.T., 84-E.T. It can, however, be placed in an alternative position below the South Gate, as in 26-E.T., 118-E.T., 171-E.T., 270-E.T., 441-E.T., and 612-E.T. Several good systems give the position in the schisma-lowered region, ie 46-E.T., 87-E.T., 99-E.T., and 152-E.T. 301-E.T. gives its own unique position for the step representing 7:4.

Two-dimensional matrix positions for ratios involving higher powers of 7 (that is, $7, 7^2, 7^3 \ldots$), showing the "vertical" orientation:

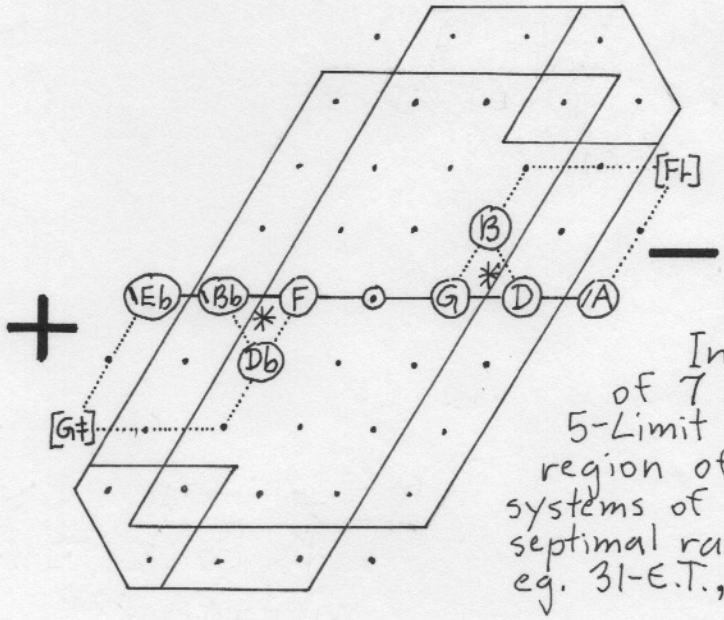


Boundary zones for Diesis resolution (and 31-E.T.) are marked.

The septimal zone ratios centre around a major and minor triad in two regions, which are either flat or sharp of their 5-Limit "cousins." For example: A# is septimal ratio 7/4 (968.826c.), which replaces just ratio 225/128 (976.537c.). The alteration is ratio 225/224 (7.712c.).



Ratios which embody 7^2 coalesce around the extremes of the Major Third Axis of JI. For example G♯ ratio 49/32 (737.652c.) is somewhat flat of the 5-Limit 192/125 (743.014c.). The alteration is 6144/6125 (5.362c.). Note that this position could also be occupied (by the matrix above) by a septimal ratio 54/35 (750.725c.) involving one power of seven.

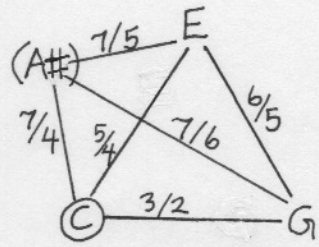


Ratios which embody 7^3 coalesce around the "comma boundary" (the meantone corridor). For example F, ratio 343/256 (506.478c.) is an alteration of Just ratio 4/3 (498.045c.). The difference is ratio 1029/1024 (8.433c.).

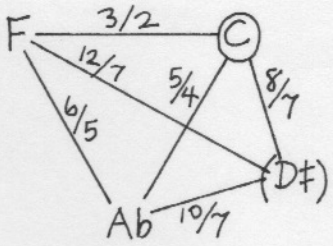
In a similar way, higher powers of 7 may be "substituted" into the 5-Limit Fabric, always within the general region of the 31-E.T. boundaries. Hence systems of temperament which represent septimal ratios well tend to be "vertical," eg. 31-E.T., 41-E.T., 53-E.T., 72-E.T., and 84-E.T.

A Three-dimensional tuning matrix for 7-Limit ratios:

Although 7-Limit ratios may be adequately integrated into the 5-Limit tuning matrix, and although this approach is preferable from the standpoint of practicality, a 7-Limit matrix can be consistently constructed by converting the triangular array into a tetrahedral array, based on the Harmonic Series tetrad and its reciprocal:

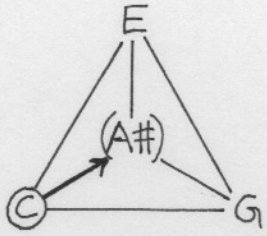


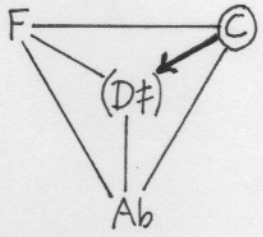
Harmonic Series Tetrahedron



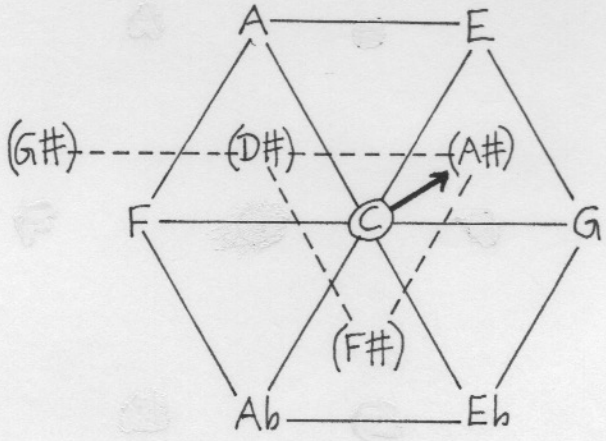
Sub-harmonic Series Tetrahedron

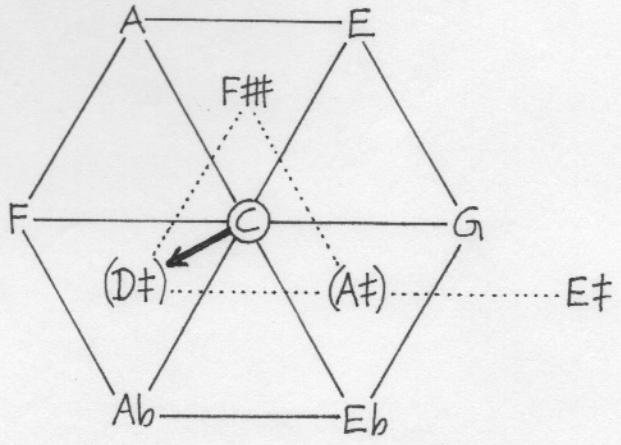
The mapping of this three-dimensional matrix may be simplified by using a "vertical orientation" so that the above harmonies become:

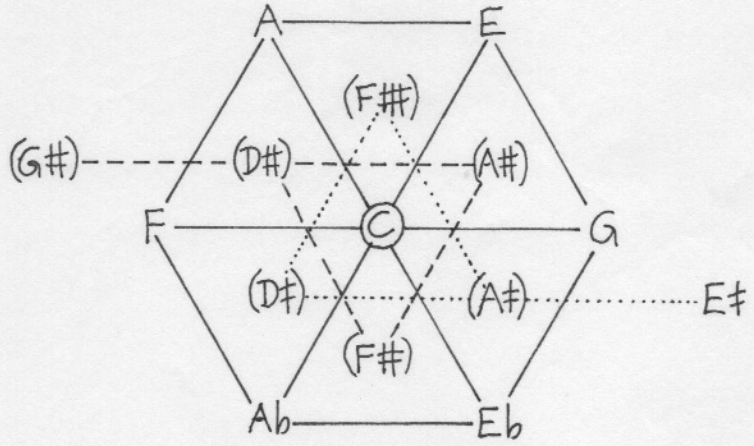




These relations may be extended in a similar manner to the 5-Limit matrix:







As shown left, the over and under series may be combined so that the 5-Limit region and the two 7-Limit regions may be integrated into a close-packed array.

Ratios using further powers of 7 (eg. 7×7) add further levels of complexity.

Numbers of the family grouping $2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot 11^e$ (**11-Limit** or Partchian)

1	*
2	*
3	*
4	2^2
5	*
6	$2 \cdot 3$
7	*
8	2^3
9	3^2
10	$2 \cdot 5$
11	*
12	$2^2 \cdot 3$
14	$2 \cdot 7$
15	$3 \cdot 5$
16	2^4
18	$2 \cdot 3^2$
20	$2^2 \cdot 5$
21	$3 \cdot 7$
22	$2 \cdot 11$
24	$2^3 \cdot 3$
25	5^2
27	3^3
28	$2^2 \cdot 7$
30	$2 \cdot 3 \cdot 5$
32	2^5
33	$3 \cdot 11$
35	$5 \cdot 7$
36	$2^2 \cdot 3^2$
40	$2^3 \cdot 5$
42	$2 \cdot 3 \cdot 7$
44	$2^2 \cdot 11$
45	$3^2 \cdot 5$
48	$2^4 \cdot 3$
49	7^2
50	$2 \cdot 5^2$
54	$2 \cdot 3^3$
55	$5 \cdot 11$
56	$2^3 \cdot 7$
60	$2^2 \cdot 3 \cdot 5$
63	$3^2 \cdot 7$
64	2^6
66	$2 \cdot 3 \cdot 11$
70	$2 \cdot 5 \cdot 7$
72	$2^3 \cdot 3^2$
75	$3 \cdot 5^2$
77	$7 \cdot 11$
80	$2^4 \cdot 5$
81	3^4
84	$2^2 \cdot 3 \cdot 7$
88	$2^3 \cdot 11$
90	$2 \cdot 3^2 \cdot 5$
96	$2^5 \cdot 3$
98	$2 \cdot 7^2$
99	$3^2 \cdot 11$
100	$2^2 \cdot 5^2$
105	$3 \cdot 5 \cdot 7$
108	$2^2 \cdot 3^3$
110	$2 \cdot 5 \cdot 11$
112	$2^4 \cdot 7$
120	$2^3 \cdot 3 \cdot 5$
121	11^2
125	5^3
126	$2 \cdot 3^2 \cdot 7$
128	2^7
132	$2^2 \cdot 3 \cdot 11$
135	$3^3 \cdot 5$
140	$2^2 \cdot 5 \cdot 7$
144	$2^4 \cdot 3^2$
147	$3 \cdot 7^2$
150	$2 \cdot 3 \cdot 5^2$
154	$2 \cdot 7 \cdot 11$
160	$2^5 \cdot 5$
162	$2 \cdot 3^4$
165	$3 \cdot 5 \cdot 11$
168	$2^3 \cdot 3 \cdot 7$
175	$5^2 \cdot 7$
176	$2^4 \cdot 11$
180	$2^2 \cdot 3^2 \cdot 5$
189	$3^3 \cdot 7$
192	$2^6 \cdot 3$
196	$2^2 \cdot 7^2$
198	$2 \cdot 3^2 \cdot 11$
200	$2^3 \cdot 5^2$
210	$2 \cdot 3 \cdot 5 \cdot 7$
216	$2^3 \cdot 3^3$
220	$2^2 \cdot 5 \cdot 11$
224	$2^5 \cdot 7$
225	$3^2 \cdot 5^2$
231	$3 \cdot 7 \cdot 11$
240	$2^4 \cdot 3 \cdot 5$
242	$2 \cdot 11^2$
243	3^5
245	$5 \cdot 7^2$
250	$2 \cdot 5^3$
252	$2^2 \cdot 3^2 \cdot 7$
256	2^8
264	$2^3 \cdot 3 \cdot 11$
270	$2 \cdot 3^3 \cdot 5$
275	$5^2 \cdot 11$
280	$2^3 \cdot 5 \cdot 7$
288	$2^5 \cdot 3^2$
294	$2 \cdot 3 \cdot 7^2$
297	$3^3 \cdot 11$
300	$2^2 \cdot 3 \cdot 5^2$
308	$2^2 \cdot 7 \cdot 11$
315	$3^2 \cdot 5 \cdot 7$
320	$2^6 \cdot 5$
324	$2^2 \cdot 3^4$
330	$2 \cdot 3 \cdot 5 \cdot 11$
336	$2^4 \cdot 3 \cdot 7$
343	7^3
350	$2 \cdot 5^2 \cdot 7$
352	$2^5 \cdot 11$
360	$2^3 \cdot 3^2 \cdot 5$
363	$3 \cdot 11^2$
375	$3 \cdot 5^3$
378	$2 \cdot 3^3 \cdot 7$
384	$2^7 \cdot 3$
385	$5 \cdot 7 \cdot 11$
392	$2^3 \cdot 7^2$
396	$2^2 \cdot 3^2 \cdot 11$
400	$2^4 \cdot 5^2$
405	$3^4 \cdot 5$
420	$2^2 \cdot 3 \cdot 5 \cdot 7$
432	$2^4 \cdot 3^3$
440	$2^3 \cdot 5 \cdot 11$
448	$2^6 \cdot 7$
450	$2 \cdot 3^2 \cdot 5^2$
462	$2 \cdot 3 \cdot 7 \cdot 11$
480	$2^5 \cdot 3 \cdot 5$
484	$2^2 \cdot 11^2$
486	$2 \cdot 3^5$
490	$2 \cdot 5 \cdot 7^2$
495	$3^2 \cdot 5 \cdot 11$
500	$2^2 \cdot 5^3$
504	$2^3 \cdot 3^2 \cdot 7$
512	2^9
525	$3 \cdot 5^2 \cdot 7$
528	$2^4 \cdot 3 \cdot 11$
539	$7^2 \cdot 11$
540	$2^2 \cdot 3^3 \cdot 5$
550	$2 \cdot 5^2 \cdot 11$
560	$2^4 \cdot 5 \cdot 7$
567	$3^4 \cdot 7$
576	$2^6 \cdot 3^2$
588	$2^2 \cdot 3 \cdot 7^2$
594	$2 \cdot 3^3 \cdot 11$
600	$2^3 \cdot 3 \cdot 5^2$
605	$5 \cdot 11^2$
616	$2^3 \cdot 7 \cdot 11$
625	5^4
630	$2 \cdot 3^2 \cdot 5 \cdot 7$
640	$2^7 \cdot 5$
648	$2^3 \cdot 3^4$
660	$2^2 \cdot 3 \cdot 5 \cdot 11$
672	$2^5 \cdot 3 \cdot 7$
675	$3^3 \cdot 5^2$
686	$2 \cdot 7^3$
693	$3^2 \cdot 7 \cdot 11$
700	$2^2 \cdot 5^2 \cdot 7$
704	$2^6 \cdot 11$
720	$2^4 \cdot 3^2 \cdot 5$
726	$2 \cdot 3 \cdot 11^2$
729	3^6
735	$3 \cdot 5 \cdot 7^2$
750	$2 \cdot 3 \cdot 5^3$
756	$2^2 \cdot 3^3 \cdot 7$
768	$2^8 \cdot 3$
770	$2 \cdot 5 \cdot 7 \cdot 11$
784	$2^4 \cdot 7^2$
792	$2^3 \cdot 3^2 \cdot 11$
800	$2^5 \cdot 5^2$
810	$2 \cdot 3^4 \cdot 5$
825	$3 \cdot 5^2 \cdot 11$
840	$2^3 \cdot 3 \cdot 5 \cdot 7$
847	$7 \cdot 11^2$
864	$2^5 \cdot 3^3$
875	$5^3 \cdot 7$
880	$2^4 \cdot 5 \cdot 11$
882	$2 \cdot 3^2 \cdot 7^2$
891	$3^4 \cdot 11$
896	$2^7 \cdot 7$
900	$2^2 \cdot 3^2 \cdot 5^2$
924	$2^2 \cdot 3 \cdot 7 \cdot 11$
945	$3^3 \cdot 5 \cdot 7$
960	$2^6 \cdot 3 \cdot 5$
968	$2^3 \cdot 11^2$
972	$2^2 \cdot 3^5$
980	$2^2 \cdot 5 \cdot 7^2$
990	$2 \cdot 3^2 \cdot 5 \cdot 11$
1000	$2^3 \cdot 5^3$
1008	$2^4 \cdot 3^2 \cdot 7$
1024	2^{10}
1029	$3 \cdot 7^3$
1050	$2 \cdot 3 \cdot 5^2 \cdot 7$
1056	$2^5 \cdot 3 \cdot 11$
1078	$2 \cdot 7^2 \cdot 11$
1080	$2^3 \cdot 3^3 \cdot 5$
1089	$3^2 \cdot 11^2$
1100	$2^2 \cdot 5^2 \cdot 11$
1120	$2^5 \cdot 5 \cdot 7$
1125	$3^2 \cdot 5^3$
1134	$2 \cdot 3^4 \cdot 7$
1152	$2^7 \cdot 3^2$
1155	$3 \cdot 5 \cdot 7 \cdot 11$
1176	$2^3 \cdot 3 \cdot 7^2$
1188	$2^2 \cdot 3^3 \cdot 11$
1200	$2^4 \cdot 3 \cdot 5^2$

$441 = 3^2 \cdot 7^2$

11-Limit Ratios (between 1/1 and 2/1) not found in the 7-Limit or 5-Limit:

Ratio	Prime Factors	Cents	Pitch
11/6	$11/2\cdot3$	1049.363	
11/7	$11/7$	782.492	
11/8	$11/2^3$	551.318	
11/9	$11/3^2$	347.408	
11/10	$11/2\cdot5$	165.004	
12/11	$2^2\cdot3/11$	150.637	
14/11	$2\cdot7/11$	417.508	
15/11	$3\cdot5/11$	536.951	
16/11	$2^4/11$	648.682	
18/11	$2\cdot3^2/11$	852.592	
20/11	$2^2\cdot5/11$	1034.996	
21/11	$3\cdot7/11$	1119.463	
22/15	$2\cdot11/3\cdot5$	663.049	
22/21	$2\cdot11/3\cdot7$	80.537	
25/22	$5^2/2\cdot11$	221.309	
27/22	$3^3/2\cdot11$	354.547	
33/20	$3\cdot11/2^2\cdot5$	866.959	
33/25	$3\cdot11/5^2$	480.646	
33/28	$3\cdot11/2^2\cdot7$	284.447	
33/32	$3\cdot11/2^5$	53.273	
35/22	$5\cdot7/2\cdot11$	803.822	
35/33	$5\cdot7/3\cdot11$	101.867	
40/33	$2^3\cdot5/3\cdot11$	333.041	
44/25	$2^2\cdot11/5^2$	978.690	
44/27	$2^2\cdot11/3^3$	845.453	
44/35	$2^2\cdot11/5\cdot7$	396.178	
45/44	$3^2\cdot5/2^2\cdot11$	38.906	
49/33	$7^2/3\cdot11$	684.379	
49/44	$7^2/2^2\cdot11$	186.334	
50/33	$2\cdot5^2/3\cdot11$	719.354	
55/28	$5\cdot11/2^2\cdot7$	1168.806	
55/32	$5\cdot11/2^5$	937.632	
55/36	$5\cdot11/2^2\cdot3^2$	733.722	
55/42	$5\cdot11/2\cdot3\cdot7$	466.851	
55/48	$5\cdot11/2^4\cdot3$	235.677	
55/49	$5\cdot11/7^2$	199.980	
55/54	$5\cdot11/2\cdot3^3$	31.767	
56/33	$2^3\cdot7/3\cdot11$	915.553	
56/55	$2^3\cdot7/5\cdot11$	31.194	
63/44	$3^2\cdot7/2^2\cdot11$	621.418	
63/55	$3^2\cdot7/5\cdot11$	235.104	
64/33	$2^6/3\cdot11$	1146.727	
64/55	$2^6/5\cdot11$	262.368	
66/35	$2\cdot3\cdot11/5\cdot7$	1098.133	
66/49	$2\cdot3\cdot11/7^2$	515.621	
72/55	$2^3\cdot3^2/5\cdot11$	466.278	
75/44	$3\cdot5^2/2^2\cdot11$	923.264	
77/40	$7\cdot11/2^3\cdot5$	1133.830	
77/45	$7\cdot11/3^2\cdot5$	929.920	
77/48	$7\cdot11/2^4\cdot3$	818.189	
77/50	$7\cdot11/2\cdot5^2$	747.516	
77/54	$7\cdot11/2\cdot3^3$	614.279	
77/60	$7\cdot11/2^2\cdot3\cdot5$	431.875	
77/64	$7\cdot11/2^6$	320.144	
77/72	$7\cdot11/2^3\cdot3^2$	116.234	
77/75	$7\cdot11/3\cdot5^2$	45.561	
80/77	$2^4\cdot5/7\cdot11$	66.170	
81/44	$3^4/2^2\cdot11$	1056.502	
81/55	$3^4/5\cdot11$	670.188	
81/77	$3^4/7\cdot11$	87.676	
84/55	$2^2\cdot3\cdot7/5\cdot11$	733.149	
88/45	$2^3\cdot11/3^2\cdot5$	1161.094	
88/49	$2^3\cdot11/7^2$	1013.666	
88/63	$2^3\cdot11/3^2\cdot7$	578.582	
88/75	$2^3\cdot11/3\cdot5^2$	276.736	
88/81	$2^3\cdot11/3^4$	143.498	
90/77	$2\cdot3^2\cdot5/7\cdot11$	270.080	
96/55	$2^5\cdot3/5\cdot11$	964.323	
96/77	$2^5\cdot3/7\cdot11$	381.811	
98/55	$2\cdot7^2/5\cdot11$	1000.020	
99/50	$3^2\cdot11/2\cdot5^2$	1182.601	
99/54	$3^2\cdot11/2\cdot3^3$	1049.363	
99/56	$3^2\cdot11/2^3\cdot7$	986.402	
99/64	$3^2\cdot11/2^6$	755.228	
99/70	$3^2\cdot11/2\cdot5\cdot7$	600.088	
99/80	$3^2\cdot11/2^4\cdot5$	368.914	
99/98	$3^2\cdot11/2\cdot7^2$	17.576	
100/77	$2^2\cdot5^2/7\cdot11$	452.484	
100/99	$2^2\cdot5^2/3^2\cdot11$	17.399	
105/88	$3\cdot5\cdot7/2^3\cdot11$	305.777	
108/55	$2^2\cdot3^3/5\cdot11$	1168.233	
108/77	$2^2\cdot3^3/7\cdot11$	585.721	
110/63	$2\cdot5\cdot11/3^2\cdot7$	964.896	
110/81	$2\cdot5\cdot11/3^4$	529.812	
112/99	$2^4\cdot7/3^2\cdot11$	213.598	
120/77	$2^3\cdot3\cdot5/7\cdot11$	768.125	
121/63	$11^2/3^2\cdot7$	1129.900	
121/64	$11^2/2^6$	1102.636	
121/70	$11^2/2\cdot5\cdot7$	947.496	
121/72	$11^2/2^3\cdot3^2$	898.726	
121/75	$11^2/3\cdot5^2$	828.053	
121/80	$11^2/2^4\cdot5$	716.322	
121/81	$11^2/3^4$	694.816	

continued over page

121/84	$11^2/2^2\cdot3\cdot7$	631.855
121/90	$11^2/2\cdot3^2\cdot5$	512.412
121/96	$11^2/2^5\cdot3$	400.681
121/98	$11^2/2\cdot7^2$	364.984
121/100	$11^2/2^2\cdot5^2$	330.008
121/105	$11^2/3\cdot5\cdot7$	245.541
121/108	$11^2/2^2\cdot3^3$	196.771
121/112	$11^2/2^4\cdot7$	133.810
121/120	$11^2/2^3\cdot3\cdot5$	14.367
125/66	$5^3/2\cdot3\cdot11$	1105.668
125/77	$5^3/7\cdot11$	838.797
125/88	$5^3/2^3\cdot11$	607.623
125/99	$5^3/3^2\cdot11$	403.713
125/121	$5^3/11^2$	56.305
126/121	$2\cdot3^2\cdot7/11^2$	70.100
128/77	$2^7/7\cdot11$	879.856
128/99	$2^7/3^2\cdot11$	444.772
128/121	$2^7/11^2$	97.364
132/125	$2^2\cdot3\cdot11/5^3$	94.332
135/77	$3^3\cdot5/7\cdot11$	972.035
135/88	$3^3\cdot5/2^3\cdot11$	740.861
135/121	$3^3\cdot5/11^2$	189.543
140/99	$2^2\cdot5\cdot7/3^2\cdot11$	599.912
140/121	$2^2\cdot5\cdot7/11^2$	252.504
144/77	$2^4\cdot3^2/7\cdot11$	1083.766
144/121	$2^4\cdot3^2/11^2$	301.274
147/88	$3\cdot7^2/2^3\cdot11$	888.289
147/110	$3\cdot7^2/2\cdot5\cdot11$	501.975
147/121	$3\cdot7^2/11^2$	336.971
150/77	$2\cdot3\cdot5^2/7\cdot11$	1154.439
150/121	$2\cdot3\cdot5^2/11^2$	371.947
154/81	$2\cdot7\cdot11/3^4$	1112.324
154/125	$2\cdot7\cdot11/5^3$	361.203
154/135	$2\cdot7\cdot11/3^3\cdot5$	227.965
160/99	$2^5\cdot5/3^2\cdot11$	831.086
160/121	$2^5\cdot5/11^2$	483.678
162/121	$2\cdot3^4/11^2$	505.184
165/98	$3\cdot5\cdot11/2\cdot7^2$	901.935
165/112	$3\cdot5\cdot11/2^4\cdot7$	670.761
165/128	$3\cdot5\cdot11/2^7$	439.587
168/121	$2^3\cdot3\cdot7/11^2$	568.145
175/88	$5^2\cdot7/2^3\cdot11$	1190.135
175/99	$5^2\cdot7/3^2\cdot11$	986.225
175/121	$5^2\cdot7/11^2$	638.817
175/132	$5^2\cdot7/2^2\cdot3\cdot11$	488.180
176/105	$2^4\cdot11/3\cdot5\cdot7$	894.223
176/125	$2^4\cdot11/5^3$	592.377
176/135	$2^4\cdot11/3^3\cdot5$	459.139
176/147	$2^4\cdot11/3\cdot7^2$	311.711
176/175	$2^4\cdot11/5^2\cdot7$	9.865
180/121	$2^2\cdot3^2\cdot5/11^2$	687.588
189/110	$3^3\cdot7/2\cdot5\cdot11$	937.059
189/121	$3^3\cdot7/11^2$	772.055
189/176	$3^3\cdot7/2^4\cdot11$	123.373
192/121	$2^6\cdot3/11^2$	799.319
196/99	$2^2\cdot7^2/3^2\cdot11$	1182.424
196/121	$2^2\cdot7^2/11^2$	835.016
196/165	$2^2\cdot7^2/3\cdot5\cdot11$	298.065
198/125	$2\cdot3^2\cdot11/5^3$	796.287
198/175	$2\cdot3^2\cdot11/5^2\cdot7$	213.775
200/121	$2^3\cdot5^2/11^2$	869.992
210/121	$2\cdot3\cdot5\cdot7/11^2$	954.459
216/121	$2^3\cdot3^3/11^2$	1003.229
220/147	$2^2\cdot5\cdot11/3\cdot7^2$	698.025
220/189	$2^2\cdot5\cdot11/3^3\cdot7$	262.941
224/121	$2^5\cdot7/11^2$	1066.190
224/165	$2^5\cdot7/3\cdot5\cdot11$	529.239
225/121	$3^2\cdot5^2/11^2$	1073.902
225/154	$3^2\cdot5^2/2\cdot7\cdot11$	656.394
225/176	$3^2\cdot5^2/2^4\cdot11$	425.219
231/125	$3\cdot7\cdot11/5^3$	1063.158
231/128	$3\cdot7\cdot11/2^7$	1022.099
231/160	$3\cdot7\cdot11/2^5\cdot5$	635.785
231/200	$3\cdot7\cdot11/2^3\cdot5^2$	249.471
240/121	$2^4\cdot3\cdot5/11^2$	1185.633
242/125	$2\cdot11^2/5^3$	1143.695
242/135	$2\cdot11^2/3^3\cdot5$	1010.457
242/147	$2\cdot11^2/3\cdot7^2$	863.029
242/175	$2\cdot11^2/5^2\cdot7$	561.183
242/189	$2\cdot11^2/3^3\cdot7$	427.945
242/225	$2\cdot11^2/3^2\cdot5^2$	126.098
243/154	$3^5/2\cdot7\cdot11$	789.631
243/176	$3^5/2^4\cdot11$	558.457
243/220	$3^5/2^2\cdot5\cdot11$	172.143
243/242	$3^5/2\cdot11^2$	7.139
245/132	$5\cdot7^2/2^2\cdot3\cdot11$	1070.693
245/176	$5\cdot7^2/2^4\cdot11$	572.648
245/198	$5\cdot7^2/2\cdot3^2\cdot11$	368.738
245/242	$5\cdot7^2/2\cdot11^2$	21.330
250/231	$2\cdot5^3/3\cdot7\cdot11$	136.842
256/165	$2^8/3\cdot5\cdot11$	760.413
256/231	$2^8/3\cdot7\cdot11$	177.901
264/175	$2^3\cdot3\cdot11/5^2\cdot7$	711.820
264/245	$2^3\cdot3\cdot11/5\cdot7^2$	129.307
275/144	$5^2\cdot11/2^4\cdot3^2$	1120.035
275/147	$5^2\cdot11/3\cdot7^2$	1084.339
275/162	$5^2\cdot11/2\cdot3^4$	916.125
275/168	$5^2\cdot11/2^3\cdot3\cdot7$	853.164

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275/189	$5^2 \cdot 11 / 3^3 \cdot 7$	649.254
275/192	$5^2 \cdot 11 / 2^6 \cdot 3$	621.990
275/196	$5^2 \cdot 11 / 2^2 \cdot 7^2$	586.294
275/216	$5^2 \cdot 11 / 2^3 \cdot 3^3$	418.080
275/224	$5^2 \cdot 11 / 2^5 \cdot 7$	355.119
275/243	$5^2 \cdot 11 / 3^5$	214.170
275/252	$5^2 \cdot 11 / 2^2 \cdot 3^2 \cdot 7$	151.209
275/256	$5^2 \cdot 11 / 2^8$	123.945
288/275	$2^5 \cdot 3^2 / 5^2 \cdot 11$	79.965
294/275	$2 \cdot 3 \cdot 7^2 / 5^2 \cdot 11$	115.661
297/160	$3^3 \cdot 11 / 2^5 \cdot 5$	1070.869
297/175	$3^3 \cdot 11 / 5^2 \cdot 7$	915.730
297/196	$3^3 \cdot 11 / 2^2 \cdot 7^2$	719.531
297/200	$3^3 \cdot 11 / 2^3 \cdot 5^2$	684.555
297/224	$3^3 \cdot 11 / 2^5 \cdot 7$	488.357
297/245	$3^3 \cdot 11 / 5 \cdot 7^2$	333.217
297/250	$3^3 \cdot 11 / 2 \cdot 5^3$	298.242
297/256	$3^3 \cdot 11 / 2^8$	257.183
297/280	$3^3 \cdot 11 / 2^3 \cdot 5 \cdot 7$	102.043
308/225	$2^2 \cdot 7 \cdot 11 / 3^2 \cdot 5^2$	543.606
308/243	$2^2 \cdot 7 \cdot 11 / 3^5$	410.369
315/176	$3^2 \cdot 5 \cdot 7 / 2^4 \cdot 11$	1007.732
315/242	$3^2 \cdot 5 \cdot 7 / 2 \cdot 11^2$	456.414
320/231	$2^6 \cdot 5 / 3 \cdot 7 \cdot 11$	564.215
320/297	$2^6 \cdot 5 / 3^3 \cdot 11$	129.131
324/275	$2^2 \cdot 3^4 / 5^2 \cdot 11$	283.875
336/275	$2^4 \cdot 3 \cdot 7 / 5^2 \cdot 11$	346.836
343/176	$7^3 / 2^4 \cdot 11$	1155.160
343/198	$7^3 / 2 \cdot 3^2 \cdot 11$	951.250
343/220	$7^3 / 2^2 \cdot 5 \cdot 11$	768.846
343/242	$7^3 / 2 \cdot 11^2$	603.842
343/264	$7^3 / 2^3 \cdot 3 \cdot 11$	453.205
343/275	$7^3 / 5^2 \cdot 11$	382.532
343/297	$7^3 / 3^3 \cdot 11$	249.295
343/330	$7^3 / 2 \cdot 3 \cdot 5 \cdot 11$	66.891
350/297	$2 \cdot 5^2 \cdot 7 / 3^3 \cdot 11$	284.270
352/189	$2^5 \cdot 11 / 3^3 \cdot 7$	1076.627
352/225	$2^5 \cdot 11 / 3^2 \cdot 5^2$	774.780
352/243	$2^5 \cdot 11 / 3^5$	641.543
352/245	$2^5 \cdot 11 / 5 \cdot 7^2$	627.352
352/315	$2^5 \cdot 11 / 3^2 \cdot 5 \cdot 7$	192.268
352/343	$2^5 \cdot 11 / 7^3$	44.840
363/196	$3 \cdot 11^2 / 2^2 \cdot 7^2$	1066.939
363/200	$3 \cdot 11^2 / 2^3 \cdot 5^2$	1031.963
363/224	$3 \cdot 11^2 / 2^5 \cdot 7$	835.765
363/245	$3 \cdot 11^2 / 5 \cdot 7^2$	680.625
363/250	$3 \cdot 11^2 / 2 \cdot 5^3$	645.650
363/256	$3 \cdot 11^2 / 2^8$	604.591
363/280	$3 \cdot 11^2 / 2^3 \cdot 5 \cdot 7$	449.451
363/320	$3 \cdot 11^2 / 2^6 \cdot 5$	218.277
363/343	$3 \cdot 11^2 / 7^3$	98.113
363/350	$3 \cdot 11^2 / 2 \cdot 5^2 \cdot 7$	63.138
375/242	$3 \cdot 5^3 / 2 \cdot 11^2$	758.260
375/308	$3 \cdot 5^3 / 2^2 \cdot 7 \cdot 11$	340.752
375/352	$3 \cdot 5^3 / 2^5 \cdot 11$	109.578
378/275	$2 \cdot 3^3 \cdot 7 / 5^2 \cdot 11$	550.746
384/275	$2^7 \cdot 3 / 5^2 \cdot 11$	578.010
385/216	$5 \cdot 7 \cdot 11 / 2^3 \cdot 3^3$	1000.593
385/243	$5 \cdot 7 \cdot 11 / 3^5$	796.683
385/256	$5 \cdot 7 \cdot 11 / 2^8$	706.458
385/288	$5 \cdot 7 \cdot 11 / 2^5 \cdot 3^2$	502.548
385/324	$5 \cdot 7 \cdot 11 / 2^2 \cdot 3^4$	298.638
385/384	$5 \cdot 7 \cdot 11 / 2^7 \cdot 3$	4.503
392/275	$2^3 \cdot 7^2 / 5^2 \cdot 11$	613.706
392/297	$2^3 \cdot 7^2 / 3^3 \cdot 11$	480.469
392/363	$2^3 \cdot 7^2 / 3 \cdot 11^2$	133.061
396/245	$2^2 \cdot 3^2 \cdot 11 / 5 \cdot 7^2$	831.262
396/343	$2^2 \cdot 3^2 \cdot 11 / 7^3$	248.750
400/231	$2^4 \cdot 5^2 / 3 \cdot 7 \cdot 11$	950.529
400/297	$2^4 \cdot 5^2 / 3^3 \cdot 11$	515.444
400/363	$2^4 \cdot 5^2 / 3 \cdot 11^2$	168.037
405/242	$3^4 \cdot 5 / 2 \cdot 11^2$	891.498
405/308	$3^4 \cdot 5 / 2^2 \cdot 7 \cdot 11$	473.990
405/352	$3^4 \cdot 5 / 2^5 \cdot 11$	242.816
432/275	$2^4 \cdot 3^3 / 5^2 \cdot 11$	781.920
432/385	$2^4 \cdot 3^3 / 5 \cdot 7 \cdot 11$	199.407
440/243	$2^3 \cdot 5 \cdot 11 / 3^5$	1027.857
440/343	$2^3 \cdot 5 \cdot 11 / 7^3$	431.154
448/275	$2^6 \cdot 7 / 5^2 \cdot 11$	844.881
448/297	$2^6 \cdot 7 / 3^3 \cdot 11$	711.643
448/363	$2^6 \cdot 7 / 3 \cdot 11^2$	364.235
484/243	$2^2 \cdot 11^2 / 3^5$	1192.861
484/245	$2^2 \cdot 11^2 / 5 \cdot 7^2$	1178.670
484/315	$2^2 \cdot 11^2 / 3^2 \cdot 5 \cdot 7$	743.586
484/343	$2^2 \cdot 11^2 / 7^3$	596.158
484/375	$2^2 \cdot 11^2 / 3 \cdot 5^3$	441.740
484/405	$2^2 \cdot 11^2 / 3^4 \cdot 5$	308.502
486/275	$2 \cdot 3^5 / 5^2 \cdot 11$	985.830
486/385	$2 \cdot 3^5 / 5 \cdot 7 \cdot 11$	403.317
490/297	$2 \cdot 5 \cdot 7^2 / 3^3 \cdot 11$	866.783
490/363	$2 \cdot 5 \cdot 7^2 / 3 \cdot 11^2$	519.375
495/256	$3^2 \cdot 5 \cdot 11 / 2^8$	1141.542
495/343	$3^2 \cdot 5 \cdot 11 / 7^3$	635.064
495/392	$3^2 \cdot 5 \cdot 11 / 2^3 \cdot 7^2$	403.890
495/448	$3^2 \cdot 5 \cdot 11 / 2^6 \cdot 7$	172.716
500/297	$2^2 \cdot 5^3 / 3^3 \cdot 11$	901.758
500/363	$2^2 \cdot 5^3 / 3 \cdot 11^2$	554.350
504/275	$2^3 \cdot 3^2 \cdot 7 / 5^2 \cdot 11$	1048.791

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512/275	$2^9/5^2\cdot 11$	1076.055
512/297	$2^9/3^3\cdot 11$	942.817
512/363	$2^9/3\cdot 11^2$	595.409
512/385	$2^9/5\cdot 7\cdot 11$	493.542
512/495	$2^9/3^2\cdot 5\cdot 11$	58.458
525/352	$3\cdot 5^2\cdot 7/2^5\cdot 11$	692.090
525/484	$3\cdot 5^2\cdot 7/2^2\cdot 11^2$	140.772
528/343	$2^4\cdot 3\cdot 11/7^3$	746.795
539/270	$7^2\cdot 11/2\cdot 3^3\cdot 5$	1196.791
539/288	$7^2\cdot 11/2^5\cdot 3^2$	1085.060
539/300	$7^2\cdot 11/2^2\cdot 3\cdot 5^2$	1014.387
539/320	$7^2\cdot 11/2^6\cdot 5$	902.656
539/324	$7^2\cdot 11/2^2\cdot 3^4$	881.150
539/360	$7^2\cdot 11/2^3\cdot 3^2\cdot 5$	698.746
539/375	$7^2\cdot 11/3\cdot 5^3$	628.074
539/384	$7^2\cdot 11/2^7\cdot 3$	587.015
539/400	$7^2\cdot 11/2^4\cdot 5^2$	516.342
539/405	$7^2\cdot 11/3^4\cdot 5$	494.836
539/432	$7^2\cdot 11/2^4\cdot 3^3$	383.105
539/480	$7^2\cdot 11/2^5\cdot 3\cdot 5$	200.701
539/486	$7^2\cdot 11/2\cdot 3^5$	179.195
539/500	$7^2\cdot 11/2^2\cdot 5^3$	130.029
539/512	$7^2\cdot 11/2^9$	88.970
540/539	$2^2\cdot 3^3\cdot 5/7^2\cdot 11$	3.209
550/343	$2\cdot 5^2\cdot 11/7^3$	817.468
560/297	$2^4\cdot 5\cdot 7/3^3\cdot 11$	1097.957
560/363	$2^4\cdot 5\cdot 7/3\cdot 11^2$	750.549
567/352	$3^4\cdot 7/2^5\cdot 11$	825.328
567/440	$3^4\cdot 7/2^3\cdot 5\cdot 11$	439.014
567/484	$3^4\cdot 7/2^2\cdot 11^2$	274.010
567/550	$3^4\cdot 7/2\cdot 5^2\cdot 11$	52.701
576/385	$2^6\cdot 3^2/5\cdot 7\cdot 11$	697.452
576/539	$2^6\cdot 3^2/7^2\cdot 11$	114.940
594/343	$2\cdot 3^3\cdot 11/7^3$	950.705
600/539	$2^3\cdot 3\cdot 5^2/7^2\cdot 11$	185.613
605/336	$5\cdot 11^2/2^4\cdot 3\cdot 7$	1018.169
605/343	$5\cdot 11^2/7^3$	982.472
605/378	$5\cdot 11^2/2\cdot 3^3\cdot 7$	814.259
605/384	$5\cdot 11^2/2^7\cdot 3$	786.995
605/392	$5\cdot 11^2/2^3\cdot 7^2$	751.298
605/432	$5\cdot 11^2/2^4\cdot 3^3$	583.085
605/448	$5\cdot 11^2/2^6\cdot 7$	520.124
605/486	$5\cdot 11^2/2\cdot 3^5$	379.175
605/504	$5\cdot 11^2/2^3\cdot 3^2\cdot 7$	316.214
605/512	$5\cdot 11^2/2^9$	288.950
605/567	$5\cdot 11^2/3^4\cdot 7$	112.304
605/576	$5\cdot 11^2/2^6\cdot 3^2$	85.040
605/588	$5\cdot 11^2/2^2\cdot 3\cdot 7^2$	49.343
616/375	$2^3\cdot 7\cdot 11/3\cdot 5^3$	859.248
616/405	$2^3\cdot 7\cdot 11/3^4\cdot 5$	726.010
625/352	$5^4/2^5\cdot 11$	993.937
625/363	$5^4/3\cdot 11^2$	940.664
625/396	$5^4/2^2\cdot 3^2\cdot 11$	790.027
625/462	$5^4/2\cdot 3\cdot 7\cdot 11$	523.156
625/484	$5^4/2^2\cdot 11^2$	442.619
625/528	$5^4/2^4\cdot 3\cdot 11$	291.982
625/539	$5^4/7^2\cdot 11$	256.285
625/594	$5^4/2\cdot 3^3\cdot 11$	88.072
625/616	$5^4/2^3\cdot 7\cdot 11$	25.111
640/363	$2^7\cdot 5/3\cdot 11^2$	981.723
640/539	$2^7\cdot 5/7^2\cdot 11$	297.344
648/385	$2^3\cdot 3^4/5\cdot 7\cdot 11$	901.362
648/539	$2^3\cdot 3^4/7^2\cdot 11$	318.850
648/605	$2^3\cdot 3^4/5\cdot 11^2$	118.870
660/343	$2^2\cdot 3\cdot 5\cdot 11/7^3$	1133.109
672/605	$2^5\cdot 3\cdot 7/5\cdot 11^2$	181.831
675/352	$3^3\cdot 5^2/2^5\cdot 11$	1127.175
675/484	$3^3\cdot 5^2/2^2\cdot 11^2$	575.857
675/539	$3^3\cdot 5^2/7^2\cdot 11$	389.523
675/616	$3^3\cdot 5^2/2^3\cdot 7\cdot 11$	158.349
686/363	$2\cdot 7^3/3\cdot 11^2$	1101.887
686/495	$2\cdot 7^3/3^2\cdot 5\cdot 11$	564.936
686/605	$2\cdot 7^3/5\cdot 11^2$	217.528
693/400	$3^2\cdot 7\cdot 11/2^4\cdot 5^2$	951.426
693/500	$3^2\cdot 7\cdot 11/2^2\cdot 5^3$	565.113
693/512	$3^2\cdot 7\cdot 11/2^9$	524.054
693/625	$3^2\cdot 7\cdot 11/5^4$	178.799
693/640	$3^2\cdot 7\cdot 11/2^7\cdot 5$	137.740
700/363	$2^2\cdot 5^2\cdot 7/3\cdot 11^2$	1136.862
704/375	$2^6\cdot 11/3\cdot 5^3$	1090.422
704/405	$2^6\cdot 11/3^4\cdot 5$	957.184
704/525	$2^6\cdot 11/3\cdot 5^2\cdot 7$	507.910
704/567	$2^6\cdot 11/3^4\cdot 7$	374.672
704/625	$2^6\cdot 11/5^4$	206.063
704/675	$2^6\cdot 11/3^3\cdot 5^2$	72.826
720/539	$2^4\cdot 3^2\cdot 5/7^2\cdot 11$	501.254
726/625	$2\cdot 3\cdot 11^2/5^4$	259.336
729/385	$3^6/5\cdot 7\cdot 11$	1105.272
729/440	$3^6/2^3\cdot 5\cdot 11$	874.098
729/484	$3^6/2^2\cdot 11^2$	709.094
729/539	$3^6/7^2\cdot 11$	522.760
729/550	$3^6/2\cdot 5^2\cdot 11$	487.785
729/605	$3^6/5\cdot 11^2$	322.780
729/616	$3^6/2^3\cdot 7\cdot 11$	291.586
729/704	$3^6/2^6\cdot 11$	60.412
735/484	$3\cdot 5\cdot 7^2/2^2\cdot 11^2$	723.285
735/704	$3\cdot 5\cdot 7^2/2^6\cdot 11$	74.603
750/539	$2\cdot 3\cdot 5^3/7^2\cdot 11$	571.926

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756/605	$2^2 \cdot 3^3 \cdot 7 / 5 \cdot 11^2$	385.741
768/385	$2^8 \cdot 3 / 5 \cdot 7 \cdot 11$	1195.497
768/539	$2^8 \cdot 3 / 7^2 \cdot 11$	612.985
768/605	$2^8 \cdot 3 / 5 \cdot 11^2$	413.005
770/729	$2 \cdot 5 \cdot 7 \cdot 11 / 3^6$	94.728
784/495	$2^4 \cdot 7^2 / 3^2 \cdot 5 \cdot 11$	796.110
784/605	$2^4 \cdot 7^2 / 5 \cdot 11^2$	448.702
792/625	$2^3 \cdot 3^2 \cdot 11 / 5^4$	409.973
800/539	$2^5 \cdot 5^2 / 7^2 \cdot 11$	683.658
800/693	$2^5 \cdot 5^2 / 3^2 \cdot 7 \cdot 11$	248.574
810/539	$2 \cdot 3^4 \cdot 5 / 7^2 \cdot 11$	705.164
825/448	$3 \cdot 5^2 \cdot 11 / 2^6 \cdot 7$	1057.074
825/512	$3 \cdot 5^2 \cdot 11 / 2^9$	825.900
825/686	$3 \cdot 5^2 \cdot 11 / 2 \cdot 7^3$	319.423
825/784	$3 \cdot 5^2 \cdot 11 / 2^4 \cdot 7^2$	88.249
847/432	$7 \cdot 11^2 / 2^4 \cdot 3^3$	1165.597
847/450	$7 \cdot 11^2 / 2 \cdot 3^2 \cdot 5^2$	1094.924
847/480	$7 \cdot 11^2 / 2^5 \cdot 3 \cdot 5$	983.193
847/486	$7 \cdot 11^2 / 2 \cdot 3^5$	961.687
847/500	$7 \cdot 11^2 / 2^2 \cdot 5^3$	912.521
847/512	$7 \cdot 11^2 / 2^9$	871.462
847/540	$7 \cdot 11^2 / 2^2 \cdot 3^3 \cdot 5$	779.283
847/576	$7 \cdot 11^2 / 2^6 \cdot 3^2$	667.552
847/600	$7 \cdot 11^2 / 2^3 \cdot 3 \cdot 5^2$	596.879
847/625	$7 \cdot 11^2 / 5^4$	526.207
847/640	$7 \cdot 11^2 / 2^7 \cdot 5$	485.148
847/648	$7 \cdot 11^2 / 2^3 \cdot 3^4$	463.642
847/675	$7 \cdot 11^2 / 3^3 \cdot 5^2$	392.969
847/720	$7 \cdot 11^2 / 2^4 \cdot 3^2 \cdot 5$	281.238
847/729	$7 \cdot 11^2 / 3^6$	259.732
847/750	$7 \cdot 11^2 / 2 \cdot 3 \cdot 5^3$	210.566
847/768	$7 \cdot 11^2 / 2^8 \cdot 3$	169.507
847/800	$7 \cdot 11^2 / 2^5 \cdot 5^2$	98.834
847/810	$7 \cdot 11^2 / 2 \cdot 3^4 \cdot 5$	77.328
864/539	$2^5 \cdot 3^3 / 7^2 \cdot 11$	816.895
864/605	$2^5 \cdot 3^3 / 5 \cdot 11^2$	616.915
864/847	$2^5 \cdot 3^3 / 7 \cdot 11^2$	34.403
875/484	$5^3 \cdot 7 / 2^2 \cdot 11^2$	1025.131
875/528	$5^3 \cdot 7 / 2^4 \cdot 3 \cdot 11$	874.494
875/594	$5^3 \cdot 7 / 2 \cdot 3^3 \cdot 11$	670.584
875/704	$5^3 \cdot 7 / 2^6 \cdot 11$	376.449
875/726	$5^3 \cdot 7 / 2 \cdot 3 \cdot 11^2$	323.176
875/792	$5^3 \cdot 7 / 2^3 \cdot 3^2 \cdot 11$	172.539
880/567	$2^4 \cdot 5 \cdot 11 / 3^4 \cdot 7$	760.986
880/729	$2^4 \cdot 5 \cdot 11 / 3^6$	325.902
882/605	$2 \cdot 3^2 \cdot 7^2 / 5 \cdot 11^2$	652.612
891/448	$3^4 \cdot 11 / 2^6 \cdot 7$	1190.312
891/490	$3^4 \cdot 11 / 2 \cdot 5 \cdot 7^2$	1035.172
891/500	$3^4 \cdot 11 / 2^2 \cdot 5^3$	1000.197
891/512	$3^4 \cdot 11 / 2^9$	959.138
891/560	$3^4 \cdot 11 / 2^4 \cdot 5 \cdot 7$	803.998
891/625	$3^4 \cdot 11 / 5^4$	613.883
891/640	$3^4 \cdot 11 / 2^7 \cdot 5$	572.824
891/686	$3^4 \cdot 11 / 2 \cdot 7^3$	452.660
891/700	$3^4 \cdot 11 / 2^2 \cdot 5^2 \cdot 7$	417.685
891/784	$3^4 \cdot 11 / 2^4 \cdot 7^2$	221.486
891/800	$3^4 \cdot 11 / 2^5 \cdot 5^2$	186.511
891/875	$3^4 \cdot 11 / 5^3 \cdot 7$	31.371
896/495	$2^7 \cdot 7 / 3^2 \cdot 5 \cdot 11$	1027.284
896/605	$2^7 \cdot 7 / 5 \cdot 11^2$	679.876
896/825	$2^7 \cdot 7 / 3 \cdot 5^2 \cdot 11$	142.926
896/891	$2^7 \cdot 7 / 3^4 \cdot 11$	9.688
900/539	$2^2 \cdot 3^2 \cdot 5^2 / 7^2 \cdot 11$	887.568
900/847	$2^2 \cdot 3^2 \cdot 5^2 / 7 \cdot 11^2$	105.076
924/625	$2^2 \cdot 3 \cdot 7 \cdot 11 / 5^4$	676.844
945/484	$3^3 \cdot 5 \cdot 7 / 2^2 \cdot 11^2$	1158.369
945/704	$3^3 \cdot 5 \cdot 7 / 2^6 \cdot 11$	509.687
960/539	$2^6 \cdot 3 \cdot 5 / 7^2 \cdot 11$	999.299
960/847	$2^6 \cdot 3 \cdot 5 / 7 \cdot 11^2$	216.807
968/525	$2^3 \cdot 11^2 / 3 \cdot 5^2 \cdot 7$	1059.228
968/567	$2^3 \cdot 11^2 / 3^4 \cdot 7$	925.990
968/625	$2^3 \cdot 11^2 / 5^4$	757.381
968/675	$2^3 \cdot 11^2 / 3^3 \cdot 5^2$	624.143
968/729	$2^3 \cdot 11^2 / 3^6$	490.906
968/735	$2^3 \cdot 11^2 / 3 \cdot 5 \cdot 7^2$	476.715
968/875	$2^3 \cdot 11^2 / 5^3 \cdot 7$	174.869
968/945	$2^3 \cdot 11^2 / 3^3 \cdot 5 \cdot 7$	41.631
972/539	$2^2 \cdot 3^5 / 7^2 \cdot 11$	1020.805
972/605	$2^2 \cdot 3^5 / 5 \cdot 11^2$	820.825
972/847	$2^2 \cdot 3^5 / 7 \cdot 11^2$	238.313
980/891	$2^2 \cdot 5 \cdot 7^2 / 3^4 \cdot 11$	164.828
1000/539	$2^3 \cdot 5^3 / 7^2 \cdot 11$	1069.971
1000/693	$2^3 \cdot 5^3 / 3^2 \cdot 7 \cdot 11$	634.887
1000/847	$2^3 \cdot 5^3 / 7 \cdot 11^2$	287.479
1000/891	$2^3 \cdot 5^3 / 3^4 \cdot 11$	199.803
1024/539	$2^{10} / 7^2 \cdot 11$	1111.030
1024/605	$2^{10} / 5 \cdot 11^2$	911.050
1024/693	$2^{10} / 3^2 \cdot 7 \cdot 11$	675.946
1024/825	$2^{10} / 3 \cdot 5^2 \cdot 11$	374.100
1024/847	$2^{10} / 7 \cdot 11^2$	328.538
1024/891	$2^{10} / 3^4 \cdot 11$	240.862
1029/550	$3 \cdot 7^3 / 2 \cdot 5^2 \cdot 11$	1084.487
1029/605	$3 \cdot 7^3 / 5 \cdot 11^2$	919.483
1029/704	$3 \cdot 7^3 / 2^6 \cdot 11$	657.115
1029/880	$3 \cdot 7^3 / 2^4 \cdot 5 \cdot 11$	270.801
1029/968	$3 \cdot 7^3 / 2^3 \cdot 11^2$	105.797
1056/625	$2^5 \cdot 3 \cdot 11 / 5^4$	908.018
1056/875	$2^5 \cdot 3 \cdot 11 / 5^3 \cdot 7$	325.506

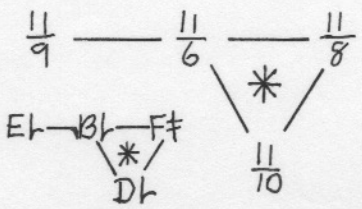
continued over page

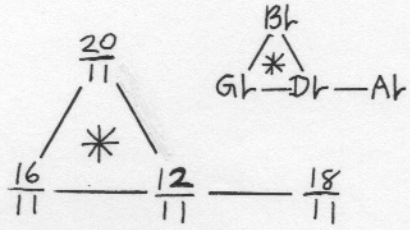
1078/625	$2 \cdot 7^2 \cdot 11 / 5^4$	943.715
1078/675	$2 \cdot 7^2 \cdot 11 / 3^3 \cdot 5^2$	810.477
1078/729	$2 \cdot 7^2 \cdot 11 / 3^6$	677.240
1080/847	$2^3 \cdot 3^3 \cdot 5 / 7 \cdot 11^2$	420.717
1089/560	$3^2 \cdot 11^2 / 2^4 \cdot 5 \cdot 7$	1151.406
1089/625	$3^2 \cdot 11^2 / 5^4$	961.291
1089/640	$3^2 \cdot 11^2 / 2^7 \cdot 5$	920.232
1089/686	$3^2 \cdot 11^2 / 2 \cdot 7^3$	800.068
1089/800	$3^2 \cdot 11^2 / 2^5 \cdot 5^2$	533.918
1089/875	$3^2 \cdot 11^2 / 5^3 \cdot 7$	378.779
1089/896	$3^2 \cdot 11^2 / 2^7 \cdot 7$	337.720
1089/980	$3^2 \cdot 11^2 / 2^2 \cdot 5 \cdot 7^2$	182.580
1089/1000	$3^2 \cdot 11^2 / 2^3 \cdot 5^3$	147.605
1089/1024	$3^2 \cdot 11^2 / 2^{10}$	106.546
1100/567	$2^2 \cdot 5^2 \cdot 11 / 3^4 \cdot 7$	1147.299
1100/729	$2^2 \cdot 5^2 \cdot 11 / 3^6$	712.215
1100/1029	$2^2 \cdot 5^2 \cdot 11 / 3 \cdot 7^3$	115.513
1120/891	$2^5 \cdot 5 \cdot 7 / 3^4 \cdot 11$	396.002
1120/1089	$2^5 \cdot 5 \cdot 7 / 3^2 \cdot 11^2$	48.594
1125/616	$3^2 \cdot 5^3 / 2^3 \cdot 7 \cdot 11$	1042.707
1125/704	$3^2 \cdot 5^3 / 2^6 \cdot 11$	811.533
1125/847	$3^2 \cdot 5^3 / 7 \cdot 11^2$	491.389
1125/968	$3^2 \cdot 5^3 / 2^3 \cdot 11^2$	260.215
1125/1078	$3^2 \cdot 5^3 / 2 \cdot 7^2 \cdot 11$	73.881
1134/605	$2 \cdot 3^4 \cdot 7 / 5 \cdot 11^2$	1087.696
1152/605	$2^7 \cdot 3^2 / 5 \cdot 11^2$	1114.960
1152/847	$2^7 \cdot 3^2 / 7 \cdot 11^2$	532.448
1155/1024	$3 \cdot 5 \cdot 7 \cdot 11 / 2^{10}$	208.413
1176/605	$2^3 \cdot 3 \cdot 7^2 / 5 \cdot 11^2$	1150.657
1188/625	$2^2 \cdot 3^3 \cdot 11 / 5^4$	1111.928
1188/875	$2^2 \cdot 3^3 \cdot 11 / 5^3 \cdot 7$	529.416
1200/847	$2^4 \cdot 3 \cdot 5^2 / 7 \cdot 11^2$	603.121



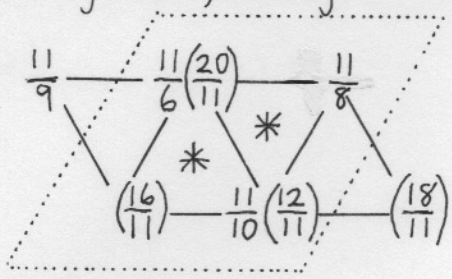
Mapping 11-Limit Ratios

The two regions (inversions to each other) embodying the most important 11-Limit ratios:



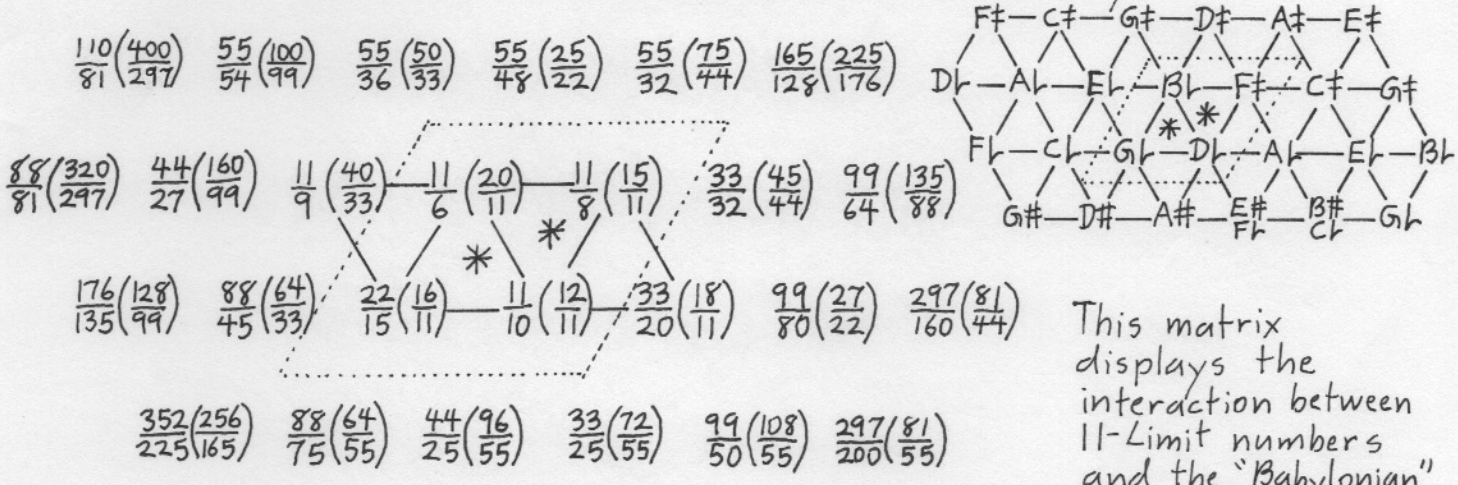


The ratio 11/10 (165.004c.) is almost identical to ratio 12/11 (150.637c.). (The difference is ratio 121/120 - 14.367c.). Hence the two regions can be integrated, using a bracket to differentiate "near misses:"



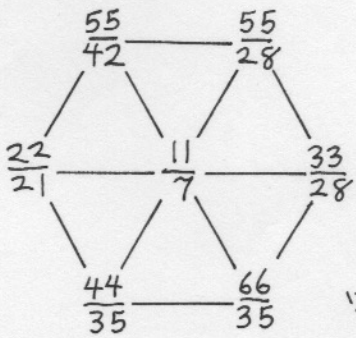
The dotted line represents the position of these ratios within the region of the Harmonic Antipodes (see over page).

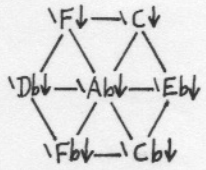
This "combined" 5-Limit matrix can be expanded to display many of the 11-Limit ratios of low numerosity:

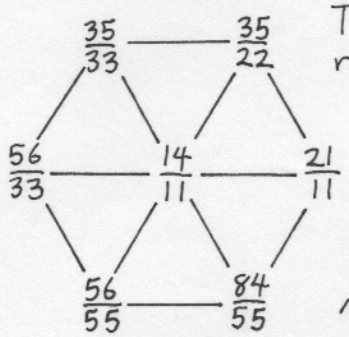


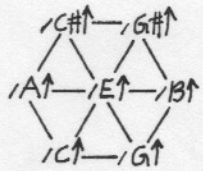
This matrix displays the interaction between 11-Limit numbers and the "Babylonian" 3-Limit + 5-Limit numbers.

It does not indicate the interaction with 7-Limit numbers. This Further triad Field coalesces around ratios 11/7 (782.492c.) and its inversion, the ratio 14/11 (417.508c.) Note how close the latter ratio approximates the 3-Limit 'E (407.820c.) The difference is ratio 896/891 (9.688c.) Hence the pitch notation suggested below.



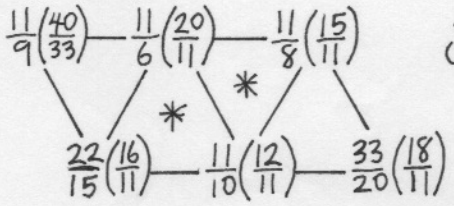




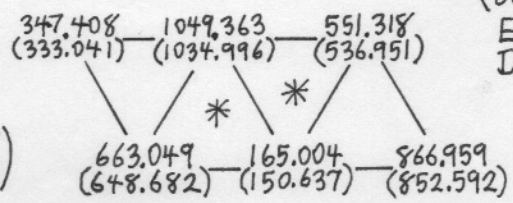


Why **11-Limit ratios** are **associated** with "**neutral**" **intervals** and with the region of the Harmonic Antipodes:

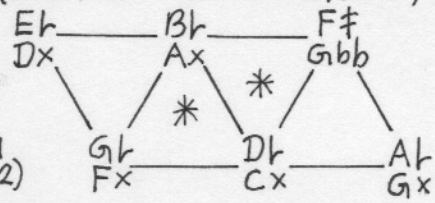
Principal Ratios:

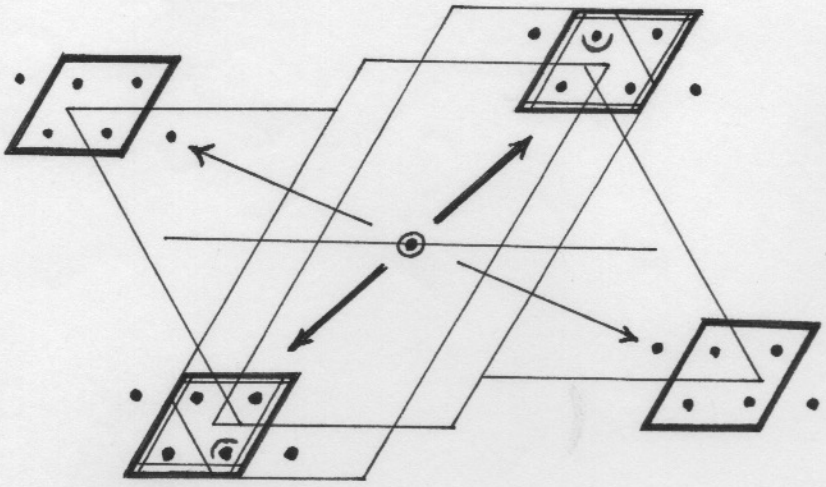


Size in Cents:



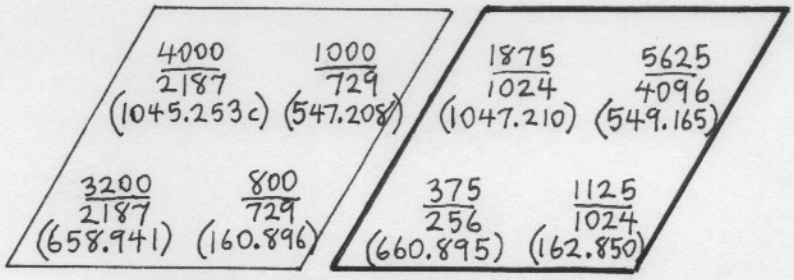
Suggested Pitch names:
(derived from Meantone)

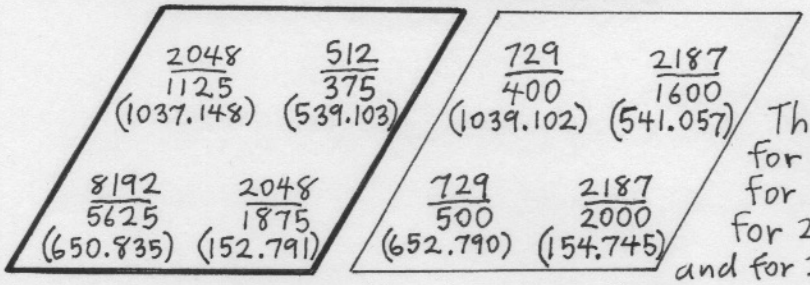




Location of the four regions collectively called the Harmonic Antipodes of 5-Limit Just Intonation. This region presents neutral intervals at the greatest "harmonic remove" from the musical consonances.

Ratios and cents (bracketed) presented below for comparison with the 11-Limit ratios above:



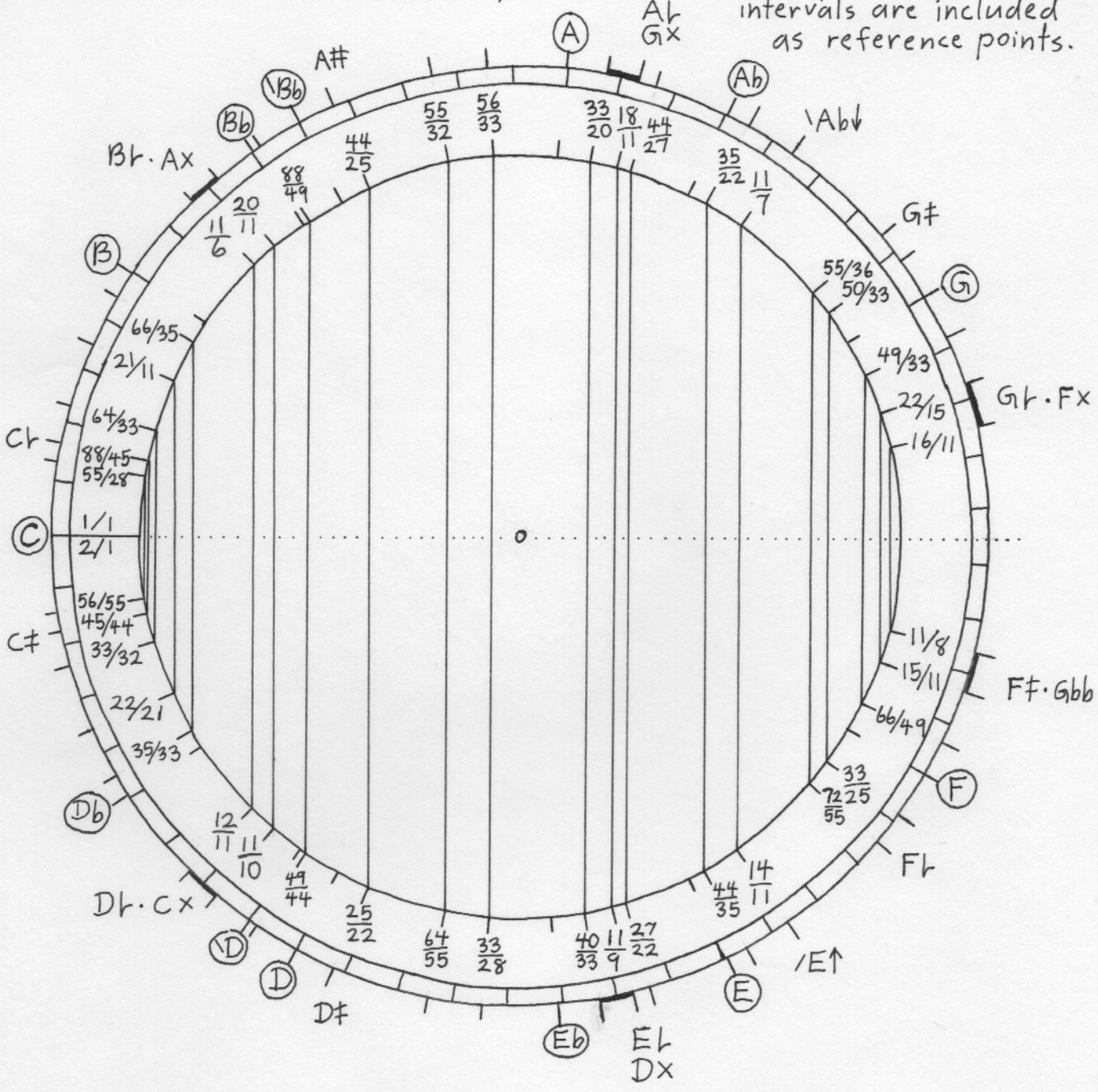


For example, compare the 11-Limit ratio 11/6 (165.004c.) to 5-Limit 1125/1024 (162.850c.) Both represent C𝄪 or D♭, about half-way between D♭ and D♮. The three alternative approximations are progressively more flat ie 800/729 (160.896 cents), 2187/2000 (154.745 cents), and 2048/1874 (152.791 cents). The deviation from 11/10 is:
for 1125/1024 − 5632/5625 (2.154 c.).
for 800/729 − 8019/8000 (4.107c.)
for 2187/2000 − 2200/2187 (10.260 c.)
and for 2048/1875 − 4125/4096 (12.214 c.)

If we now choose the "partner" 12/11 (150.637c.) and compare the four approximations at the Harmonic Antipodes, the deviations run in the opposite direction as the deviations from 11/10:
- deviation sharp for 1125/1024 − 4125/4096 (12.214c)..... 6s
- deviation sharp for 800/729 − 2200/2187 (10.260c)..... 5s
- deviation sharp for 2187/2000 − 8019/8000 (4.107c)..... 2s
- deviation sharp for 2048/1875 − 5632/5625 (2.154c)..... 1s

The principal 11-Limit ratios drawn on the circular graph:

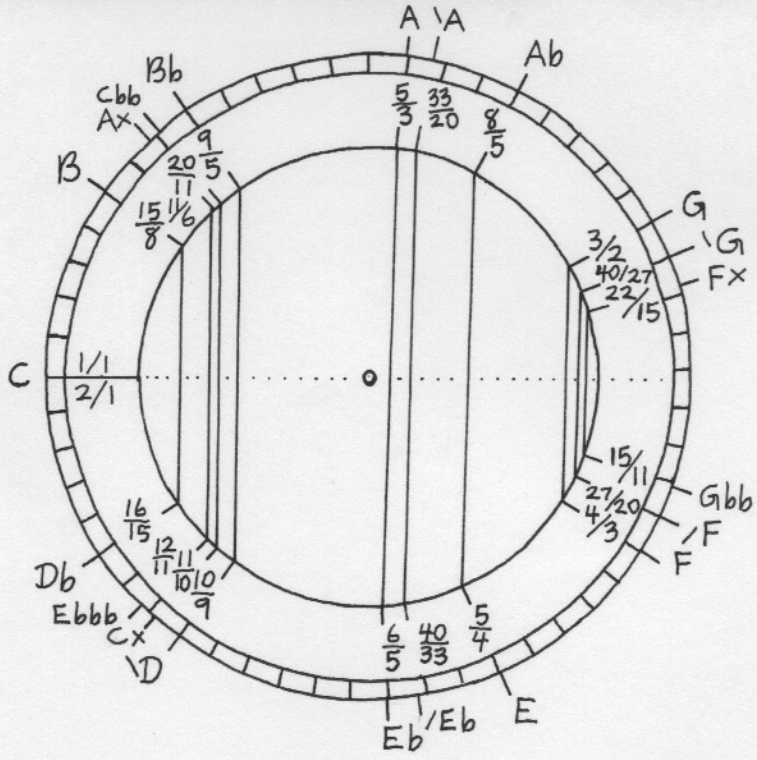
Interval pairs are connected by solid lines. Some prominant 5-Limit intervals are included as reference points.



Numbers of the family grouping $2^a \cdot 3^b \cdot 5^c \cdot 11^d$ (Subset of 11-Limit)

1	*	50	$2 \cdot 5^2$	165	$3 \cdot 5 \cdot 11$	384	$2^7 \cdot 3$	726	$2 \cdot 3 \cdot 11^2$
2	*	54	$2 \cdot 3^3$	176	$2^4 \cdot 11$	396	$2^2 \cdot 3^2 \cdot 11$	729	3^6
3	*	55	$5 \cdot 11$	180	$2^2 \cdot 3^2 \cdot 5$	400	$2^4 \cdot 5$	750	$2 \cdot 3 \cdot 5^3$
4	2^2	60	$2^2 \cdot 3 \cdot 5$	192	$2^6 \cdot 3$	405	$3^4 \cdot 5$	768	$2^8 \cdot 3$
5	*	64	2^6	198	$2 \cdot 3^2 \cdot 11$	432	$2^4 \cdot 3^3$	792	$2^3 \cdot 3^2 \cdot 11$
6	$2 \cdot 3$	66	$2 \cdot 3 \cdot 11$	200	$2^3 \cdot 5^2$	440	$2^3 \cdot 5 \cdot 11$	800	$2^5 \cdot 5^2$
8	2^3	72	$2^3 \cdot 3^2$	216	$2^3 \cdot 3^3$	450	$2 \cdot 3^2 \cdot 5^2$	810	$2 \cdot 3^4 \cdot 5$
9	3^2	75	$3 \cdot 5^2$	220	$2^2 \cdot 5 \cdot 11$	480	$2^5 \cdot 3 \cdot 5$	825	$3 \cdot 5^2 \cdot 11$
10	$2 \cdot 5$	80	$2^4 \cdot 5$	225	$3^2 \cdot 5^2$	484	$2^2 \cdot 11^2$	864	$2^5 \cdot 3^3$
11	*	81	3^4	240	$2^4 \cdot 3 \cdot 5$	486	$2 \cdot 3^5$	880	$2^4 \cdot 5 \cdot 11$
12	$2^2 \cdot 3$	88	$2^3 \cdot 11$	242	$2 \cdot 11^2$	495	$3^2 \cdot 5 \cdot 11$	891	$3^4 \cdot 11$
15	$3 \cdot 5$	90	$2 \cdot 3^2 \cdot 5$	243	3^5	500	$2^2 \cdot 5^3$	900	$2^2 \cdot 3^2 \cdot 5^2$
16	2^4	96	$2^5 \cdot 3$	250	$2 \cdot 5^3$	512	2^9	960	$2^6 \cdot 3 \cdot 5$
18	$2 \cdot 3^2$	99	$3^2 \cdot 11$	256	2^8	528	$2^4 \cdot 3 \cdot 11$	968	$2^3 \cdot 11^2$
20	$2^2 \cdot 5$	100	$2^2 \cdot 5^2$	264	$2^3 \cdot 3 \cdot 11$	540	$2^2 \cdot 3^3 \cdot 5$	972	$2^2 \cdot 3^5$
22	$2 \cdot 11$	108	$2^2 \cdot 3^3$	270	$2 \cdot 3^3 \cdot 5$	550	$2 \cdot 5^2 \cdot 11$	990	$2 \cdot 3^2 \cdot 5 \cdot 11$
24	$2^3 \cdot 3$	110	$2 \cdot 5 \cdot 11$	275	$5^2 \cdot 11$	576	$2^6 \cdot 3^2$	1000	$2^3 \cdot 5^3$
25	5^2	120	$2^3 \cdot 3 \cdot 5$	288	$2^5 \cdot 3^2$	594	$2 \cdot 3^3 \cdot 11$	1024	2^{10}
27	3^3	121	11^2	297	$3^3 \cdot 11$	600	$2^3 \cdot 3 \cdot 5^2$	1080	$2^3 \cdot 3^3 \cdot 5$
30	$2 \cdot 3 \cdot 5$	125	5^3	300	$2^2 \cdot 3 \cdot 5^2$	605	$5 \cdot 11^2$	1089	$3^2 \cdot 11^2$
32	2^5	128	2^7	320	$2^6 \cdot 5$	625	5^4	1100	$2^2 \cdot 5^2 \cdot 11$
33	$3 \cdot 11$	132	$2^2 \cdot 3 \cdot 11$	324	$2^2 \cdot 3^4$	640	$2^7 \cdot 5$	1125	$3^2 \cdot 5^3$
36	$2^2 \cdot 3^2$	135	$3^3 \cdot 5$	330	$2 \cdot 3 \cdot 5 \cdot 11$	648	$2^3 \cdot 3^4$	1152	$2^7 \cdot 3^2$
40	$2^3 \cdot 5$	144	$2^4 \cdot 3^2$	352	$2^5 \cdot 11$	660	$2^2 \cdot 3 \cdot 5 \cdot 11$	1188	$2^2 \cdot 3^3 \cdot 11$
44	$2^2 \cdot 11$	150	$2 \cdot 3 \cdot 5^2$	360	$2^3 \cdot 3^2 \cdot 5$	675	$3^3 \cdot 5^2$	1200	$2^4 \cdot 3 \cdot 5^2$
45	$3^2 \cdot 5$	160	$2^5 \cdot 5$	363	$3 \cdot 11^2$	704	$2^6 \cdot 11$		↓
48	$2^4 \cdot 3$	162	$2 \cdot 3^4$	375	$3 \cdot 5^3$	720	$2^4 \cdot 3^2 \cdot 5$		

A characteristic monochord: ruled by the double 60:120



60 : 64 : 66 : 72 : 75 :
o – C Db Cx Eb E
u – C B Cbb A Ab

80 : 81 : 88 : 90 : 96 :
F /F Fx G Ab
G \G Gbb F E

99 : 100 : 108 : 110 : 120
\A A Bb Ax C
/Eb Eb \D Ebbb C

A complex 11-Limit Monochord using double 360:720

() bracket gives 7-Limit element, [] bracket gives 11-Limit element

Number Series	360	[363]	375	(378)	384	[385]
Over Pitch	C		C#	/C#	Db	
Ratio	1/1	121/120	25/24	21/20	16/15	77/72
Under Pitch	C		Cb	\Cb	B	
Ratio	2/1	240/121	48/25	40/21	15/8	144/77

(392)	[396]	400	405	(420)	432	[440]	(441)
		\D	D	D#	Eb		
49/45	11/10	10/9	9/8	7/6	6/5	11/9	49/40
		Bb	\Bb	A‡	A		
90/49	20/11	9/5	16/9	12/7	5/3	18/11	80/49

(448)	450	[462]	480	[484]	486	(490)	[495]
	E		F		/F	F#	
56/45	5/4	77/60	4/3	121/90	27/20	49/36	99/72
	Ab		G		\G	Gb	
45/28	8/5	120/77	3/2	180/121	40/27	72/49	144/99

500	(504)	512	(525)	[528]	[539]	540	[550]
\F#	F#	\Gb	Fx			G	
25/18	7/5	64/45	35/24	22/15	539/360	3/2	55/36
Gb	Gb	F#	Gbb			F	
36/25	10/7	45/32	48/35	15/11	720/539	4/3	72/55

(560)	(567)	576	(588)	[594]	600	[605]	[616]
G#	/G#	Ab	Ab		A		
14/9	63/40	8/5	49/30	33/20	5/3	121/72	77/45
Fb	\Fb	E	Eb		Eb		
9/7	80/63	5/4	60/49	40/33	6/5	144/121	90/77

625	(630)	640	648	[660]	(672)	675	(686)
\A#	A#	\Bb	Bb		B	B	B‡
125/72	7/4	16/9	9/5	11/6	25/18	15/8	343/180
Ebb	D‡	D	\D		Db	Db	Dbb
144/125	8/7	9/8	10/9	12/11	15/14	16/15	360/343

[693]	(700)	[704]	720
	B#		C
77/40	35/18	88/45	2/1
	Dbb		C
80/77	36/35	45/44	1/1

Numbers of the family grouping $2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot 11^e \cdot 13^f$ (**13-Limit**):

1	*	72	$2^3 \cdot 3^2$	198	$2 \cdot 3^2 \cdot 11$	390	$2 \cdot 3 \cdot 5 \cdot 13$	650	$2 \cdot 5^2 \cdot 13$	990	$2 \cdot 3^2 \cdot 5 \cdot 11$				
2	*	75	$3 \cdot 5^2$	200	$2^3 \cdot 5^2$	392	$2^3 \cdot 7^2$	660	$2^2 \cdot 3 \cdot 5 \cdot 11$	1000	$2^3 \cdot 5^3$				
3	*	77	$7 \cdot 11$	208	$2^4 \cdot 13$	396	$2^2 \cdot 3^2 \cdot 11$	672	$2^5 \cdot 3 \cdot 7$	1001	$7 \cdot 11 \cdot 13$				
4	2^2	78	$2 \cdot 3 \cdot 13$	210	$2 \cdot 3 \cdot 5 \cdot 7$	400	$2^4 \cdot 5^2$	675	$3^3 \cdot 5^2$	1008	$2^4 \cdot 3^2 \cdot 7$				
5	*	80	$2^4 \cdot 5$	216	$2^3 \cdot 3^3$	405	$3^4 \cdot 5$	676	$2^2 \cdot 13^2$	1014	$2 \cdot 3 \cdot 13^2$				
6	$2 \cdot 3$	81	3^4	220	$2^2 \cdot 5 \cdot 11$	416	$2^5 \cdot 13$	686	$2 \cdot 7^3$	1024	2^{10}				
7	*	84	$2^2 \cdot 3 \cdot 7$	224	$2^5 \cdot 7$	420	$2^2 \cdot 3 \cdot 5 \cdot 7$	693	$3^2 \cdot 7 \cdot 11$	1029	$3 \cdot 7^3$				
8	2^3	88	$2^3 \cdot 11$	225	$3^2 \cdot 5^2$	429	$3 \cdot 11 \cdot 13$	700	$2^2 \cdot 5^2 \cdot 7$	1040	$2^4 \cdot 5 \cdot 13$				
9	3^2	90	$2 \cdot 3^2 \cdot 5$	231	$3 \cdot 7 \cdot 11$	432	$2^4 \cdot 3^3$	702	$2 \cdot 3^3 \cdot 13$	1050	$2 \cdot 3 \cdot 5^2 \cdot 7$				
10	$2 \cdot 5$	91	$7 \cdot 13$	234	$2 \cdot 3^2 \cdot 13$	440	$2^3 \cdot 5 \cdot 11$	704	$2^6 \cdot 11$	1053	$3^4 \cdot 13$				
11	*	96	$2^5 \cdot 3$	240	$2^4 \cdot 3 \cdot 5$	441	$3^2 \cdot 7^2$	715	$5 \cdot 11 \cdot 13$	1056	$2^5 \cdot 3 \cdot 11$				
12	$2^2 \cdot 3$	98	$2 \cdot 7^2$	242	$2 \cdot 11^2$	448	$2^6 \cdot 7$	720	$2^4 \cdot 3^2 \cdot 5$	1078	$2 \cdot 7^2 \cdot 11$				
13	*	99	$3^2 \cdot 11$	243	3^5	450	$2 \cdot 3^2 \cdot 5^2$	726	$2 \cdot 3 \cdot 11^2$	1080	$2^3 \cdot 3^2 \cdot 5$				
14	$2 \cdot 7$	100	$2^2 \cdot 5^2$	245	$5 \cdot 7^2$	455	$5 \cdot 7 \cdot 13$	728	$2^3 \cdot 7 \cdot 13$	1089	$3^2 \cdot 11^2$				
15	$3 \cdot 5$	104	$2^3 \cdot 13$	250	$2 \cdot 5^3$	462	$2 \cdot 3 \cdot 7 \cdot 11$	729	3^6	1092	$2^2 \cdot 3 \cdot 7 \cdot 13$				
16	2^4	105	$3 \cdot 5 \cdot 7$	252	$2^2 \cdot 3^2 \cdot 7$	468	$2^2 \cdot 3^2 \cdot 13$	735	$3 \cdot 5 \cdot 7^2$	1100	$2^2 \cdot 5^2 \cdot 11$				
18	$2 \cdot 3^2$	108	$2^2 \cdot 3^3$	256	2^8	480	$2^5 \cdot 3 \cdot 5$	750	$2 \cdot 3 \cdot 5^3$	1120	$2^5 \cdot 5 \cdot 7$				
20	$2^2 \cdot 5$	110	$2 \cdot 5 \cdot 11$	260	$2^2 \cdot 5 \cdot 13$	484	$2^2 \cdot 11^2$	756	$2^2 \cdot 3^3 \cdot 7$	1125	$3^2 \cdot 5^3$				
21	$3 \cdot 7$	112	$2^4 \cdot 7$	264	$2^3 \cdot 3 \cdot 11$	486	$2 \cdot 3^5$	768	$2^8 \cdot 3$	1134	$2 \cdot 3^4 \cdot 7$				
22	$2 \cdot 11$	117	$3^2 \cdot 13$	270	$2 \cdot 3^3 \cdot 5$	490	$2 \cdot 5 \cdot 7^2$	770	$2 \cdot 5 \cdot 7 \cdot 11$	1144	$2^3 \cdot 11 \cdot 13$				
24	$2^3 \cdot 3$	120	$2^3 \cdot 3 \cdot 5$	273	$3 \cdot 7 \cdot 13$	495	$3^2 \cdot 5 \cdot 11$	780	$2^2 \cdot 3 \cdot 5 \cdot 13$	1152	$2^7 \cdot 3^2$				
25	5^2	121	11^2	275	$5^2 \cdot 11$	500	$2^2 \cdot 5^3$	784	$2^4 \cdot 7^2$	1155	$3 \cdot 5 \cdot 7 \cdot 11$				
26	$2 \cdot 13$	125	5^3	280	$2^3 \cdot 5 \cdot 7$	504	$2^3 \cdot 3^2 \cdot 7$	792	$2^3 \cdot 3^2 \cdot 11$	1170	$2 \cdot 3^2 \cdot 5 \cdot 13$				
27	3^3	126	$2 \cdot 3^2 \cdot 7$	286	$2 \cdot 11 \cdot 13$	507	$3 \cdot 13^2$	800	$2^5 \cdot 5^2$	1176	$2^3 \cdot 3 \cdot 7^2$				
28	$2^2 \cdot 7$	128	2^7	288	$2^5 \cdot 3^2$	512	2^9	810	$2 \cdot 3^4 \cdot 5$	1183	$7 \cdot 13^2$				
30	$2 \cdot 3 \cdot 5$	130	$2 \cdot 5 \cdot 13$	294	$2 \cdot 3 \cdot 7^2$	520	$2^3 \cdot 5 \cdot 13$	819	$3^2 \cdot 7 \cdot 13$	1188	$2^2 \cdot 3^3 \cdot 11$				
32	2^5	132	$2^2 \cdot 3 \cdot 11$	297	$3^3 \cdot 11$	525	$3 \cdot 5^2 \cdot 7$	825	$3 \cdot 5^2 \cdot 11$	1200	$2^4 \cdot 3 \cdot 5^2$				
33	$3 \cdot 11$	135	$3^3 \cdot 5$	300	$2^2 \cdot 3 \cdot 5^2$	528	$2^4 \cdot 3 \cdot 11$	832	$2^6 \cdot 13$		↓				
35	$5 \cdot 7$	140	$2^2 \cdot 5 \cdot 7$	308	$2^2 \cdot 7 \cdot 11$	539	$7^2 \cdot 11$	840	$2^3 \cdot 3 \cdot 5 \cdot 7$						
36	$2^2 \cdot 3^2$	143	$11 \cdot 13$	312	$2^3 \cdot 3 \cdot 13$	540	$2^2 \cdot 3^3 \cdot 5$	845	$5 \cdot 13^2$						
39	$3 \cdot 13$	144	$2^4 \cdot 3^2$	315	$3^2 \cdot 5 \cdot 7$	546	$2 \cdot 3 \cdot 7 \cdot 13$	847	$7 \cdot 11^2$						
40	$2^3 \cdot 5$	147	$3 \cdot 7^2$	320	$2^6 \cdot 5$	550	$2 \cdot 5^2 \cdot 11$	858	$2 \cdot 3 \cdot 11 \cdot 13$						
42	$2 \cdot 3 \cdot 7$	150	$2 \cdot 3 \cdot 5^2$	324	$2^2 \cdot 3^4$	560	$2^4 \cdot 5 \cdot 7$	864	$2^5 \cdot 3^3$						
44	$2^2 \cdot 11$	154	$2 \cdot 7 \cdot 11$	325	$5^2 \cdot 13$	567	$3^4 \cdot 7$	875	$5^3 \cdot 7$						
45	$3^2 \cdot 5$	156	$2^2 \cdot 3 \cdot 13$	330	$2 \cdot 3 \cdot 5 \cdot 11$	572	$2^2 \cdot 11 \cdot 13$	880	$2^4 \cdot 5 \cdot 11$						
48	$2^4 \cdot 3$	160	$2^5 \cdot 5$	336	$2^4 \cdot 3 \cdot 7$	576	$2^6 \cdot 3^2$	882	$2 \cdot 3^2 \cdot 7^2$						
49	7^2	162	$2 \cdot 3^4$	338	$2 \cdot 13^2$	585	$3^2 \cdot 5 \cdot 13$	891	$3^4 \cdot 11$						
50	$2 \cdot 5^2$	165	$3 \cdot 5 \cdot 11$	343	7^3	588	$2^2 \cdot 3 \cdot 7^2$	896	$2^7 \cdot 7$						
52	$2^2 \cdot 13$	168	$2^3 \cdot 3 \cdot 7$	350	$2 \cdot 5^2 \cdot 7$	594	$2 \cdot 3^3 \cdot 11$	900	$2^2 \cdot 3^2 \cdot 5^2$						
54	$2 \cdot 3^3$	169	13^2	351	$3^3 \cdot 13$	600	$2^3 \cdot 3 \cdot 5^2$	910	$2 \cdot 5 \cdot 7 \cdot 13$						
55	$5 \cdot 11$	175	$5^2 \cdot 7$	352	$2^5 \cdot 11$	605	$5 \cdot 11^2$	924	$2^2 \cdot 3 \cdot 7 \cdot 11$						
56	$2^3 \cdot 7$	176	$2^4 \cdot 11$	360	$2^3 \cdot 3^2 \cdot 5$	616	$2^3 \cdot 7 \cdot 11$	936	$2^3 \cdot 3^2 \cdot 13$						
60	$2^2 \cdot 3 \cdot 5$	180	$2^2 \cdot 3^2 \cdot 5$	363	$3 \cdot 11^2$	624	$2^4 \cdot 3 \cdot 13$	945	$3^3 \cdot 5 \cdot 7$						
63	$3^2 \cdot 7$	182	$2 \cdot 7 \cdot 13$	364	$2^2 \cdot 7 \cdot 13$	625	5^4	960	$2^6 \cdot 3 \cdot 5$						
64	2^6	189	$3^3 \cdot 7$	375	$3 \cdot 5^3$	630	$2 \cdot 3^2 \cdot 5 \cdot 7$	968	$2^3 \cdot 11^2$						
65	$5 \cdot 13$	192	$2^6 \cdot 3$	378	$2 \cdot 3^3 \cdot 7$	637	$7^2 \cdot 13$	972	$2^2 \cdot 3^5$						
66	$2 \cdot 3 \cdot 11$	195	$3 \cdot 5 \cdot 13$	384	$2^7 \cdot 3$	640	$2^7 \cdot 5$	975	$3 \cdot 5^2 \cdot 13$						
70	$2 \cdot 5 \cdot 7$	196	$2^2 \cdot 7^2$	385	$5 \cdot 7 \cdot 11$	648	$2^3 \cdot 3^4$	980	$2^2 \cdot 5 \cdot 7^2$						

13-Limit Ratios (between 1/1 and 2/1) not Found in 11-, 7-, or 5-Limit:

Ratio	Prime Factors	Cents	Pitch
13/7	13/7	1071.702	
13/8	13/2^3	840.528	
13/9	13/3^2	636.618	
13/10	13/2·5	454.214	
13/11	13/11	289.210	
13/12	13/2^2·3	138.573	
14/13	2·7/13	128.298	
15/13	3·5/13	247.741	
16/13	2^4/13	359.472	
18/13	2·3^2/13	563.382	
20/13	2^2·5/13	745.786	
21/13	3·7/13	830.253	
22/13	2·11/13	910.790	
24/13	2^3·3/13	1061.427	
25/13	5^2/13	1132.100	
26/15	2·13/3·5	952.259	
26/21	2·13/3·7	369.747	
26/25	2·13/5^2	67.900	
27/26	3^3/2·13	65.337	
33/26	3·11/2·13	412.745	
35/26	5·7/2·13	514.612	
39/20	3·13/2^2·5	1156.169	
39/22	3·13/2·11	991.165	
39/25	3·13/5^2	769.855	
39/28	3·13/2^2·7	573.657	
39/35	3·13/5·7	187.343	
40/39	2^3·5/3·13	43.831	
44/39	2^2·11/3·13	208.835	
45/26	3^2·5/2·13	949.696	
49/26	7^2/2·13	1097.124	
49/39	7^2/3·13	395.169	
50/39	2·5^2/3·13	430.145	
52/27	2^2·13/3^3	1134.663	
52/33	2^2·13/3·11	787.255	
52/35	2^2·13/5·7	685.388	
52/45	2^2·13/3^2·5	250.304	
52/49	2^2·13/7^2	102.876	
55/39	5·11/3·13	595.149	
55/52	5·11/2^2·13	97.104	
56/39	2^3·7/3·13	626.343	
63/52	3^2·7/2^2·13	332.208	
64/39	2^6/3·13	857.517	
65/33	5·13/3·11	1173.568	
65/36	5·13/2^2·3^2	1022.931	
65/42	5·13/2·3·7	756.060	
65/44	5·13/2^2·11	675.523	
65/48	5·13/2^4·3	524.886	
65/49	5·13/7^2	489.190	
65/54	5·13/2·3^3	320.976	
65/56	5·13/2^3·7	258.015	
65/63	5·13/3^2·7	54.105	
65/64	5·13/2^6	26.841	
66/65	2·3·11/5·13	26.432	
70/39	2·5·7/3·13	1012.657	
72/65	2^3·3^2/5·13	177.069	
75/52	3·5^2/2^2·13	634.055	
77/39	7·11/3·13	1177.661	
77/52	7·11/2^2·13	679.616	
77/65	7·11/5·13	293.302	
78/49	2·3·13/7^2	804.831	
78/55	2·3·13/5·11	604.851	
78/77	2·3·13/7·11	22.339	
81/52	3^4/2^2·13	767.292	
81/65	3^4/5·13	380.979	
84/65	2^2·3·7/5·13	443.940	
88/65	2^3·11/5·13	524.477	
91/48	7·13/2^4·3	1107.399	
91/50	7·13/2·5^2	1036.726	
91/54	7·13/2·3^3	903.489	
91/55	7·13/5·11	871.722	
91/60	7·13/2^2·3·5	721.085	
91/64	7·13/2^6	609.354	
91/66	7·13/2·3·11	556.081	
91/72	7·13/2^3·3^2	405.444	
91/75	7·13/3·5^2	334.771	
91/80	7·13/2^4·5	223.040	
91/81	7·13/3^4	201.534	
91/88	7·13/2^3·11	58.036	
91/90	7·13/2·3^2·5	19.130	
96/65	2^5·3/5·13	675.114	
96/91	2^5·3/7·13	92.601	
98/65	2·7^2/5·13	710.810	
99/52	3^2·11/2^2·13	1114.700	
99/65	3^2·11/5·13	728.387	
99/91	3^2·11/7·13	145.874	
100/91	2^2·5^2/7·13	163.274	
104/55	2^3·13/5·11	1102.896	
104/63	2^3·13/3^2·7	867.792	
104/75	2^3·13/3·5^2	565.945	
104/77	2^3·13/7·11	520.384	
104/81	2^3·13/3^4	432.708	
104/99	2^3·13/3^2·11	85.300	
105/104	3·5·7/2^3·13	16.567	

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108/65	$2^2 \cdot 3^3 / 5 \cdot 13$	879.024
108/91	$2^2 \cdot 3^3 / 7 \cdot 13$	296.511
110/91	$2 \cdot 5 \cdot 11 / 7 \cdot 13$	328.278
112/65	$2^4 \cdot 7 / 5 \cdot 13$	941.985
117/64	$3^2 \cdot 13 / 2^6$	1044.438
117/70	$3^2 \cdot 13 / 2 \cdot 5 \cdot 7$	889.298
117/77	$3^2 \cdot 13 / 7 \cdot 11$	724.294
117/80	$3^2 \cdot 13 / 2^4 \cdot 5$	658.124
117/88	$3^2 \cdot 13 / 2^3 \cdot 11$	493.120
117/98	$3^2 \cdot 13 / 2 \cdot 7^2$	306.786
117/100	$3^2 \cdot 13 / 2^2 \cdot 5^2$	271.810
117/110	$3^2 \cdot 13 / 2 \cdot 5 \cdot 11$	106.806
117/112	$3^2 \cdot 13 / 2^4 \cdot 7$	75.612
120/91	$2^3 \cdot 3 \cdot 5 / 7 \cdot 13$	478.915
121/65	$11^2 / 5 \cdot 13$	1075.795
121/78	$11^2 / 2 \cdot 3 \cdot 13$	760.153
121/91	$11^2 / 7 \cdot 13$	493.282
121/104	$11^2 / 2^3 \cdot 13$	262.108
121/117	$11^2 / 3^2 \cdot 13$	58.198
125/78	$5^3 / 2 \cdot 3 \cdot 13$	816.458
125/91	$5^3 / 7 \cdot 13$	549.588
125/104	$5^3 / 2^3 \cdot 13$	318.413
125/117	$5^3 / 3^2 \cdot 13$	114.503
126/65	$2 \cdot 3^2 \cdot 7 / 5 \cdot 13$	1145.895
128/65	$2^7 / 5 \cdot 13$	1173.159
128/91	$2^7 / 7 \cdot 13$	590.646
128/117	$2^7 / 3^2 \cdot 13$	155.562
130/77	$2 \cdot 5 \cdot 13 / 7 \cdot 11$	906.698
130/81	$2 \cdot 5 \cdot 13 / 3^4$	819.021
130/99	$2 \cdot 5 \cdot 13 / 3^2 \cdot 11$	471.613
130/121	$2 \cdot 5 \cdot 13 / 11^2$	124.205
132/91	$2^2 \cdot 3 \cdot 11 / 7 \cdot 13$	643.919
135/91	$3^3 \cdot 5 / 7 \cdot 13$	682.825
135/104	$3^3 \cdot 5 / 2^3 \cdot 13$	451.651
140/117	$2^2 \cdot 5 \cdot 7 / 3^2 \cdot 13$	310.702
143/72	$11 \cdot 13 / 2^3 \cdot 3^2$	1187.936
143/75	$11 \cdot 13 / 3 \cdot 5^2$	1117.263
143/80	$11 \cdot 13 / 2^4 \cdot 5$	1005.532
143/81	$11 \cdot 13 / 3^4$	984.026
143/84	$11 \cdot 13 / 2^2 \cdot 3 \cdot 7$	921.065
143/90	$11 \cdot 13 / 2 \cdot 3^2 \cdot 5$	801.622
143/96	$11 \cdot 13 / 2^5 \cdot 3$	689.891
143/98	$11 \cdot 13 / 2 \cdot 7^2$	654.194
143/100	$11 \cdot 13 / 2^2 \cdot 5^2$	619.218
143/105	$11 \cdot 13 / 3 \cdot 5 \cdot 7$	534.751
143/108	$11 \cdot 13 / 2^2 \cdot 3^3$	485.981
143/112	$11 \cdot 13 / 2^4 \cdot 7$	423.020
143/120	$11 \cdot 13 / 2^3 \cdot 3 \cdot 5$	303.577
143/125	$11 \cdot 13 / 5^3$	232.904
143/126	$11 \cdot 13 / 2 \cdot 3^2 \cdot 7$	219.110
143/128	$11 \cdot 13 / 2^7$	191.846
143/135	$11 \cdot 13 / 3^3 \cdot 5$	99.667
143/140	$11 \cdot 13 / 2^2 \cdot 5 \cdot 7$	36.706
144/91	$2^4 \cdot 3^2 / 7 \cdot 13$	794.556
144/143	$2^4 \cdot 3^2 / 11 \cdot 13$	12.064
147/104	$3 \cdot 7^2 / 2^3 \cdot 13$	599.079
147/130	$3 \cdot 7^2 / 2 \cdot 5 \cdot 13$	212.765
147/143	$3 \cdot 7^2 / 11 \cdot 13$	47.761
150/91	$2 \cdot 3 \cdot 5^2 / 7 \cdot 13$	865.229
150/143	$2 \cdot 3 \cdot 5^2 / 11 \cdot 13$	82.737
154/117	$2 \cdot 7 \cdot 11 / 3^2 \cdot 13$	475.706
156/121	$2^2 \cdot 3 \cdot 13 / 11^2$	439.847
156/125	$2^2 \cdot 3 \cdot 13 / 5^3$	383.542
160/91	$2^5 \cdot 5 / 7 \cdot 13$	976.960
160/117	$2^5 \cdot 5 / 3^2 \cdot 13$	541.876
160/143	$2^5 \cdot 5 / 11 \cdot 13$	194.468
162/91	$2 \cdot 3^4 / 7 \cdot 13$	998.466
162/143	$2 \cdot 3^4 / 11 \cdot 13$	215.974
165/91	$3 \cdot 5 \cdot 11 / 7 \cdot 13$	1030.233
165/104	$3 \cdot 5 \cdot 11 / 2^3 \cdot 13$	799.059
168/143	$2^3 \cdot 3 \cdot 7 / 11 \cdot 13$	278.935
169/88	$13^2 / 2^3 \cdot 11$	1129.737
169/90	$13^2 / 2 \cdot 3^2 \cdot 5$	1090.832
169/96	$13^2 / 2^5 \cdot 3$	979.100
169/98	$13^2 / 2 \cdot 7^2$	943.403
169/99	$13^2 / 3^2 \cdot 11$	925.827
169/100	$13^2 / 2^2 \cdot 5^2$	908.428
169/105	$13^2 / 3 \cdot 5 \cdot 7$	823.961
169/108	$13^2 / 2^2 \cdot 3^3$	775.190
169/110	$13^2 / 2 \cdot 5 \cdot 11$	743.424
169/112	$13^2 / 2^4 \cdot 7$	712.229
169/120	$13^2 / 2^3 \cdot 3 \cdot 5$	592.787
169/121	$13^2 / 11^2$	578.419
169/125	$13^2 / 5^3$	522.114
169/126	$13^2 / 2 \cdot 3^2 \cdot 7$	508.319
169/128	$13^2 / 2^7$	481.055
169/132	$13^2 / 2^2 \cdot 3 \cdot 11$	427.782
169/135	$13^2 / 3^3 \cdot 5$	388.877
169/140	$13^2 / 2^2 \cdot 5 \cdot 7$	325.916
169/144	$13^2 / 2^4 \cdot 3^2$	277.145
169/147	$13^2 / 3 \cdot 7^2$	241.449
169/150	$13^2 / 2 \cdot 3 \cdot 5^2$	206.473
169/154	$13^2 / 2 \cdot 7 \cdot 11$	160.911
169/160	$13^2 / 2^5 \cdot 5$	94.742
169/162	$13^2 / 2 \cdot 3^4$	73.235
169/165	$13^2 / 3 \cdot 5 \cdot 11$	41.469
169/168	$13^2 / 2^3 \cdot 3 \cdot 7$	10.274
175/104	$5^2 \cdot 7 / 2^3 \cdot 13$	900.926

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175/117	$5^2 \cdot 7 / 3^2 \cdot 13$	697.016
175/143	$5^2 \cdot 7 / 11 \cdot 13$	349.608
175/156	$5^2 \cdot 7 / 2^2 \cdot 3 \cdot 13$	198.971
175/169	$5^2 \cdot 7 / 13^2$	60.398
176/91	$2^4 \cdot 11 / 7 \cdot 13$	1141.964
176/117	$2^4 \cdot 11 / 3^2 \cdot 13$	706.880
176/169	$2^4 \cdot 11 / 13^2$	70.263
180/91	$2^2 \cdot 3^2 \cdot 5 / 7 \cdot 13$	1180.870
180/143	$2^2 \cdot 3^2 \cdot 5 / 11 \cdot 13$	398.378
180/169	$2^2 \cdot 3^2 \cdot 5 / 13^2$	109.168
182/99	$2 \cdot 7 \cdot 13 / 3^2 \cdot 11$	1054.126
182/121	$2 \cdot 7 \cdot 13 / 11^2$	706.718
182/125	$2 \cdot 7 \cdot 13 / 5^3$	650.412
182/135	$2 \cdot 7 \cdot 13 / 3^3 \cdot 5$	517.175
182/165	$2 \cdot 7 \cdot 13 / 3 \cdot 5 \cdot 11$	169.767
189/104	$3^3 \cdot 7 / 2^3 \cdot 13$	1034.163
189/130	$3^3 \cdot 7 / 2 \cdot 5 \cdot 13$	647.850
189/143	$3^3 \cdot 7 / 11 \cdot 13$	482.845
189/169	$3^3 \cdot 7 / 13^2$	193.636
192/143	$2^6 \cdot 3 / 11 \cdot 13$	510.109
192/169	$2^6 \cdot 3 / 13^2$	220.900
195/98	$3 \cdot 5 \cdot 13 / 2 \cdot 7^2$	1191.145
195/112	$3 \cdot 5 \cdot 13 / 2^4 \cdot 7$	959.970
195/121	$3 \cdot 5 \cdot 13 / 11^2$	826.160
195/128	$3 \cdot 5 \cdot 13 / 2^7$	728.796
195/154	$3 \cdot 5 \cdot 13 / 2 \cdot 7 \cdot 11$	408.653
195/176	$3 \cdot 5 \cdot 13 / 2^4 \cdot 11$	177.478
196/117	$2^2 \cdot 7^2 / 3^2 \cdot 13$	893.214
196/143	$2^2 \cdot 7^2 / 11 \cdot 13$	545.806
196/169	$2^2 \cdot 7^2 / 13^2$	256.596
196/195	$2^2 \cdot 7^2 / 3 \cdot 5 \cdot 13$	8.855
198/169	$2 \cdot 3^2 \cdot 11 / 13^2$	274.173
200/117	$2^3 \cdot 5^2 / 3^2 \cdot 13$	928.190
200/143	$2^3 \cdot 5^2 / 11 \cdot 13$	580.782
200/169	$2^3 \cdot 5^2 / 13^2$	291.572
208/105	$2^4 \cdot 13 / 3 \cdot 5 \cdot 7$	1183.433
208/121	$2^4 \cdot 13 / 11^2$	937.892
208/125	$2^4 \cdot 13 / 5^3$	881.586
208/135	$2^4 \cdot 13 / 3^3 \cdot 5$	748.349
208/147	$2^4 \cdot 13 / 3 \cdot 7^2$	600.921
208/165	$2^4 \cdot 13 / 3 \cdot 5 \cdot 11$	400.941
208/175	$2^4 \cdot 13 / 5^2 \cdot 7$	299.074
208/189	$2^4 \cdot 13 / 3^3 \cdot 7$	165.837
210/143	$2 \cdot 3 \cdot 5 \cdot 7 / 11 \cdot 13$	665.249
210/169	$2 \cdot 3 \cdot 5 \cdot 7 / 13^2$	376.039
216/143	$2^3 \cdot 3^3 / 11 \cdot 13$	714.019
216/169	$2^3 \cdot 3^3 / 13^2$	424.810
220/117	$2^2 \cdot 5 \cdot 11 / 3^2 \cdot 13$	1093.194
220/169	$2^2 \cdot 5 \cdot 11 / 13^2$	456.576
224/117	$2^5 \cdot 7 / 3^2 \cdot 13$	1124.388
224/143	$2^5 \cdot 7 / 11 \cdot 13$	776.980
224/169	$2^5 \cdot 7 / 13^2$	487.771
224/195	$2^5 \cdot 7 / 3 \cdot 5 \cdot 13$	240.030
225/143	$3^2 \cdot 5^2 / 11 \cdot 13$	784.692
225/169	$3^2 \cdot 5^2 / 13^2$	495.482
225/182	$3^2 \cdot 5^2 / 2 \cdot 7 \cdot 13$	367.184
225/208	$3^2 \cdot 5^2 / 2^4 \cdot 13$	136.010
231/130	$3 \cdot 7 \cdot 11 / 2 \cdot 5 \cdot 13$	995.257
231/169	$3 \cdot 7 \cdot 11 / 13^2$	541.044
231/208	$3 \cdot 7 \cdot 11 / 2^4 \cdot 13$	181.571
234/121	$2 \cdot 3^2 \cdot 13 / 11^2$	1141.802
234/125	$2 \cdot 3^2 \cdot 13 / 5^3$	1085.497
234/175	$2 \cdot 3^2 \cdot 13 / 5^2 \cdot 7$	502.984
240/143	$2^4 \cdot 3 \cdot 5 / 11 \cdot 13$	896.423
240/169	$2^4 \cdot 3 \cdot 5 / 13^2$	607.213
242/169	$2 \cdot 11^2 / 13^2$	621.581
242/195	$2 \cdot 11^2 / 3 \cdot 5 \cdot 13$	373.840
243/130	$3^5 / 2 \cdot 5 \cdot 13$	1082.934
243/143	$3^5 / 11 \cdot 13$	917.929
243/169	$3^5 / 13^2$	628.720
243/182	$3^5 / 2 \cdot 7 \cdot 13$	500.421
243/208	$3^5 / 2^4 \cdot 13$	269.247
245/143	$5 \cdot 7^2 / 11 \cdot 13$	932.120
245/156	$5 \cdot 7^2 / 2^2 \cdot 3 \cdot 13$	781.483
245/169	$5 \cdot 7^2 / 13^2$	642.910
245/208	$5 \cdot 7^2 / 2^4 \cdot 13$	283.438
245/234	$5 \cdot 7^2 / 2 \cdot 3^2 \cdot 13$	79.528
250/143	$2 \cdot 5^3 / 11 \cdot 13$	967.096
250/169	$2 \cdot 5^3 / 13^2$	677.886
252/143	$2^2 \cdot 3^2 \cdot 7 / 11 \cdot 13$	980.890
252/169	$2^2 \cdot 3^2 \cdot 7 / 13^2$	691.681
256/143	$2^8 / 11 \cdot 13$	1008.154
256/169	$2^8 / 13^2$	718.945
256/195	$2^8 / 3 \cdot 5 \cdot 13$	471.204
260/147	$2^2 \cdot 5 \cdot 13 / 3 \cdot 7^2$	987.235
260/189	$2^2 \cdot 5 \cdot 13 / 3^3 \cdot 7$	552.150
260/231	$2^2 \cdot 5 \cdot 13 / 3 \cdot 7 \cdot 11$	204.743
260/243	$2^2 \cdot 5 \cdot 13 / 3^5$	117.066
264/169	$2^3 \cdot 3 \cdot 11 / 13^2$	772.218
270/143	$2 \cdot 3^3 \cdot 5 / 11 \cdot 13$	1100.333
270/169	$2 \cdot 3^3 \cdot 5 / 13^2$	811.123
273/160	$3 \cdot 7 \cdot 13 / 2^5 \cdot 5$	924.995
273/176	$3 \cdot 7 \cdot 13 / 2^4 \cdot 11$	759.991
273/200	$3 \cdot 7 \cdot 13 / 2^3 \cdot 5^2$	538.681
273/220	$3 \cdot 7 \cdot 13 / 2^2 \cdot 5 \cdot 11$	373.677
273/242	$3 \cdot 7 \cdot 13 / 2 \cdot 11^2$	208.673
273/250	$3 \cdot 7 \cdot 13 / 2 \cdot 5^3$	152.367
273/256	$3 \cdot 7 \cdot 13 / 2^8$	111.309

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Numbers of the family grouping $2^a \cdot 3^b \cdot 5^c \cdot 13^d$ (Mayan) 13-Limit Subset

1	*
2	*
3	*
4	2^2
5	*
6	$2 \cdot 3$
8	2^3
9	3^2
10	$2 \cdot 5$
12	$2^2 \cdot 3$
13	*
15	$3 \cdot 5$
16	2^4
18	$2 \cdot 3^2$
20	$2^2 \cdot 5$
24	$2^3 \cdot 3$
25	5^2
26	$2 \cdot 13$
27	3^3
30	$2 \cdot 3 \cdot 5$
32	2^5
36	$2^2 \cdot 3^2$
39	$3 \cdot 13$
40	$2^3 \cdot 5$
45	$3^2 \cdot 5$
48	$2^4 \cdot 3$
50	$2 \cdot 5^2$
52	$2^2 \cdot 13$
54	$2 \cdot 3^3$
60	$2^2 \cdot 3 \cdot 5$
64	2^6
65	$5 \cdot 13$
72	$2^3 \cdot 3^2$
75	$3 \cdot 5^2$
78	$2 \cdot 3 \cdot 13$
80	$2^4 \cdot 5$
81	3^4
90	$2 \cdot 3^2 \cdot 5$
96	$2^5 \cdot 3$
100	$2^2 \cdot 5^2$
104	$2^3 \cdot 13$
108	$2^2 \cdot 3^3$
117	$3^2 \cdot 13$
120	$2^3 \cdot 3 \cdot 5$
125	5^3
128	2^7
130	$2 \cdot 5 \cdot 13$
135	$3^3 \cdot 5$
144	$2^4 \cdot 3^2$
150	$2 \cdot 3 \cdot 5^2$
156	$2^2 \cdot 3 \cdot 13$
160	$2^5 \cdot 5$
162	$2 \cdot 3^4$
169	13^2
180	$2^2 \cdot 3^2 \cdot 5$
192	$2^6 \cdot 3$
195	$3 \cdot 5 \cdot 13$
200	$2^3 \cdot 5^2$
208	$2^4 \cdot 13$
216	$2^3 \cdot 3^3$
225	$3^2 \cdot 5^2$
234	$2 \cdot 3^2 \cdot 13$
240	$2^4 \cdot 3 \cdot 5$
243	3^5
250	$2 \cdot 5^3$
256	2^8
260	$2^2 \cdot 5 \cdot 13$
270	$2 \cdot 3^3 \cdot 5$
288	$2^5 \cdot 3^2$
300	$2^2 \cdot 3 \cdot 5^2$
312	$2^3 \cdot 3 \cdot 13$
320	$2^6 \cdot 5$
324	$2^2 \cdot 3^4$
325	$5^2 \cdot 13$
338	$2 \cdot 13^2$
351	$3^3 \cdot 13$
360	$2^3 \cdot 3^2 \cdot 5$
375	$3 \cdot 5^3$
384	$2^7 \cdot 3$
390	$2 \cdot 3 \cdot 5 \cdot 13$
400	$2^4 \cdot 5^2$
405	$3^4 \cdot 5$
416	$2^5 \cdot 13$
432	$2^4 \cdot 3^3$
450	$2 \cdot 3^2 \cdot 5^2$
468	$2^2 \cdot 3^2 \cdot 13$
480	$2^5 \cdot 3 \cdot 5$
486	$2 \cdot 3^5$
500	$2^2 \cdot 5^3$
507	$3 \cdot 13^2$
512	2^9
520	$2^3 \cdot 5 \cdot 13$
540	$2^2 \cdot 3^3 \cdot 5$
576	$2^6 \cdot 3^2$
585	$3^2 \cdot 5 \cdot 13$
600	$2^3 \cdot 3 \cdot 5^2$
624	$2^4 \cdot 3 \cdot 13$
625	5^4
640	$2^7 \cdot 5$
648	$2^3 \cdot 3^4$
650	$2 \cdot 5^2 \cdot 13$
675	$3^3 \cdot 5^2$
676	$2^2 \cdot 13^2$
702	$2 \cdot 3^3 \cdot 13$
720	$2^4 \cdot 3^2 \cdot 5$
729	3^6
750	$2 \cdot 3 \cdot 5^3$
768	$2^8 \cdot 3$
780	$2^2 \cdot 3 \cdot 5 \cdot 13$
800	$2^5 \cdot 5^2$
810	$2 \cdot 3^4 \cdot 5$
832	$2^6 \cdot 13$
845	$5 \cdot 13^2$
864	$2^5 \cdot 3^3$
900	$2^2 \cdot 3^2 \cdot 5^2$
936	$2^3 \cdot 3^2 \cdot 13$
960	$2^6 \cdot 3 \cdot 5$
972	$2^2 \cdot 3^5$
975	$3 \cdot 5^2 \cdot 13$
1000	$2^3 \cdot 5^3$
1014	$2 \cdot 3 \cdot 13^2$
1040	$2^4 \cdot 5 \cdot 13$
1053	$3^4 \cdot 13$
1080	$2^3 \cdot 3^3 \cdot 5$
1125	$3^2 \cdot 5^3$
1152	$2^7 \cdot 3^2$
1170	$2 \cdot 3^2 \cdot 5 \cdot 13$
1200	$2^4 \cdot 3 \cdot 5^2$

Numbers of the family grouping $2^a \cdot 5^b \cdot 13^c$ (Mayan) 13-Limit subset

1	*
2	*
4	2^2
5	*
8	2^3
10	$2 \cdot 5$
13	*
16	2^4
20	$2^2 \cdot 5$
25	5^2
26	$2 \cdot 13$
32	2^5
40	$2^3 \cdot 5$
50	$2 \cdot 5^2$
52	$2^2 \cdot 13$
64	2^6
65	$5 \cdot 13$
80	$2^4 \cdot 5$
100	$2^2 \cdot 5^2$
104	$2^3 \cdot 13$
125	5^3
128	2^7
130	$2 \cdot 5 \cdot 13$
160	$2^5 \cdot 5$
169	13^2
200	$2^3 \cdot 5^2$
208	$2^4 \cdot 13$
250	$2 \cdot 5^3$
256	2^8
260	$2^2 \cdot 5 \cdot 13$
320	$2^6 \cdot 5$
325	$5^2 \cdot 13$
338	$2 \cdot 13^2$
400	$2^4 \cdot 5^2$
416	$2^5 \cdot 13$
500	$2^2 \cdot 5^3$
512	2^9
520	$2^3 \cdot 5 \cdot 13$
625	5^4
640	$2^7 \cdot 5$
650	$2 \cdot 5^2 \cdot 13$
676	$2^2 \cdot 13^2$
800	$2^5 \cdot 5^2$
832	$2^6 \cdot 13$
845	$5 \cdot 13^2$
1000	$2^3 \cdot 5^3$
1040	$2^4 \cdot 5 \cdot 13$

A complex 13-Limit Monochord using double 360:720 and based on 'Mayan' number family $2^g \cdot 3^b \cdot 5^c \cdot 13^d$

The bracket indicates the 13-Limit elements

Number Series	360	:	375	:	384	:	(390)	:	400	:	405	:
Over Pitch——	C		C#		Db				\D		D	
Ratio——	1/1		25/24		16/15		13/12		10/9		9/8	
Under Pitch——	C		Cb		B				Bb		\Bb	
Ratio——	2/1		48/25		15/8		24/13		9/5		16/9	

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(416)	:	432	:	450	:	(468)	:	480	:	486	:	500	:	507	:
		Eb		E				F		/F		\F#			
52/45		6/5		5/4		13/10		4/3		27/20		25/18		169/120	
		A		Ab				G		\G		Gb			
45/26		5/3		8/5		20/13		3/2		40/27		36/25		240/169	

512	:	(520)	:	540	:	576	:	(585)	:	600	:	(624)	:	625	:
\Gb				G		Ab				A				\A#	
64/45		13/9		3/2		8/5		13/8		5/3		26/15		125/72	
F#				F		E				Eb				Ebb	
45/32		18/13		4/3		5/4		16/13		6/5		15/13		144/125	

640	:	648	:	(650)	:	675	:	(676)	:	(702)	:	720
\Bb		Bb				B						C
16/9		9/5		65/36		15/8		169/90		39/20		2/1
D		\D				Db						C
9/8		10/9		72/65		16/15		180/169		40/39		1/1

Shades of the Genera (53-E.T. division): A systematic generation with the diesis as the minimum scalar element. (Altogether 153 patterns).

Pitch → / Number →	G 31	32	33	G# 34	\Ab 35	Ab 36	37	38	A 39	/A 40	41	42	A# 43	\Bb 44	Bb 45	46	47	B 48	/B 49	Cb 50	51	52	C 53	Name of Tetrachord
2·2·18	●		●		●																		●	Enharmonic (Pythagorean)
2·3·17	●		●			●																	●	Enharmonic (Just)
3·2·17	●			●		●																	●	Enharmonic (Archytas)
2·4·16	●		●				●																●	
3·3·16	●			●			●																●	
4·2·16	●				●		●																●	
2·5·15	●		●					●															●	
3·4·15	●			●				●															●	
4·3·15	●				●			●															●	
5·2·15	●					●		●															●	
2·6·14	●		●						●														●	
3·5·14	●			●					●														●	Just Chromatic (Ptolemy)
4·4·14	●				●				●														●	
5·3·14	●					●			●														●	Just Chromatic (Classical Gk.)
6·2·14	●						●		●														●	
2·7·13	●		●							●													●	
3·6·13	●			●						●													●	
4·5·13	●				●					●													●	Pythagorean Chromatic
5·4·13	●					●				●													●	
6·3·13	●						●			●													●	
7·2·13	●							●		●													●	
2·8·12	●		●								●												●	
3·7·12	●			●							●												●	
4·6·12	●				●						●												●	
5·5·12	●					●					●												●	
6·4·12	●						●				●												●	
7·3·12	●							●			●												●	
8·2·12	●								●		●												●	
2·9·11	●		●									●											●	
3·8·11	●			●								●											●	

(continued over page).....

Pitch→ / Number→	G 31	32	33	G# 34	Ab 35	Ab 36	37	38	A 39	/A 40	41	42	A# 43	Bb 44	Bb 45	46	47	B 48	/B 49	Cb 50	51	52	C 53	Name of Tetrachord
4·7·11	●				●							●											●	
5·6·11	●					●						●											●	
6·5·11	●						●					●											●	
7·4·11	●							●				●											●	
8·3·11	●								●			●											●	
9·2·11	●									●		●											●	
2·10·10	●		●										●										●	
3·9·10	●			●									●										●	
4·8·10	●				●								●										●	Ptolemy Soft diatonic
5·7·10	●					●							●										●	
6·6·10	●						●						●										●	
7·5·10	●							●					●										●	
8·4·10	●								●				●										●	
9·3·10	●									●			●										●	
10·2·10	●										●		●										●	
2·11·9	●		●											●									●	
3·10·9	●			●										●									●	Septimal Minor Diatonic (Archytas middle soft diatonic)
4·9·9	●				●									●									●	Pythagorean Minor Diatonic
5·8·9	●					●								●									●	Just Minor Diatonic (Didymus)
6·7·9	●						●							●									●	
7·6·9	●							●						●									●	
8·5·9	●								●					●									●	Just non-symmetrical 5,6,7□
9·4·9	●									●				●									●	Pythagorean 5,6,7□ (symmetrical)
10·3·9	●										●			●									●	
11·2·9	●											●		●									●	
2·12·8	●		●												●								●	
3·11·8	●			●											●								●	
4·10·8	●				●										●								●	
5·9·8	●					●									●								●	Just Minor Diatonic
6·8·8	●						●								●								●	
7·7·8	●							●							●								●	Gk. Equal Diatonic (11th sub-harmonic element)
8·6·8	●								●						●								●	Just Symmetrical 5,6,7□ (classical Greek Phrygian)

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Pitch→ / Number→	G 31	32	33	G# 34	Ab 35	Ab 36	37	38	A 39	↑A 40	41	42	A# 43	↓Bb 44	Bb 45	46	47	B 48	↑B 49	Cb 50	51	52	C 53	Name of Tetrachord
9·5·8	●									●					●								●	Just non-symmetrical 5,6,7□
10·4·8	●										●				●								●	
11·3·8	●											●			●								●	
12·2·8	●												●		●								●	
2·13·7	●		●													●							●	
3·12·7	●			●												●							●	
4·11·7	●				●											●							●	
5·10·7	●					●										●							●	
6·9·7	●						●									●							●	
7·8·7	●							●								●							●	
8·7·7	●								●							●							●	Inverse Gk. Equal Diatonic (11th harmonic element)
9·6·7	●									●						●							●	
10·5·7	●										●					●							●	
11·4·7	●											●				●							●	
12·3·7	●												●			●							●	
13·2·7	●													●		●							●	
2·14·6	●		●														●						●	
3·13·6	●			●													●						●	
4·12·6	●				●												●						●	
5·11·6	●					●											●						●	
6·10·6	●						●										●						●	
7·9·6	●							●									●						●	
8·8·6	●								●								●						●	
9·7·6	●									●							●						●	
10·6·6	●										●						●						●	
11·5·6	●											●					●						●	
12·4·6	●												●				●						●	
13·3·6	●													●			●						●	
14·2·6	●														●		●						●	
2·15·5	●		●															●					●	
3·14·5	●			●														●					●	
4·13·5	●				●													●					●	

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Pitch→ / Number→	G 31	32	33	G# 34	Ab 35	Ab 36	37	38	A 39	/A 40	41	42	A# 43	Bb 44	Bb 45	46	47	B 48	/B 49	Cb 50	51	52	C 53	Name of Tetrachord
5·12·5	●					●												●					●	Just Symmetrical M7b6 □
6·11·5	●						●											●					●	
7·10·5	●							●										●					●	
8·9·5	●								●									●					●	Just Major Diatonic
9·8·5	●									●								●					●	Just Major Diatonic (Inverse Didymus)
10·7·5	●										●							●					●	
11·6·5	●											●						●					●	
12·5·5	●												●					●					●	
13·4·5	●													●				●					●	Inverse Pythagorean Chromatic
14·3·5	●														●			●					●	Inverse Gk. Chromatic
15·2·5	●															●		●					●	
2·16·4	●		●																●				●	
3·15·4	●			●															●				●	
4·14·4	●				●														●				●	
5·13·4	●					●													●				●	
6·12·4	●						●												●				●	
7·11·4	●							●											●				●	
8·10·4	●								●										●				●	
9·9·4	●									●									●				●	Pythagorean Major Diatonic
10·8·4	●										●								●				●	Inverse Ptolemy Soft Diatonic
11·7·4	●											●							●				●	
12·6·4	●												●						●				●	
13·5·4	●													●					●				●	
14·4·4	●														●				●				●	
15·3·4	●															●			●				●	
16·2·4	●																●		●				●	
2·17·3	●		●																	●			●	
3·16·3	●			●																●			●	
4·15·3	●				●															●			●	
5·14·3	●					●														●			●	
6·13·3	●						●													●			●	
7·12·3	●							●												●			●	

Pitch → / Number →	G 31	32	33	G# 34	Ab 35	Ab 36	37	38	A 39	/A 40	41	42	A# 43	Bb 44	Bb 45	46	47	B 48	/B 49	Cb 50	51	52	C 53	Name of Tetrachord
8·11·3	●								●											●			●	
9·10·3	●									●										●			●	Septimal Major Diatonic (Inverse Archytas Soft Diatonic)
10·9·3	●										●									●			●	
11·8·3	●											●								●			●	
12·7·3	●												●							●			●	
13·6·3	●													●						●			●	
14·5·3	●														●					●			●	Inverse Ptolemy Chromatic
15·4·3	●															●				●			●	
16·3·3	●																●			●			●	
17·2·3	●																	●		●			●	Inverse Archytas Enharmonic
2·18·2	●		●																		●		●	
3·17·2	●			●																	●		●	
4·16·2	●				●																●		●	
5·15·2	●					●															●		●	
6·14·2	●						●														●		●	
7·13·2	●							●													●		●	
8·12·2	●								●												●		●	
9·11·2	●									●											●		●	
10·10·2	●										●										●		●	
11·9·2	●											●									●		●	
12·8·2	●												●								●		●	
13·7·2	●													●							●		●	
14·6·2	●														●						●		●	
15·5·2	●															●					●		●	
16·4·2	●																●				●		●	
17·3·2	●																	●			●		●	Inverse Just Enharmonic
18·2·2	●																		●		●		●	Inverse Pythagorean Enharmonic

Shades of the Genera (31-E.T. division) A systematic generation

Pitch → / Number →	G 18	G♯ 19	G# 20	Ab 21	A♭ 22	A 23	A♯ 24	A# 25	Bb 26	B♭ 27	B 28	B♯ 29	C♭ 30	C 31	Name of Tetrachord
1 · 1 · 11	●	●	●											●	Enharmonic
1 · 2 · 10	●	●		●										●	Enharmonic
2 · 1 · 10	●		●	●										●	Enharmonic (Archytas)
1 · 3 · 9	●	●			●									●	
2 · 2 · 9	●		●		●									●	
3 · 1 · 9	●			●	●									●	
1 · 4 · 8	●	●				●								●	
2 · 3 · 8	●		●			●								●	Just Chromatic (Ptolemy)
3 · 2 · 8	●			●		●								●	Just Chromatic (Classical Gk.)
4 · 1 · 8	●				●	●								●	
1 · 5 · 7	●	●					●							●	
2 · 4 · 7	●		●				●							●	
3 · 3 · 7	●			●			●							●	
4 · 2 · 7	●				●		●							●	
5 · 1 · 7	●					●	●							●	
1 · 6 · 6	●	●						●						●	
2 · 5 · 6	●		●					●						●	
3 · 4 · 6	●			●				●						●	Ptolemy Soft Diatonic
4 · 3 · 6	●				●			●						●	
5 · 2 · 6	●					●		●						●	
6 · 1 · 6	●						●	●						●	
1 · 7 · 5	●	●							●					●	
2 · 6 · 5	●		●						●					●	Septimal Minor Diatonic (Archytas Middle Soft Diatonic)
3 · 5 · 5	●			●					●					●	Minor Diatonic
4 · 4 · 5	●				●				●					●	Gk. Equal Diatonic (11th sub-harmonic element)
5 · 3 · 5	●					●			●					●	5, 6, 7□ (Classical Gk. Phrygian)
6 · 2 · 5	●						●		●					●	
7 · 1 · 5	●							●	●					●	
1 · 8 · 4	●	●								●				●	
2 · 7 · 4	●		●							●				●	

(continued over page...)

Pitch → / Number →	G 18	G♯ 19	G♯ 20	A♭ 21	A♮ 22	A 23	A♯ 24	A♯ 25	B♭ 26	B♮ 27	B 28	B♯ 29	C♮ 30	C 31	Name of Tetrachord
3 · 6 · 4	●			●						●				●	
4 · 5 · 4	●				●					●				●	
5 · 4 · 4	●					●				●				●	Inverse Gk. Equal Diatonic (11th Harmonic element)
6 · 3 · 4	●						●			●				●	
7 · 2 · 4	●							●		●				●	
8 · 1 · 4	●								●	●				●	
1 · 9 · 3	●	●									●			●	
2 · 8 · 3	●		●								●			●	
3 · 7 · 3	●			●							●			●	Symmetrical M766□
4 · 6 · 3	●				●						●			●	
5 · 5 · 3	●					●					●			●	Major Diatonic
6 · 4 · 3	●						●				●			●	Inverse Ptolemy Soft Diatonic
7 · 3 · 3	●							●			●			●	
8 · 2 · 3	●								●		●			●	Inverse Gk. Chromatic
9 · 1 · 3	●									●	●			●	
1 · 10 · 2	●	●										●		●	
2 · 9 · 2	●		●									●		●	
3 · 8 · 2	●			●								●		●	
4 · 7 · 2	●				●							●		●	
5 · 6 · 2	●					●						●		●	Septimal Major Diatonic (Inverse Archytas Soft Dia.)
6 · 5 · 2	●						●					●		●	
7 · 4 · 2	●							●				●		●	
8 · 3 · 2	●								●			●		●	Inverse Ptolemy Chromatic
9 · 2 · 2	●									●		●		●	
10 · 1 · 2	●										●	●		●	Inverse Archytas Enharmonic
1 · 11 · 1	●	●											●	●	
2 · 10 · 1	●		●										●	●	
3 · 9 · 1	●			●									●	●	
4 · 8 · 1	●				●								●	●	
5 · 7 · 1	●					●							●	●	
6 · 6 · 1	●						●						●	●	
7 · 5 · 1	●							●					●	●	

Pitch → / Number →	G 18	G♯ 19	G♯ 20	Ab 21	A♭ 22	A 23	A♯ 24	A♯ 25	B♭ 26	B♭ 27	B 28	B♯ 29	C♭ 30	C 31	Name of Tetrachord
8 · 4 · 1	●								●				●	●	
9 · 3 · 1	●									●			●	●	
10 · 2 · 1	●										●		●	●	Inverse Enharmonic
11 · 1 · 1	●											●	●	●	Inverse Enharmonic

Shades of the Genera (12-E.T. division) A systematic generation

Pitch → / Number →	G 7	Ab 8	A 9	B♭ 10	B 11	C 12	Name of Tetrachord
1 · 1 · 3	●	●	●			●	Chromatic
1 · 2 · 2	●	●		●		●	Minor Diatonic
2 · 1 · 2	●		●	●		●	5,6,7 □
1 · 3 · 1	●	●			●	●	M7,b6 □
2 · 2 · 1	●		●		●	●	Major Diatonic
3 · 1 · 1	●			●	●	●	Inverse Chromatic

Shades of the Genera (19-E.T. division) A systematic generation

Pitch Number	G 11	G# 12	Ab 13	A 14	A# 15	Bb 16	B 17	B# Cb 18	C 19	Name of Tetrachord
1 · 1 · 6	●	●	●						●	Enharmonic
1 · 2 · 5	●	●		●					●	Chromatic
2 · 1 · 5	●		●	●					●	Chromatic
1 · 3 · 4	●	●			●				●	
2 · 2 · 4	●		●		●				●	
3 · 1 · 4	●			●	●				●	
1 · 4 · 3	●	●				●			●	
2 · 3 · 3	●		●			●			●	Minor Diatonic
3 · 2 · 3	●			●		●			●	5,6,7□
4 · 1 · 3	●				●	●			●	
1 · 5 · 2	●	●					●		●	
2 · 4 · 2	●		●				●		●	M7,b6□
3 · 3 · 2	●			●			●		●	Major Diatonic
4 · 2 · 2	●				●		●		●	
5 · 1 · 2	●					●	●		●	
1 · 6 · 1	●	●						●	●	
2 · 5 · 1	●		●					●	●	
3 · 4 · 1	●			●				●	●	
4 · 3 · 1	●				●			●	●	Inverse Chromatic
5 · 2 · 1	●					●		●	●	Inverse Chromatic
6 · 1 · 1	●						●	●	●	Enharmonic (Inverse)