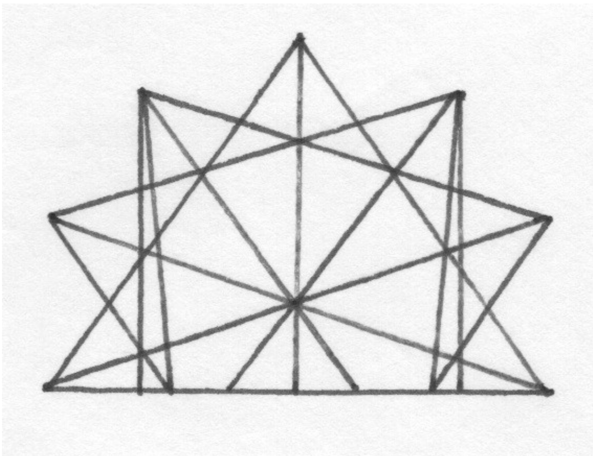
More Notes on Notation



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—on the notation of Just Intonation

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More Notes On Notation

by Siemen Terpstra

I would like to respond to the article "Just Inton(ot)ation," written by Paul Rapoport and published in **1/1**, Volume 7, Number 1 (September 1991). I was very pleased that he included my notation in his survey of the four main alternatives. He covered a lot of ground in comparing the various strengths and weaknesses of each of the models. He also gave a brief introduction to the problems of integrating higher prime ratios into the five-limit fabric. Unfortunately, I feel that Rapoport misrepresented my work in various ways. I was surprised about this since he has copies of many of my papers. I hope that he will be pleased by my brief criticism of his paper and by the various clarifications presented here.

To start, he gives my five-limit matrix on page 13 as a 90 degree grid with the line of fifths (the three-limit subset) oriented to the vertical. But I have never used this orientation, since I think that a horizontal line is much more practical and readable. Long lines of fifths are more common in tuning than long lines of thirds; therefore, the matrix tends to be expanded into a "lens" pattern which is more comfortable to handle on the horizontal than on the vertical. The other models all use the vertical orientation.

Associated with this issue is another of more importance. For not only do I use a horizontal orientation for the line of fifths, I also use a 60 degree tri-axial matrix (see Figure 1, page 4) rather than the 90 degree x and y axis matrix employed by the other three models. Rapoport must have felt that it was OK to make my grid conform to the general "90-degree mind-set."

But now you may ask: "What difference does it make to use a triangular grid rather than a square one since both give the same information?" Indeed, it is true that both approaches give the same material (the 3 × 5 Just Intonation grid). However, there is a major flaw in using the square or rectangular grid. The square grid implies that there are two tuning axes for five-limit Just Intonation: the axis of fifths/fourths, and the axis of major thirds/minor sixths. In other words, the model is a di-axial matrix. But in reality, there are three tuning axes for a five-limit just harmony, the third being the axis of minor thirds/major sixths. This third axis is skewed, hidden, or demeaned by using the square grid. On the other hand, in the tri-axial matrix, the three tuning axes of Just Intonation are given equal weight, as they should be. Thus, the line of fifths-fourths is modelled as horizontal, and the major and minor thirds are the two diagonals. After one has worked with the triangular model for a while, one can better appreciate its superior design "modelability." The triangular orientation also provides the closest packing of harmonic relations, which a square grid cannot.

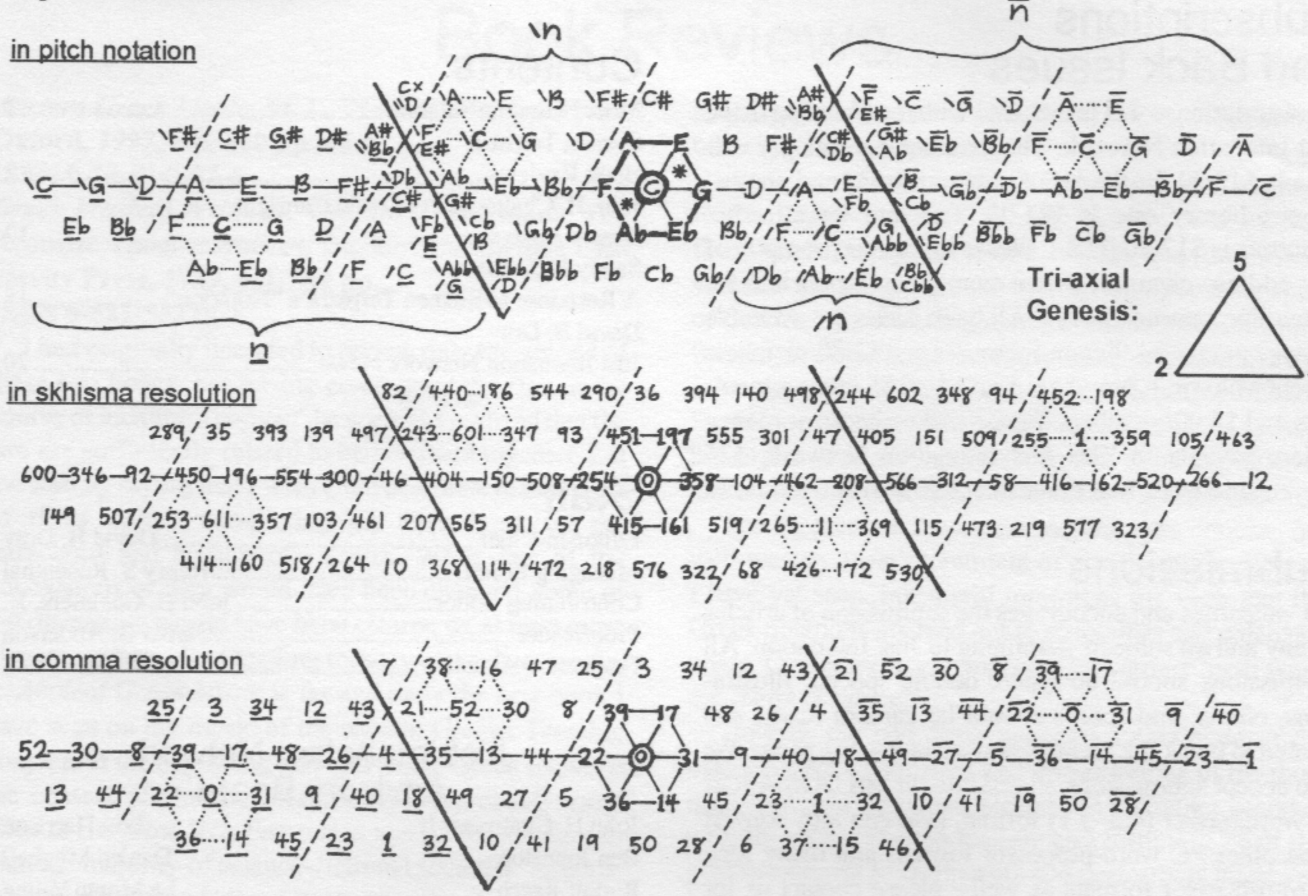
Not only does the tri-axial grid make the three axes more evident, it also highlights the six tuning directions of five-limit Just Intonation. These primary operations yield the Just Intonation hexagon, which surrounds the generator tone (C) and defines the only musical consonances. I feel that it is appropriate always to highlight the consonances, since they are such a precious rarity in the sea of dissonances which the grid generates. I also highlight the three-limit line of musical fifths (which is the three-limit subset of the five-limit field) as a major orientation in the grid.

The central hexagon is also aesthetically pleasing,

(text continued on page 4)

"Just Intonation is the best intonation" —Lou Harrison

Figure 1. Five-limit matrix



because of the way in which it presents the consonances. The perfect consonances are given by the central horizontal line. These perfect consonances are C (ratios $^1/_1$ and $^2/_1$), G (ratios $^3/_1$ and $^3/_2$), and F (ratio $^4/_3$). The medial consonances appear on the line above it. These medial consonances are A (ratio $^5/_3$) and E (ratios $^5/_1$, $^5/_2$, and $^5/_4$). Finally, the so-called imperfect consonances appear below it. These are E♭ (ratio $^6/_5$) and A♭ (ratio $^8/_5$). Remember that the matrix is octave invariant, which means that any octave of the pitch will have the same "address" in the matrix. The wonderfully balanced image presented by the consonance hexagon in the tri-axial matrix is destroyed by using the square grid, the beautiful hexagon becoming an irregular polygon.

In highlighting the central consonances and the line of fifths, we have the first examples of my consistent practice of marking in the significant boundary points on the grid. This makes it easier to read, easier to find your way around in, and clarifies its internal architecture. From what I have seen of the other versions of the grid, no one else appears to recognize the value or relevance in making the regions explicit. This may be partly because the di-axial model is not so conducive to regional differentiation. For whatever reason, the other three models do not include their own boundary markers.

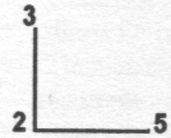
The triangular grid also makes the regional boundaries easier to draw in, and generally easier to integrate into an overall picture of the Just Intonation matrix. My particular notation of five-limit Just Intonation has evolved directly from the notion of boundaries and regions.

The triangular grid also makes the reading of just major and minor triads much simpler, since a just major triad is an upturned triangle, and a just minor triad is a down-turned triangle. Try finding triads on a square grid. Note how much more awkward it is. Once one has gotten used to the conception that each type of harmony has its own geometrical pattern on the grid, it becomes easy to find any harmony on the matrix.

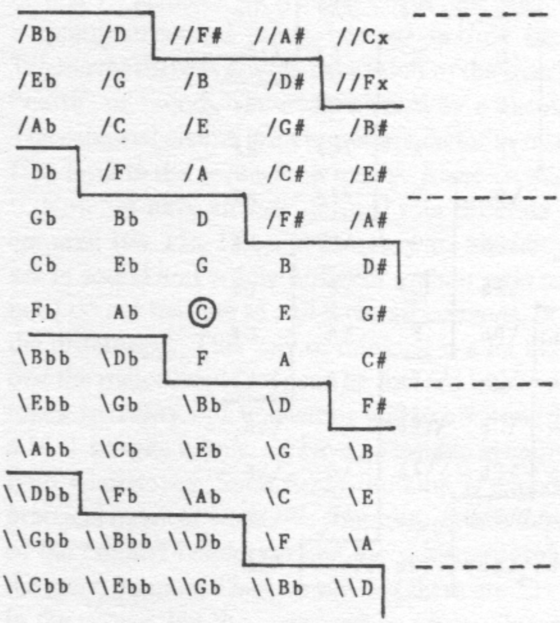
The whole architectural concept of boundaries and regions stems from the encounter with two commanding micro-intervals on the grid. These key micro-intervals are the skhisma (s for short) and the syntonic comma (c

Figure 2. Five-limit pitch matrix: di-axial model

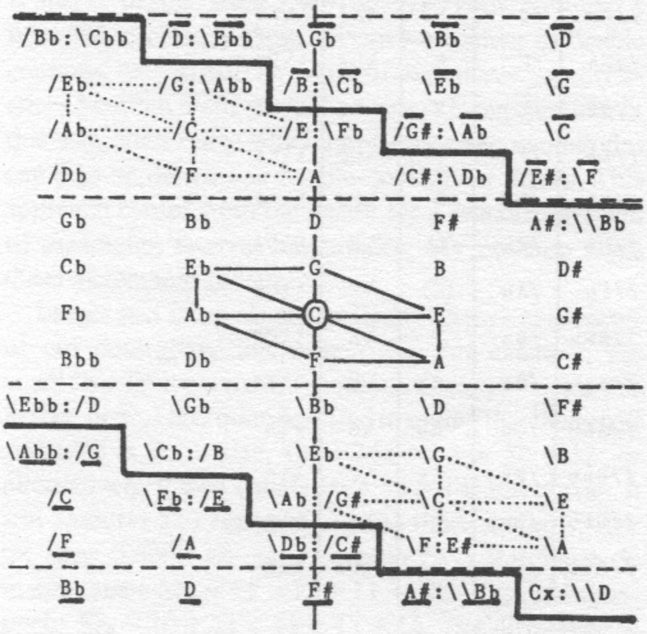
Genesis:



Ben Johnston and others



Siemen Terpstra, translated to 90-degree axis with various regional boundaries marked



for short). Any notation for a wide expansion of the five-limit Just Intonation grid must take these two intervals into account.

The c-alteration sign is conveniently notated on the musical staff with a slash sign. Unfortunately, there is no such convenient sign for s-alteration. One possibility that has been proposed is a plus and minus sign. But this choice is not ideal. For my purposes here, I am using a line under the pitch for lowering one s, and a line above the pitch for raising one s; however, this sign is not suitable for the musical staff. In my paper on Irregular Temperaments and Well-temperaments, I used superscripts. For example, the just major third (5:4) may be raised (tempered) by three s, and may then be written E^{+3}.

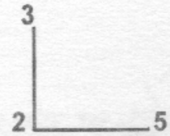
I have raised the specter of temperament here, and it may be offensive to some of you! Perhaps it is best to let you know where I stand. My effort has been to try to understand the structure of tuning systems in general, with a minimum of polemics for or against particular intonations. With this attitude in mind, I have endeavored to evolve a notation which can be used for Just Intonation as well as various tempered systems, especially the ones that closely approximate Just Intonation. Of course, the focus here is on Just Intonation and not on temperament.

Rapoport presents four alternative notations for the matrix, one of which is a highly modified version of mine. Unfortunately, he also got a number of the pitch notations wrong. Also, I feel that he could have made the boundary positions of the various notations clearer and easier to read by making them explicit on the grid. He did not do this for my grid or for the others. Perhaps this is in deference to the other three models, which do not exhibit their regions. In the interest of clarifying the differences between our various notations, I have redrawn the four matrices (Figures 2 and 3), filling in their boundaries where appropriate for all of the models. I have also included the 90 degree twisted version of my grid (even though I find it repulsive) in order to compare the solutions of our various efforts. I will refer to the Eitz, Tenney, and Johnston solutions from time to time. It is useful to compare my 90 degree version with my proper 60 degree version to see which has superior readability.

Anyone who has actually spent time setting just scales quickly becomes familiar with the comma. It is practically unavoidable, unless one restricts the number of pitches radically. If we are setting a chord or a simple

Figure 3. Five-limit pitch matrix: di-axial model

Genesis:



Blackwood, Eitz, Helmholtz, Ellis, Barbour, and others

//Bb	/D	F#	\A#	\\Cx
//Eb	/G	B	\D#	\\Fx
//Ab	/C	E	\G#	\\B#
//Db	/F	A	\C#	\\E#
//Gb	/Bb	D	\F#	\\A#
//Cb	/Eb	G	\B	\\D#
//Fb	/Ab	Ⓒ	\E	\\G#
//Bbb	/Db	F	\A	\\C#
//Ebb	/Gb	Bb	\D	\\F#
//Abb	/Cb	Eb	\G	\\B
//Dbb	/Fb	Ab	\C	\\E
//Gbb	/Bbb	Db	\F	\\A
//Cbb	/Ebb	Gb	\Bb	\\D

James Tenney and Bob Gilmore

Bb	/D	//F#	/A#	//Cx
Eb	/G	B	/D#	//Fx
Ab	/C	E	/G#	B#
Db	/F	A	/C#	E#
Gb	\Bb	D	/F#	A#
Cb	\Eb	G	\B	D#
Fb	\Ab	Ⓒ	\E	G#
\\Bbb	\Db	F	\A	C#
\\Ebb	\Gb	\\Bb	\D	F#
\\Abb	\Cb	\\Eb	\G	\\B
\\Dbb	\Fb	\\Ab	\C	\\E
\\Gbb	\\\Bbb	\\Db	\F	\\A
\\Cbb	\\\Ebb	\\Gb	\\\Bb	\\D

scale, we may not even concern ourselves with notation, since the meaning will always be clear. But for modulations and more subtle harmonies, the comma will make its appearance, to intrigue or annoy us. A Just Intonation notation must make the comma-shifted intervals explicit.

The skhisma is not so readily apparent, since one must use a wide horizontal expansion before it becomes an issue. It is also very small (around 2 cents) so that it can easily be absorbed when tuning long lines of fifths. It is more of a theoretical interval, very useful for clarifying the deep architecture of harmony. For various reasons it is very helpful to resolve the octave scale into both skhismas and commas.

Again there may be objections, since this is basically logarithmic style thinking, similar to the notion of resolving the octave into cents. We associate such concepts with temperaments. Nevertheless, I am sure that all of you have found the logarithmic concept of cents quite useful as a measuring device. The skhisma can be used in a similar way, and it is even more revealing for clarifying structure, as will become apparent shortly. The skhisma division is a deep division, like cents, but much more natural and informative than cents as a gauge of interval size. A cent is roughly a semi-skhisma. (To convert skhismas to cents, multiply by 1.9607843).

The comma resolution is also useful, since more practical just tunings employ comma-shifts; therefore, it is expedient to think of the octave as a scale of commas. Such an approach is relatively crude, but eminently practical, as well as having ancient historical precedent. The skhisma resolution, like the cents resolution, is much more precise, but more theoretical. In the accompanying diagram, I have given a segment of the five-limit tri-axial matrix in pitch notation, in skhisma-number notation, and in comma-number notation. In other words, the size of the interval in commas and skhismas is defined. As we go on, you will see why this information is useful for clarifying the regional architecture of the grid.

The most common awareness of the reality of the skhisma comes from comparing the two most famous commas. They relate by 1s, since a syntonic comma is 11s in size, and a ditonic (Pythagorean) comma is 12s in size. But in addition, the other key microtonal elements in the fabric can also be profitably related to skhisma size. For example, the diaskhisma interval (ratio 2048:2025) is 10s in size, and the septimal (seven-limit) comma (ratio 64:63) is 14s in size. This interval relates to the diaskhisma by ratio 225:224, an important structural interval which I call the septimal clisma, which is 4s in size. This is practically identical to the five-limit clisma (which has the horrendous ratio 15625:15552) and is ~~5s~~ 4s in size. The tolerances are very close: in cents, the five-limit clisma is 8.105 cents, the seven-limit clisma 7.712 cents. 4s is 7.843 cents. The clisma is also a major structural micro-interval in Just Intonation. Whenever there is a wide expansion of the matrix to the "north" or "south," intervals altered by a clisma occur. The septimal clisma is an important factor in relating the five-limit to the seven-limit matrix. More on that later.

Now we have already defined four different sizes of comma: 10s, 11s, 12s, and 14s; they are all subtly different in sound and wildly different in their ratio numbers, but I do not hesitate to call them all commas, in spite of the differences. This way of thinking is a lot like saying that the major third (5:4) can be divided into two wholetones. In reality, one wholetone will be a 9:8 and the other a 10:9. In spite of this, we have no trouble in calling them both wholetones. Such fuzzy thinking is expedient for practical musical work. We should approach the concept of the comma resolution and the skhisma resolution in the same manner. Thus we say that there are 53 commas in the octave, but they need not be identical (tempered) in size. A close look at Just Intonation always reveals a variety of sizes for any interval type.

Most musicians are used to thinking of the octave as resolved into 12 semitones. They adopted this schema from the use of equal-temperament, the use of fretted instruments and keyboards, and the influence of the musical education system. But it is just as feasible to see the octave divided into 12 semitones which are untempered (i.e., using just ratios) and somewhat different in size. The concept of a semitone as a standard of measure is thus a level of resolution and does not necessarily imply temperament. The comma and the skhisma are simply alternative levels of resolution. It is wise to be able to move back and forth between various levels of resolution, and be comfortable in each one. We shall use whatever level of resolution is most convenient for the local context, always keeping in mind that the finest level (the skhisma level) will be much more accurate in describing metrical properties than the relatively crude comma level, or the even cruder diesis and semitone levels.

The usefulness of the skhisma as a "musical interval atom" becomes more and more apparent as we examine other just intervals in the fabric. For example, the just diesis D♭♭ (ratio $^{128}/_{125}$) is 21s in size. Now 21 = 10 + 11. Thus we can say the diesis is 2c in size, even though these two commas are not identical in magnitude. The septimal diesis (ratio 36:35) is slightly wider at 25s. The difference between the two dieses is 25 – 21 = 4s, the septimal clisma. Now I would also assert that the septimal diesis is also 2c in size, since 25s = 11 + 14. Thus, although it is useful to classify these varieties of dieses as double commas, we can also be aware of the microtonal differences between them when the context is appropriate. In this way, other ratios which have very close magnitudes can also be defined as double-commas or dieses. This approach comes from the desire for a practical method of organizing interval magnitudes. My notation takes these factors into account.

Larger just intervals are generally always aggregates of our controlling micro-intervals. For example, the small five-limit chromatic semitone C♯ (ratio $^{25}/_{24}$) is 36s in size, or 3c. I feel quite justified in saying that this small semitone is 3c in size, since 36s = 12 + 12 + 12; or alternatively, it also equals 10 + 12 + 14. Either way, it still amounts to 3 commas. As one more example, take the large diatonic semitone D♭ (ratio $^{16}/_{15}$). It is 57s or 5c in size, since 57s = 11 + 11 + 11 + 12 + 12; or alternatively, 57s = 10 + 11 + 11 + 11 + 14. The difference in pitch between the diatonic semitone (D♭) and the chromatic semitone (C♯) is the diesis, since 57 – 36 = 21s. Do you see how certain key s-numbers keep recurring in the fabric? If we took the trouble to examine more just ratios, we would see the same key microtonal elements present.

I cannot speak for the other three notations given by Rapoport, but my notation rises directly from the examination of skhisma and comma relations. I use an underlying principle, which I will call the principle of optimum selection. Whenever there are various candidates for selecting a given interval, I almost always choose the one with the smallest ratio numbers (and hence the greatest consonance). Using this principle. I can justify my particular notation and criticize the others. Here are a few examples of the principle in action:

The ratio $^{16}/_{15}$ is the best candidate for D♭. Johnston and Tenney use the ratio $^{27}/_{25}$ (68s or 6c). I would reject this choice because of the higher ratio numbers. What is more, the $^{16}/_{15}$ ratio is commonly found in just diatonic scales, whereas $^{27}/_{25}$ is a bit more esoteric. Hence, I feel that the proper notation for $^{27}/_{25}$ is /D♭, or D♭ raised by a comma. In the Eitz notation, the choice is even worse, since D♭ is three-limit ratio $^{256}/_{243}$ (46s or 4c). This reflects the strong three-limit bias in this notation. Our principle would classify this semitone as a less desirable candidate than $^{16}/_{15}$. Intervals with particularly small ratio numbers act like powerful norms, so that our ears hear the more complex ratios as mistunings of the norms.

As another example, I would say that the consonant ratio $^{5}/_{3}$ is the natural candidate for pitch A; whereas, the dissonant ratio $^{27}/_{16}$ is naturally /A. It seems quite absurd to me that, in the Eitz and Tenney notations, A is $^{27}/_{16}$ and $^{5}/_{3}$ is thus /A. This flies in the face of common sense. The consonant ratios should not use comma-alteration signs. Even worse is the notation of the dissonant interval $^{81}/_{64}$ as E, while $^{5}/_{4}$ is \E. Surely the consonance $^{5}/_{4}$ is the appropriate E, and $^{81}/_{64}$ is /E. These are very clear examples of the use of the principle.

A more subtle example involves D ($^{9}/_{8}$ or 104s or 9c) and \D ($^{10}/_{9}$ or 93s or 8c). The former has the edge, but not by much. A more clear cut situation exists with its inversion. B♭ is ratio $^{9}/_{5}$ and \B♭ is ratio $^{16}/_{9}$. We can make the same argument for G♭ (ratio $^{36}/_{25}$) and \G♭ (ratio $^{64}/_{45}$). As a result of these applications of our principle, I have established the comma boundary by the broken lines on the matrix. Pitches to the left of this boundary on the tri-axial matrix are comma-lowered, and those to the right of the boundary are comma-raised. It is the application of our principle that has caused me to place the comma boundary where I did—not the notion of taking the "rows of major thirds as primary" (page 13), as Rapoport says, whatever that means. It is a natural outcome of the principle of optimum selection.

There is also another valid reason for placing the comma boundary where it is. The ratios 9:8 and 10:9 are the smallest number ratio expressions of the comma relation (81:80). The ratios 5:3 and 27:16, given above, also relate by a syntonic comma, but here larger ratio numbers are involved. In addition, (and this is the clincher), the ratios 9:8 and 10:9, as well as their inversions, sit in symmetrical array around the Tonic. We can prove this by plotting the pitches on the circular graph. The confluence of all these factors impels me to place the comma boundary where it is in the matrix. This boundary separates pitches that are natural, and pitches which are comma-altered. Note, on the tri-axial matrix, that there are two "phantom hexagons," which define the centers of the two regions of comma-alteration. Two more will appear further afield as the centers of the regions of skhisma-alteration.

Basically, my whole notation with all of its inherent boundaries can be logically generated by using the principle of optimum selection, with very few exceptions. One exception that should be noted is F♯ (ratio $^{45}/_{32}$) and \F♯ (ratio $^{25}/_{18}$). For various reasons too involved to relate here, it is expedient to prefer the higher numbered ratio as F♯. This shows that absolute consistency can sometimes be a drawback in such complex matters as Just Intonation notation. In fact, I think that Johnston goes wrong in a few places in his grid simply because he has applied his rubric in too rigorous a fashion, resulting in such oddities as $^{27}/_{25}$ for D♭. Nevertheless, his rubric (which I won't elaborate on here) is essentially correct and leads to mostly correct choices. It proves the old maxim that the exceptions prove the rule. On the other hand, the Eitz notation is absolutely consistent in its Pythagorean (three-limit) orientation, and suffers greatly for it. The Tenney notation is not much better.

In all of these notations, the boundaries are placed in a largely arbitrary fashion, the result of some pre-established set of rules, and do not reflect the actual, natural boundaries of the field. On the other hand, my notation uses these boundaries specifically to set up the most appropriate choices.

Well, I guess it is typical for me to blow my horn and assert that my notation is the best, but let me also offer some self-criticism. My notation has an extra-level of complexity, because the pitches that surround the skhisma boundary (the thick, solid lines on the tri-axial matrix) have two names instead of one. This is unfortunate, and I tried hard to eliminate this need. Nevertheless, I have come to the conclusion that it is instrumental and necessary for clarity. (Note, however, that these are the *only* pitches that need double names—until one considers temperament. Then watch out, it gets worse!). I will now do my best to justify these double names.

First of all, I need to justify the placement of the skhisma boundary itself. Believe me, the position is not arbitrary, but reflects underlying structural considerations. It sits in symmetrical fashion about the generator, just as the comma boundary does. You will see the appropriateness of its position more and more as we examine the boundary intervals.

Since we are already familiar with the C♯ and D♭ semitones, which are about three and five commas in size, let us examine the medial semitone of about four commas in size. Now there must be two candidates for this position, since the relation between C♯ and D♭ is the

[redacted]

$^9/_5$). The relation is 36:35, the septimal diesis. But using this approach, one would think that the $^7/_4$ ratio should be notated as \\B♭ or A♯, since the diesis is 2c in size, and the diesis is the difference between a sharp and a flat. I will justify this using the skhisma resolution: $^7/_4$ is 494s or 43c. $^9/_5$ is 519s or 45c. The difference is 25s or 2c—the septimal diesis. From this analysis it appears that A♯ is the most logical notation for $^7/_4$.

Blackwood derives $^7/_4$ from \B♭ (ratio $^{16}/_9$ or 508s or 44c). Now the relation here is the septimal comma (508 – 494 = 14s) of ratio 64:63. From this analysis, it again appears that the best notation for $^7/_4$ is A♯ or \\B♭. Yet both Blackwood and Johnston, as well as many others, resist using this notation for $^7/_4$. Maybe they think (wrongly) that because the number 7 appears in the ratio, it must be a form of seventh and not an augmented sixth. But the number 5 appears in the ratio $^5/_4$, yet it is not a form of fifth but a third. I cannot find any valid reason for resisting the use of A♯ to notate the $^7/_4$. In the same manner $^7/_5$ is F♯ (alteration of $^{45}/_{32}$), and $^7/_6$ is D♯ (alteration of $^{75}/_{64}$).

The approach that I take is practical in its orientation. Now it *is* possible to convert the two-dimensional five-limit tri-axial matrix into a seven-limit matrix by expanding it to three dimensions. We then have a tetrahedral grid (post Buckminster Fuller). But this is not very convenient or readable. Instead, it is better to stick to a two-dimensional grid, and substitute the seven-limit ratio for a given five-limit ratio. Essentially, this is what Blackwood and Johnston have done. The problem is that they (rather arbitrarily) started from a form of minor seventh as the place of substitution. It makes far more sense to use the A♯ as the point of substitution, since the alteration is much smaller. Specifically, the five-limit A♯ (ratio $^{225}/_{128}$ or 498s or 43c) becomes the $^7/_4$ A♯ by diminishing it by ratio 225:224—the septimal clisma of 4s. This is a much more natural and close relation than the B♭ or even \B♭. It means that the grid itself can express close septimal alternatives, simply by the use of the 4s alteration. As another example, the $^{16}/_{15}$ D♭ becomes the $^{15}/_{14}$ septimal D♭ by augmenting it 4s (ratio 225:224). This tiny microtone crops up in many contexts in the matrix, but we will not pursue it further here. Suffice it to say that the septimal clisma is the key to placing septimal substitutions into the just fabric.

The integration of ratios of 7 into the fabric thus seems fairly straightforward, but there are problems ahead. For we will need a new rubric for ratios of 7^2, then 7^3 and so on. I have solved these problems, although the solutions are beyond this paper.

I wish I could say the same for ratios of 11, 13, 17 and other high primes. For various reasons they are quite difficult to integrate into the grid. For one thing, each new layer adds a very high degree of additional complexity to the field. One ends up with what I call saturation; that is, many tones are generated that are only minutely different in pitch, all vying to occupy the same position on the grid. For example, we have the ratio $^{11}/_8$ (281s or 24+c) being close to $^{15}/_{11}$ (274s or 24c), the difference being 7s. The closest five-limit notation is G♭♭. But these ratios, and their brothers $^{11}/_9$, $^{40}/_{33}$, $^{11}/_6$, $^{20}/_{11}$, $^{22}/_{15}$, $^{16}/_{11}$, $^{11}/_{10}$, $^{12}/_{11}$, $^{33}/_{20}$, $^{18}/_{11}$, and so on, all occupy a region of the matrix which I call "the neutral zone" (after Star Trek?). These ratios are neither perfect nor augmented, neither major nor minor—they are neutral. For example, $^{11}/_9$ becomes E♭ meaning semi-flat). Additional higher primes also need new signs and thus add more notational complexities.

It is possible to substitute eleven-limit (Partchian) ratios into the five-limit fabric, by using substitutions like we did for the seven-limit. Unfortunately, these substitutions must occur in a very remote region of the matrix—the region I have called the harmonic antipodes or the neutral zone. In the tri-axial matrix diagram provided I have avoided giving the position of this region, since I did not want to magnify complexities. For most Just Intonation tuning purposes, the neutral zone is not encountered. To manifest it we must expand the grid vertically a little bit. Only a little edge of the neutral zone is actually present on the grid, marked by comma numbers 7 and 46. Comma number 7 is typical of these intervals. It is about midway between a semitone (say 5c) and a wholetone (say 9c). It could be called a three-quarter tone. Such intervals are common in Arabic music, but are foreign to Western practice. All of the important eleven-limit ratios have this quality of neutrality, making them quite exotic and strange to our ears.

As has been pointed out by Johnston and others, it is possible to lay out octave-invariant matrices for other primes besides 3× 5, for example 11 × 13, and so on. The results are somewhat strange and esoteric, since our ears want to hear simple ratios and consonances, and not continual complexities. Thus the 3 × 5 matrix has much more relevance to most music making. For this reason, it is best to start with this matrix as a norm or paradigm, and then add exotic elements where desired.

One more point about notation and the Rapoport article. In Table 1 (page 14) he presents a number of ratios which are generally called commas. I do not feel that it is wise to generalize the term "comma" too widely. Such a ratio as 50:39 is better described as a variety of double comma or diesis, since it is 22s. Again, a ratio like 26:25 is close to a chromatic semitone (25:24); consequently, it is wise to notate it as such. This helps avoid confusion. Of course, then you have a ratio like the eleven-limit 33:32. It is quite wide of a diesis, yet quite narrow of a chromatic semitone. In fact, it is somewhere around midway between the two. Such ratios are much more difficult to handle with sensitivity. One (vague) approach which has been used is D♭+ or C♯–. (The plus and minus signs have also been candidates for skhisma alteration). It makes sense to notate an exotic interval as some alteration of an interval more familiar to us. The comma resolution offers practical and powerful norms for handling exotica.

All of this goes to show that the appropriate notation must depend on some practical context. Otherwise, the grid just becomes an abstract and rather esoteric form of artwork, without relevance to musical understanding. In my work, I have sought to eliminate needless esoteric concepts, and to retain only what is useful for structural understanding. The concept of the neutral zone, or even the comma boundary, may seem strange at first, but these terms denote realities which I have encountered in actual tunings. They are ways of describing actual harmonic entities, not artificial or academic constructs having nothing to do with actual tuning work. Indeed, I evolved this grid during the '70s while trying to find ways of simplifying tuning procedures. By using the grid, I would less likely get lost in long strings of harmonic relations. Thus the grid grew out of practical concerns.

Finally, let me make one more point about the need for two signs (comma and skhisma). Although this is *the* major theoretical drawback to my system, in actual practice, it is not much of a problem. Most of the just tunings which I and most others use in the real world do not have skhisma-related pitches juxtaposed at the same time. Thus one can use a rubric for specifying which alternative is being used, and dispense with the need for the sign on the staff. In actuality, only the comma-alteration sign need be used in practically all cases. I have omitted vertical expansion because more complexities relating to the clisma ensue. Very few use extreme vertical scalings. Most scales that even go beyond the chromatic to twenty or so pitches per octave (allowing decent modulation in Just Intonation) tend to expand in a generally horizontal direction and do not need more than the comma alteration sign. Thus the occasional need for an s-alteration sign is not such a big problem. Because the comma is the only new sign needed most of the time, the comma resolution is eminently relevant, and should not automatically be tied to the 53-tone system of temperament. For most practical purposes, it *is* possible to use the same notation for 53-tone equal temperament and five-limit Just Intonation, even though the 53 temperament eliminates the skhisma altogether by using a mean comma about midway in size between the syntonic and ditonic variety. Of course, some other systems of temperament are special cases needing unique notation signs that do not concern us here.

My main point is that there does not necessarily need to be a dichotomy between Just Intonation and various systems of temperament that approximate it well. The same notation can be used for both. I have tried to show that methods of interval measurement which may be associated with various temperaments (i.e., the resolution of the octave into semitones, dieses, commas, clismas, skhismas, etc.) are also useful for handling just ratios. These methods are practical in their inception and practical in their applications. They lead to a notation that mirrors the actual regionalism of the matrix. The explicit clarification of this regionalism is my main contribution to the matrix model. The other three models do not appear to value these concepts which are implicit in the very notion of a matrix of relations.

In order to keep this article simple and short, I have focused only on the comma and skhisma resolutions, and not on the diesis or clisma resolutions, although I personally am comfortable with all of them. Each has its own perspective on the measurement of harmony.

Even though there are a number of natural resolutions for the octave matrix, in actual fact only the comma and skhisma resolutions are necessary in most contexts. Hence only the comma and skhisma resolutions are necessary for generating the notation and the correct boundary positions for the principal regions of the matrix. The comma and the skhisma are the most important, indeed the key, micro-intervals which shape the architecture of the five-limit grid. **1/1**

A Response to Siemen Terpstra's "Notation"

by David B. Doty

My primary concern here is not with Mr. Terpstra's notation as such, although I will make some remarks on that topic; rather, I am concerned mainly with the view of extended Just Intonation embodied in his lattice, a view which I think is misleading and overly restrictive.

For the record, I use Ben Johnston's notation. I'm not particularly interested in debating whether it's the "best" or not—any notation for extended Just Intonation that retains the five-line staff and the sharps and flats, with their traditional meanings, is bound to have some problems. Perhaps, by discarding the staff, the letter names, and the sharps and flats, and starting from scratch, one might invent a more logically consistent notation for extended Just Intonation, but I, for one, have no interest in doing so. Notation is far down on my list of priorities.

Not so the representation of extended Just Intonation by means of lattices. As those of you who have read my *Primer* know, I consider this device extremely valuable, if not essential, to the understanding of extended Just Intonation. However, I find Terpstra's "tri-axial matrix" unnecessarily complex and potentially misleading. Terpstra uses this lattice because it gives "equal weight" to the major third/minor sixth and the minor third/major sixth. That is exactly the problem: these two interval classes do not deserve equal weight. The major third, 5:4, is a primary interval; that is, it is the relation between a prime number and the fundamental. It is not generated by anything else, and, combined with perfect fifth/fourth, it generates all of the other intervals of the 3 × 5 fabric, including the minor third, 6:5. The minor third, on the other hand, is a secondary interval. It is simply the defect of a major third from a perfect fifth. There is no need for a separate "tuning axis" for 6:5; tune a fabric of 5:4s and 3:2s, and you will get the 6:5s whether you want them or not. The tri-axial matrix also unnecessarily complicates the representation of intervals by their lattice coordinates. On the square lattice, only two coordinates are necessary to represent any interval, and each interval is represented by only one unique coordinate pair. With the tri-axial matrix, each interval is represented by two triads of coordinates. As for the matter of finding triads on a square, two dimensional lattice, I have never found this a problem. On such a lattice, in the orientation I use, a major triad is an L, and a minor triad is an upside-down, backward L. Where's the problem?

Another matter for concern and confusion is Mr. Terpstra's concept of "boundaries" and "regions" delineated by commas, skhismas, and other microtonal intervals. Of course, it is essential to know the whereabouts on the lattice of these intervals and to understand their implications, but I cannot see in what sense they constitute "boundaries." Mr. Terpstra's lattice with its boundaries and regions suggests that tonal space has a static center (C) from which these regions and boundaries radiate outward. But the lattice has no center, except in the sense that any tone that is perceived as a tonic temporarily becomes a center. In reality, the lattice is of unlimited extent and its center is everywhere (or nowhere). *Every* tone on the lattice is bracketed symmetrically by pairs of tones that differ from it by commas, skhismas, and the like. There is no reason why a musical line or a harmonic progression should not move just as easily across these boundaries as within them. The concept of boundaries is a consequence of thinking about temperament—the boundaries represent lines along which one might fold or warp the lattice so that it forms a closed figure, i.e., they are the intervals that get "fudged" in forming a temperament.

My most serious disagreement with Terpstra is over representation of the intervals generated by seven. He gives a strong hint about his attitude toward seven very early in his article, when he identifies the five-limit intervals forming a hexagon around "C" on his tri-axial matrix as "the only musical consonances." Evidently, he does not regard 7:4, 7:5, and 7:6 as consonances, an attitude that I find baffling. According to the definition that I use, which is closely related to Arthur Benade's concept of "special relationships,"[1] 7:4, 7:5, and 7:6 are unequivocally consonant; indeed, 7:4 is a much stronger consonance than the five-limit minor sixth, 8:5. Terpstra's rejection of the consonance of seven-limit intervals may explain the rather cavalier treatment he gives them in assigning names and lattice positions. Terpstra condemns those, such as Johnston and Blackwood, who think of 7:4 "as some kind of seventh" rather than "an augmented sixth." 7:4 is not merely "some kind of seventh," it is *the* seventh, the primary interval for the prime number seven, the seven identity of a tonality. Like 5:4, it is not *derived* from anything (except $1/1$). In combination with the primes three and five, it generates a host of significant intervals that collectively constitute some of the most important resources of extended Just Intonation. Nevertheless, Terpstra gives the tone $7/4$ the obscure designation A♯ and considers it a variant of the remote pitch $225/128$. This representation disguises $7/4$'s role as seven identity of $1/1$ (that is, as the seventh of a dominant-seventh chord on C). For this to be recognized, $7/4$ must be labeled as some kind of B♭, *not* as A♯. It is for this reason that the approaches used by Johnston and Blackwood are preferable, not because $7/4$ is *derived* from either $9/5$ or $16/9$. To be sure, 7:4 is closer in pitch to $225/128$ than to either $9/5$ or $16/9$, but I don't see why this fact should be significant, except for the purpose of mapping the seven-limit intervals onto the 3×5 lattice, a practice that is best avoided in any case.

Despite his claims to the contrary, I think Mr. Terpstra's lattice and notation are more a product of his studies of temperament and of the history of Western music than of just intervals. As such, it is more likely to conceal than to reveal some of these essential properties, especially when applied to seven and higher primes.

Note:

1. Benade, Arthur H. *The Fundamentals of Musical Acoustics*. Dover, 1990. pp. 274–276 **1/1**

July 15/93

Greetings:

I got a phone-call from Mark Rankin yesterday. I had sent him a zerox of the paper MORE NOTES ON NOTATION to look over. He kindly pointed out to me that there was a source of confusion in the matrix diagram. So I have slightly amended the diagram to eliminate it. Please use this amended version rather than the one sent earlier.

The change concerns the pitches which straddle the schisma boundary. I will again use the example of the 4 comma pitch which has two names. On the right side, inside the boundary, we see the two pitch names /C# and \Db, which define the pitch which is about 92 Cents in size (47s). Then on the left side of the matrix, we have the note which is about 90 Cents in size (46s). This one is notated /C# and \Db. The bar, which indicates schisma alteration, should be under both names. In the previous version of the diagram, I only put the bar under the lower name, in order to save space in a rather cluttered diagram. Unfortunately, this created some confusion since Mark thought that the alternative name does not have a bar. In fact, any pitch name which lies beyond the schisma boundary has a schisma alteration sign. I assumed that this would be clear, but apparently this is not the case. So, even though the diagram is now a little more cluttery, it is now made explicit.

He also found a few other 'bloopers,' spelling errors, etc. I was not surprised about this, since this paper was written in a hurry (in about a week) due to various deadlines. Happily, Mark assured me that the rest of the paper was quite clear. One other problem is the length--it is probably overly long. However, I could not find a way of making it shorter and still provide clear explanations of the specifics of my notation. I hope that it is suitable for 1/1; if not, well, it was a try!

Yours respectfully...

Siem

Siemen Terpstra