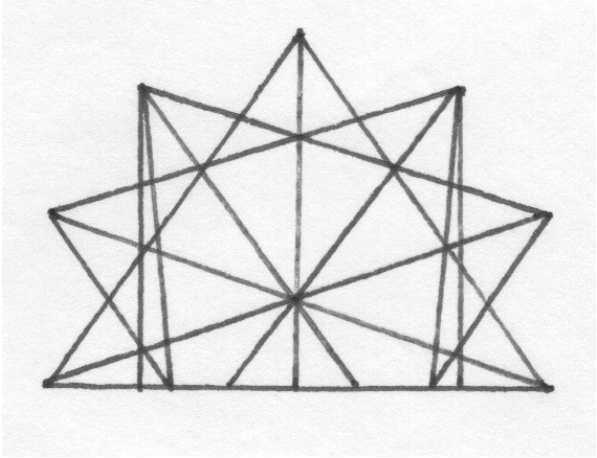
The Meaning of the Tetractys

Musical Symbolism in Pythagorean Arithmology

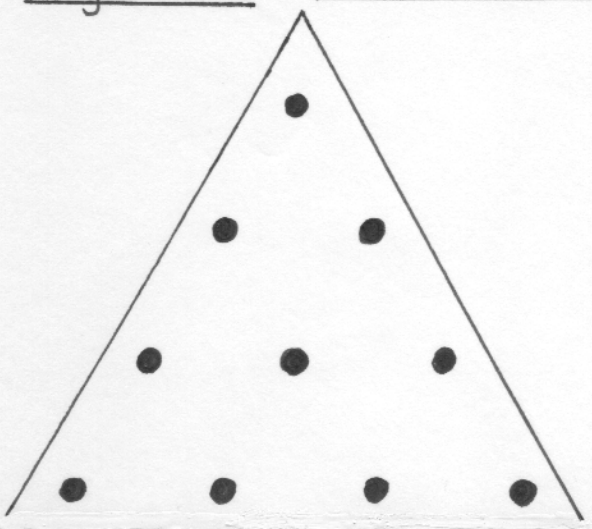
diagrams



Siemen Terpstra

www.siementerpstra.com

Figure 1: The Tetractys:



Side	Sum	Association
1	1	Point (no dimension)
+		
2	3	Line (one dimension)
+		
3	6	Surface (two dimensions)
+		
4	10	Solid (three dimensions)
=10		

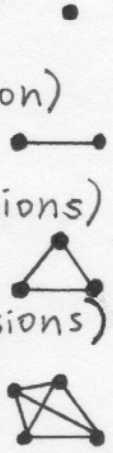
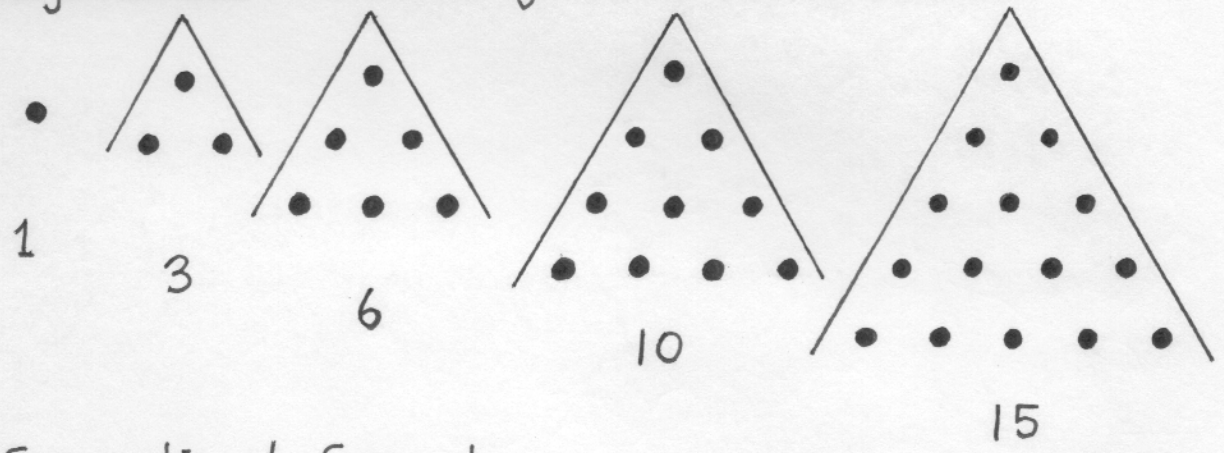


Figure 2: The Triangular Number Series:



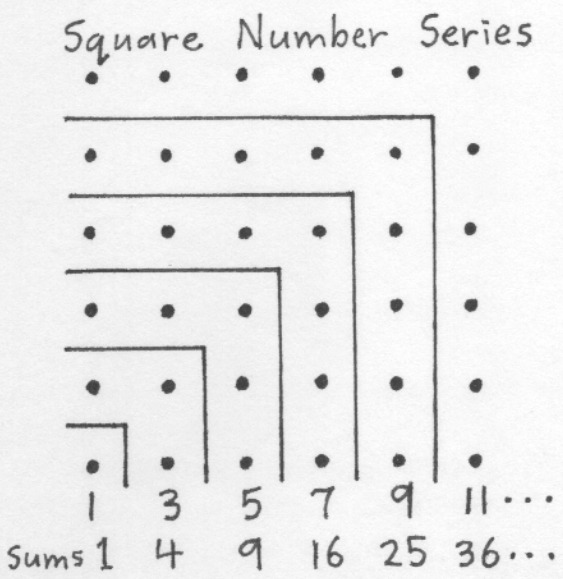
Generalized formula:

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

Segment of the series:

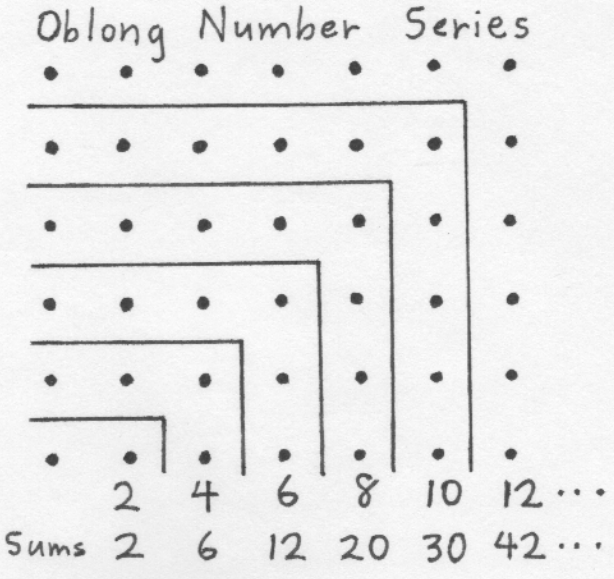
1, 3, 6, 10, 15, 21, 28, 36, 45, 55...

Figure 3: Square and Oblong Number Series:



Generalized Formula:

$$1+3+5+\cdots+(2n-1)=n^2$$

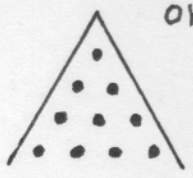


Generalized Formula:

$$2+4+6+\cdots+2n=n(n+1)$$

Figure 4: The Tetractys and Mt. Meru (Sumeru):

Mt. Meru (Sumeru)
The Holy Mountain
"Meru Prastāra"



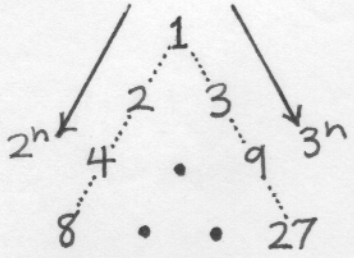
$$2^0 = 1$$
$$2^1 = 1 \quad 1$$
$$2^2 = 1 \quad 2 \quad 1$$
$$2^3 = 1 \quad 3 \quad 3 \quad 1$$
$$2^4 = 1 \quad 4 \quad 6 \quad 4 \quad 1$$
$$2^5 = 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$

Generalized expression called Binomial Theorem or Pascal's Triangle:

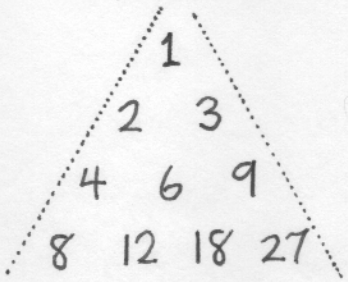
$$1 = (a+b)^0$$
$$a+b = (a+b)^1$$
$$a^2+2ab+b^2 = (a+b)^2$$
$$a^3+3a^2b+3ab^2+b^3 = (a+b)^3$$
$$a^4+4a^3b+6a^2b^2+4ab^3+b^4 = (a+b)^4$$
$$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5 = (a+b)^5$$

Figure 5: Plato's **Timaean Lambda**:

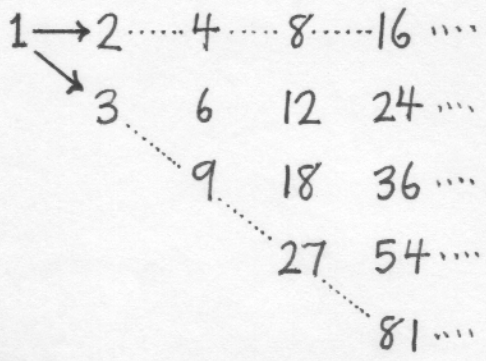
Plato's version



Crantor

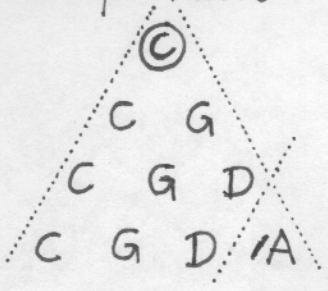


Nicomachus

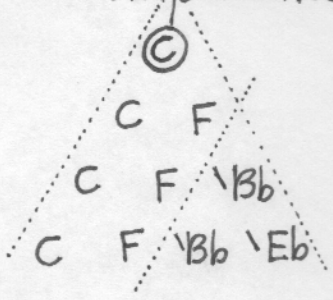


Tonal implications of the Lambda:

Harmonic Series Componants



Monochord Series Componants



Prime Factors

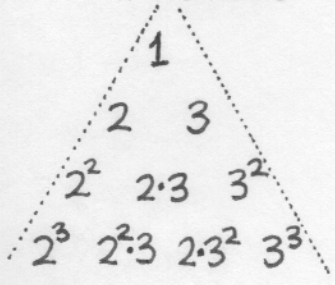
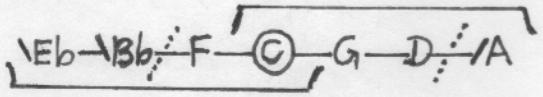


Figure 6: 3-Limit Numbers as a line of fifths:

In pitch



In ratios

$\frac{16}{27}$ $\frac{8}{9}$ $\frac{2}{3}$ ① $\frac{3}{2}$ $\frac{9}{8}$ $\frac{27}{16}$

Monochord sequence derived from Lambda numbers:

Series – 16 : 18 : 24 : 27 : 32

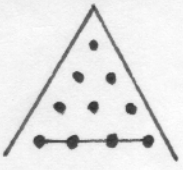
Doublings –

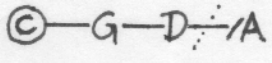
8	9	12		16
4		6		8
2		3		4
1				2
				1

Over Series (Harmonics) – C : D : G : A : C

Under Series (Monochord) – C : Bb : F : Eb : C

Double 16 : 32





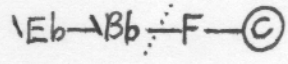


Figure 7: The Tetractys becomes the 5-Limit Matrix:

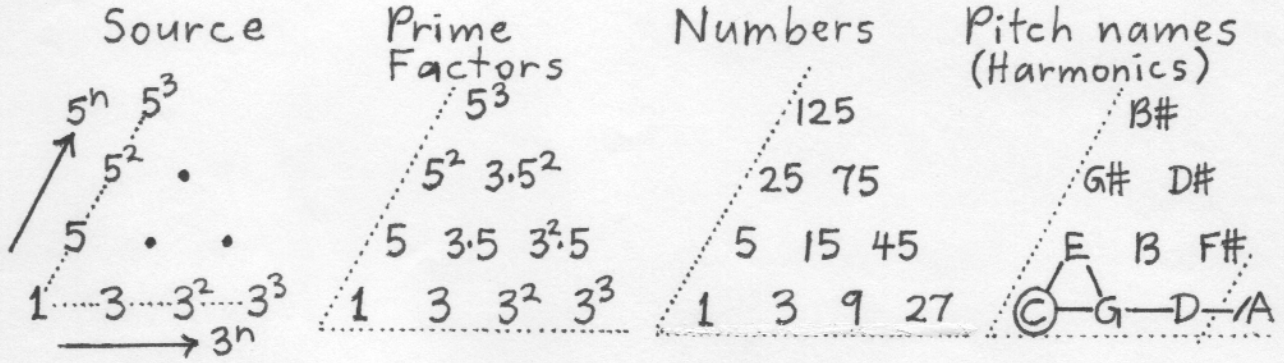
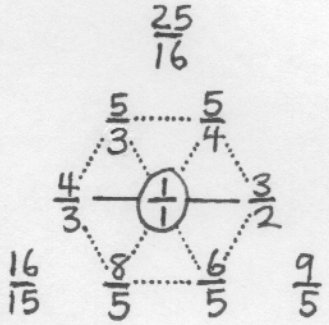


Figure 8: The Generator in the central position:

Expressed in Ratios between 1/1 and 2/1

Expressed in Pitch notation (a's Harmonic Series componants)



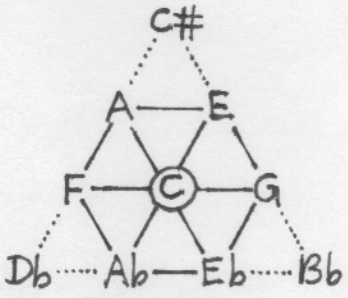


Figure 9: A Simple 5-Limit Monochord Series:

4:8

4 : 5 : 6 : 8

2 3 4

1 2

1

Over Series (Harmonics) – C : E : G : C

Under Series (Monochord) – C : Ab : F : C

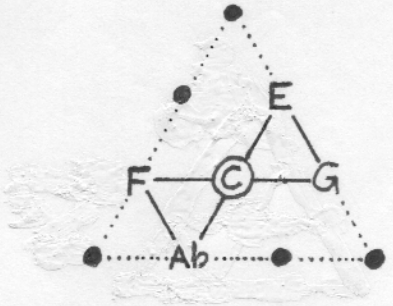
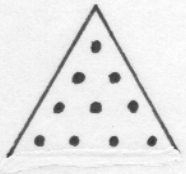


Figure 10 : The Three Roots of Sacred Geomety Generated from **One** as the Square :



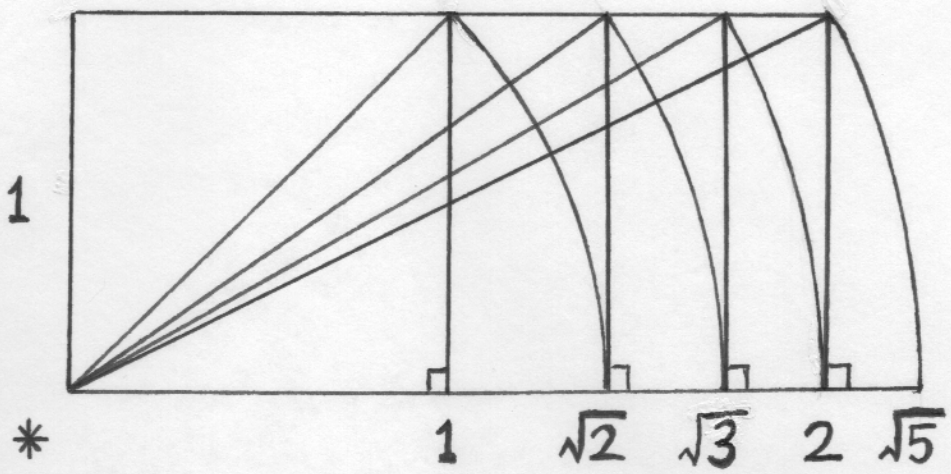


Figure 11 : The Musical Proportion :

6 : 8 :: 9 : 12

Over – C : F :: G : C

Under – C : G :: F : C

6:12

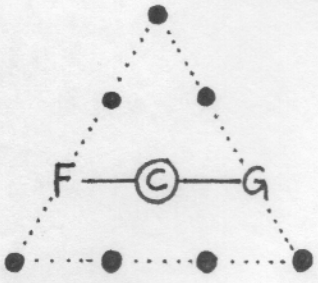
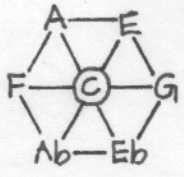


Figure 12 : The Musical Consonances :

60 : 72 : 75 : 80 :: 90 : 96 : 100 : 120

0 – C : Eb : E : F : G : Ab : A : C

U – C : A : Ab : G : F : E : Eb : C



60:120

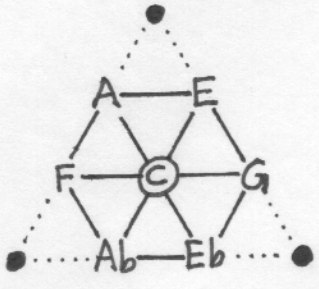
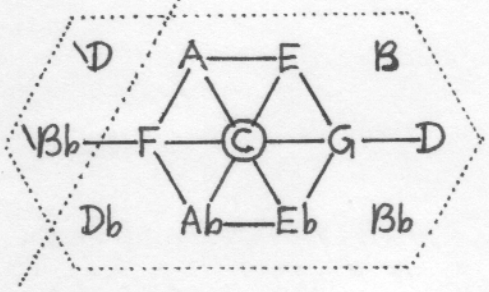


Figure 13: Two Symmetrical Cosmological Monochords:

360 : 720



4320 : 8640

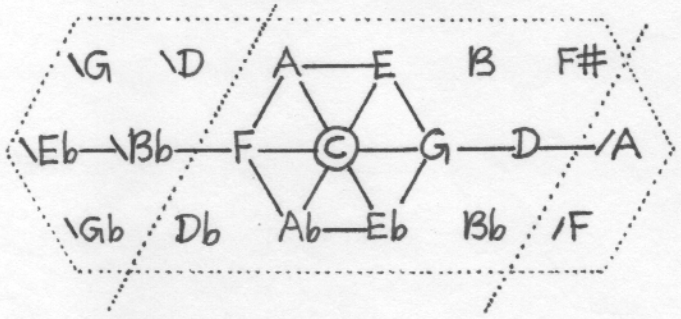
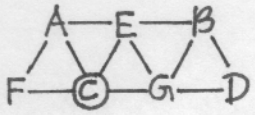


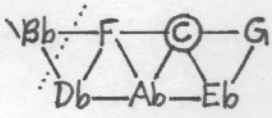
Figure 14: A Significant Non-symmetrical Monochord:

24 : 27 : 30 : 32 : 36 : 40 : 45 : 48

0 — C : D : E : F : G : A : B : C

U — C : \Bb : Ab : G : F : Eb : Db : C





24 : 48

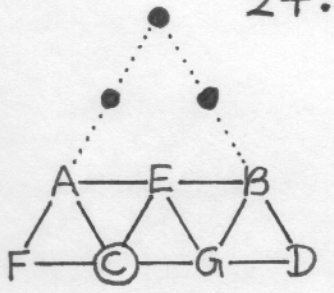
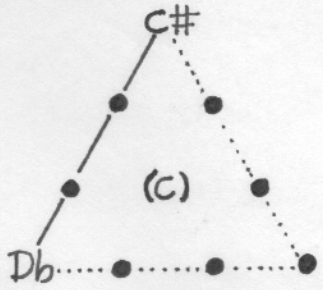
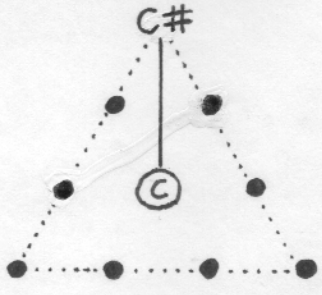


Figure 15: Some Important Musical Intervals:

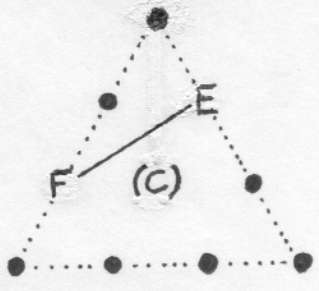
The Diesis (Enharmonic)



Chromatic Semitone



Diatonic Semitone



Interval Size in three levels of Resolution

31-E.T.	1 Diesis	2 Dieses	3 Dieses
53-E.T.	2 Commas	3 Commas	5 Commas
612-E.T.	21 Schismas	36 Schismas	57 Schismas

Figure 24: The "World Soul" Monochord Construction:

Inserting the Harmonic and Arithmetic Means: (×6)

```
   6    ::    12    ::    24    ::    48
      8 : 9:    :16 : 18:    :32 : 36:
O - C : F : G : C : F : G : C : F : G : C
U - C : G : F : C : G : F : C : G : F : C
```

F—Ⓒ—G

Circle of the Same
The Unchanging

```
   6    ::    18    ::    54    ::    162
     : 9 : 12:    : 27 : 36:    : 81 : 108:
O - C : G : C : G : D : G : D : /A : D : /A
U - C : F : C : F : \Bb : F : \Bb : \Eb : \Bb : \Eb
```

Ⓒ—G—D—/A

Circle of the Different
The Changeable

Conversion into a Monochord sequence (The Circle of Essence or Existence) in order to inspect the scale for the presence of musical fourths (ratio 4/3) (×16)

```
    96 : 108 : 128 : 144 : 162 : 192
O - C : D : F : G : /A : C
U - C : \Bb : G : F : \Eb : C
```

F—Ⓒ—G—D—/A

96:192

Filling up the musical fourths with wholetones (9/8), using the 3-Limit Tetrachord 192:216:243:256, and resulting in the sacred 3-Limit Ogdoad (×16)

```
    1536 : 1728 : 1944 : 2048 : 2187 : 2304 : 2592 : 2916 : 3072
O - C : D : /E : F : /F̄# : G : /A : /B̄ : C
U - C : \Bb : \Ab : G : F# : F : \Eb : \Db : C
```

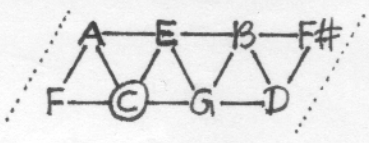
1536:3072

F—Ⓒ—G—D—/A—/E—/B̄—/F̄#

"Close cousin," the sacred 5-Limit Just Ogdoad suggested by the above construction:

```
    96 : 108 : 120 : 128 : 135 : 144 : 160 : 180 : 192
O - C : D : E : F : F# : G : A : B : C
U - C : \Bb : Ab : G : \Gb : F : Eb : Db : C
```

96:192



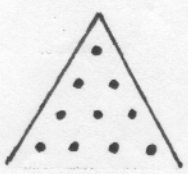
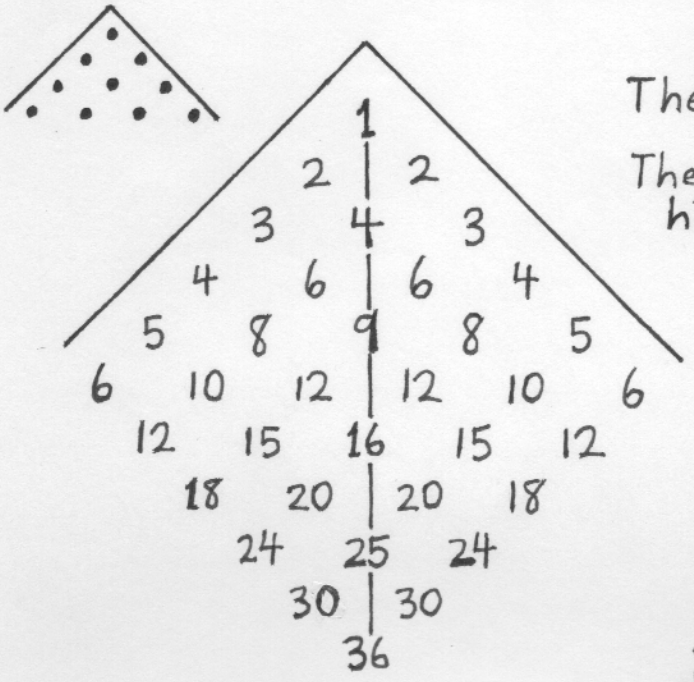


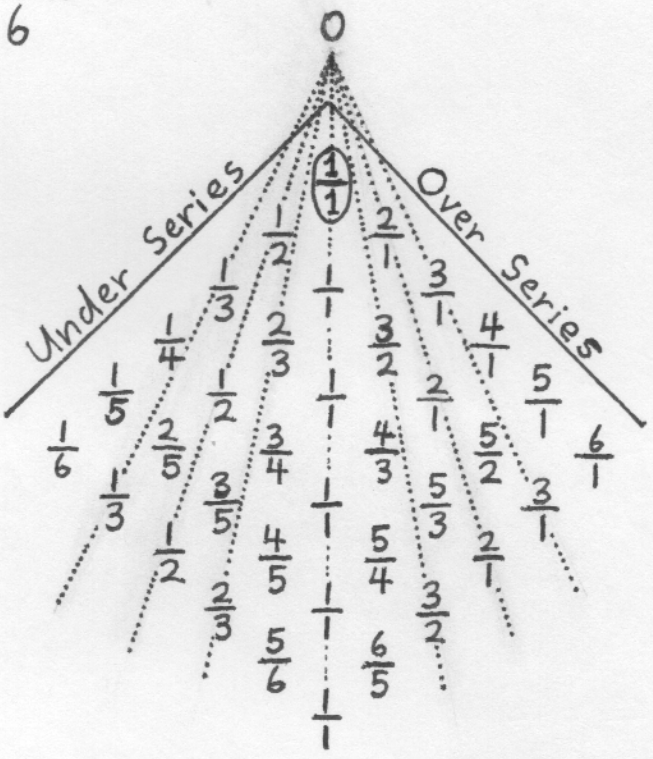
Figure 25: The Lambdoma (Multiplication Table) derived from the Tetractys:

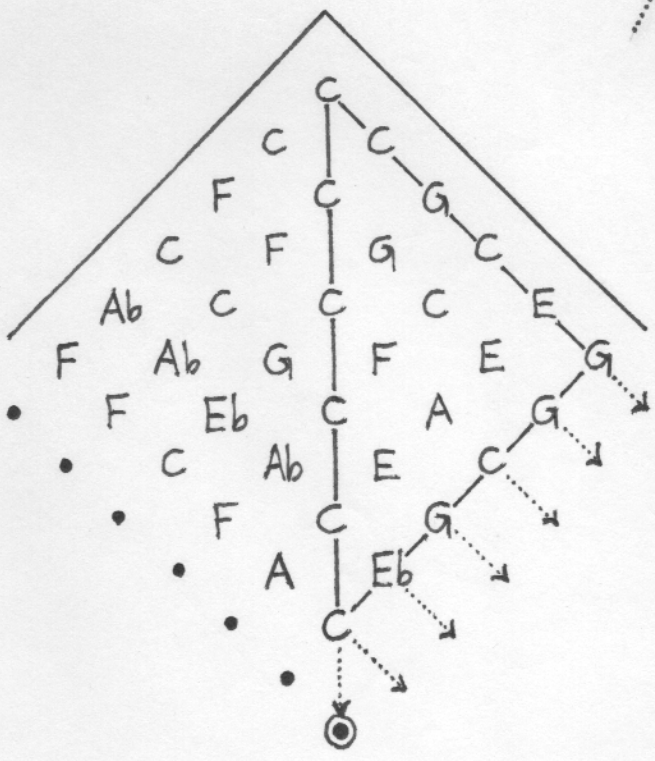


The Lambdoma with Identity Rays converging on the 0/0 or Silence (Nun)

The Multiplication Table 6x6

The Square Numbers are highlighted in the vertical.





The Lambdoma as a Pitch Array, emphasizing the Harmonic Series Triangle

Figure 26: An Example of the Geometrical Proportion on the Monochord, giving Irrational Ratios:

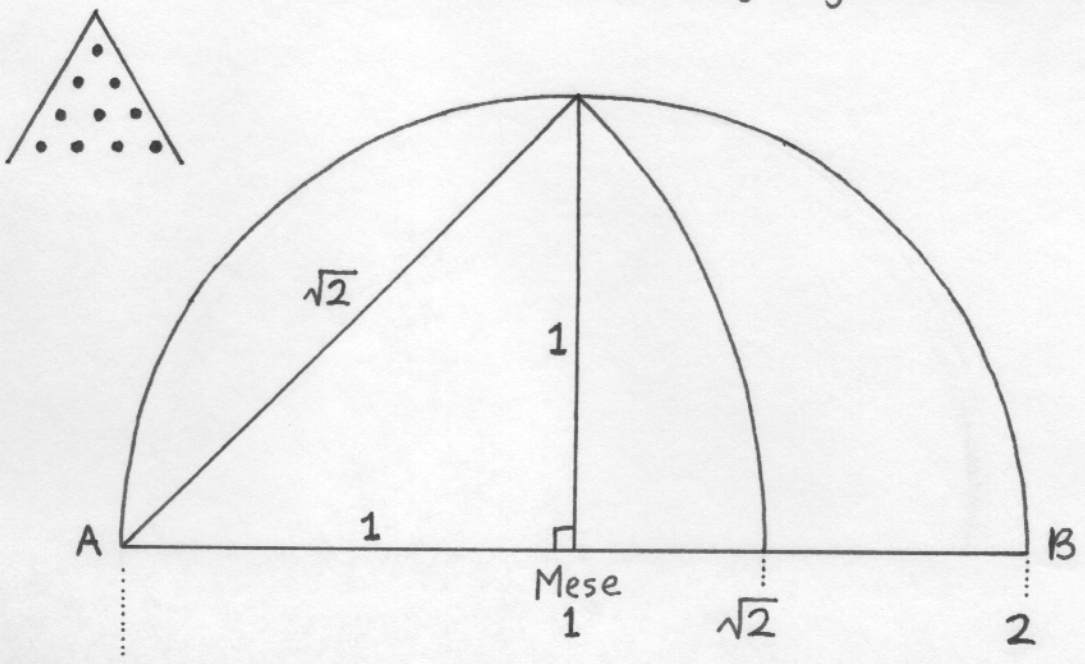
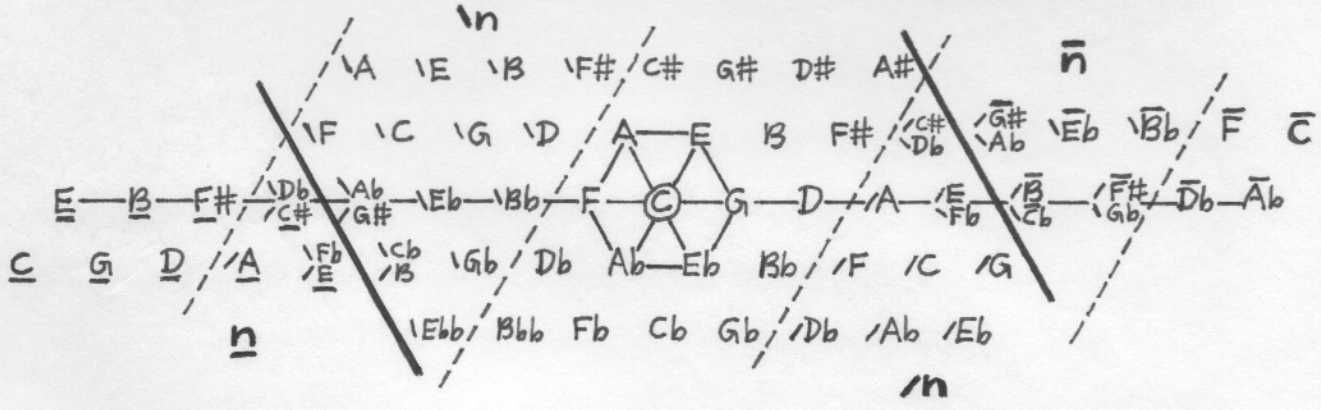


Figure 27: A Reference Guide to the Notation of 3-Limit and 5-Limit Just Intonation:



Boundary markers within the Harmony Matrix:

——— 3-Limit line of fifths plus the Consonances

- - - - - - the regions of Comma alteration (\n, /n)

▬▬▬ the regions of Schisma alteration (n, n̄)